A Simple Option Pricing Model with Heterogeneous Agents

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Abstract

The traditional valuation formulae for options are usually based on the non-arbitrage principle in markets with complete asset structures. In this paper, these assumptions are dropped. Only shares of a stock and European call options written on the stock are available on the market. Continuously trading is impossible. If, in such a case, the construction of a riskless hedge-portfolio is non-feasible, the pricing of options and underlying assets becomes a simultaneous valuation problem. This paper uses a discrete model of an economy with heterogeneous agents in order to derive the relationship between prices of stocks and options. It investigates the dynamics of the model which are driven by the agents who differ in their attitude towards risk.

By means of numerical analysis, it can be found that - in contrast to the results of the Black-Scholes-Merton theory - individual preferences have a major impact on the prices of options.

Key words: Incomplete markets, Option pricing, Heterogeneous agents
JEL classification: D52, G11

1 Introduction

The traditional theory of pricing European-style options, first introduced by Black/Scholes (1973) and extended by Merton (1973), uses an arbitragefree hedging argument. The valuation of options takes place under the assumption that the underlying asset price follows a given exogenous process. Derivative assets like options are viewed as redundant securities which payoffs can be replicated by portfolios of primary assets. Thus, the market is assumed to be complete without the option. In this approach, preferences of the investors are irrelevant. The only requirement on the preferences is that they are monotonic, such that the possibility of a riskless profit is immediately taken.

In this article we investigate the case of an option as a non-redundant asset. The payoff of the option is unique and cannot be reproduced by a hedge portfolio of a combination of primary assets. A simple economy is considered, where only shares of a stock and an option written on the stock are available in the financial market. This model is related to the one assumed by Drees/ Eckwert (1995) and is originally based on the paper of Lucas (1978). The economy is assumed to last only one period[1] and is viewed at two dates, at the beginning and at the end of the period.

That preferences have an effect on the pricing of derivative assets in a general equilibrium context has also been found by Rubinstein (1976), Brennan (1979) and Bailey/ Schutz (1989). These papers describe conditions under which valuation relationships between the underlying asset and the option do not include parameters of the preference structure of the investors.

Franke/ Stapleton /Subrahmanyam (1999) find that option prices can differ from the value of the Black-Scholes model, if the elasticity of the pricing kernel is declining. This implies that the forward price process is no longer a Brownian motion as assumed by Black-Scholes. The shape of the pricing kernel is determined by the preferences of the agents in a complete market setting. They assume the existence of a representative agent to derive this result. Our paper, in contrast, focuses on the incompleteness of the markets.

There are many reasons to concede incomplete markets. Examples are stochastic volatility, transaction costs, short sale restrictions, credit risks or the absence of securities which have payoffs in all possible states of nature in the future, so that complete risk sharing between agents is impossible. We focus on this specific point.

Most approaches of asset pricing assume homogeneous agents or the existence of a representative agent. But the occurrence of trading requires the existence of heterogeneity. Franke/ Stapleton /Subrahmanyam (1998), for example, take into consideration the existence of background risks

[1] Cox/ Ross/ Rubinstein (1979) have interpreted the Black-Scholes approach in a discrete time framework. This is more related to our investigations.

which differ across agents. They find that agents with low or no background risk sell options, whereas agents with high background risk buy options. LELAND(1980) shows that investors who have average expectations, but whose risk tolerance increases with wealth more rapidly than average, buy options, and investors who have average risk tolerance, but whose expectations of returns are more optimistic than average, buy options too. DETEMPLE/SELDEN(1991) have shown that in an incomplete market setting primary and derivative assets interact. This interaction increases when heterogeneous agents are introduced. They assume agents who have mean-variance utility and who differ in their expectations with respect to the future variance of the market. Agents who assess high future volatility buy options, whereas those who assess low volatility sell options.

In this paper we show that differences in the investors´ preferences have an impact on the relationship between the asset prices and the amount of trading in the market. We do so by first assuming homogeneous agents. It can be found that the options are not traded, but that there exists an equilibrium price of the option. Because of the homogeneous structure of the agents, every deviation from the equilibrium price induces an immediate demand or supply only on one side of the market. Thus, only one equilibrium price of the option can exist for each kind of homogeneous agent. Variation of the preferences of the homogeneous agents leads to a change in the price relation of the share and the option.

Finally, we introduce agents which differ in their preferences. They have different degrees of risk aversion. Under this assumption trading takes place. The agents with a higher degree of risk aversion sell options and buy the safer asset, that is i.e. the share. The agents with a lower degree of risk aversion buy those options and sell stocks.

We find out that the amount of trading and the price of the option grows with the increasing difference in the risk aversion. We use the Arrow-Pratt coefficient of relative risk aversion as an indicator for the varying attitudes towards risk..

The paper is organized as follows: In section 2, the theoretical model is introduced and the equilibrium asset prices are derived when all agents are homogeneous. Price relations are simulated given the assumption of isoelastic preferences with alternative constant relative risk aversion. In section 3, heterogeneous agents are introduced. The influence of heterogeneity on the asset prices and on the amount of shares and options traded in the market is shown. In section 4, the main conclusions of the paper are summarized.

2 The model

We consider a model with an exchange economy with only one single perishable good. The economy is populated by N agents. In the first step they are assumed to be homogeneous with respect to their risk aversion. Later in this paper this assumption is relaxed.

The economy is viewed in one period at two dates, the beginning $t = 0$ and the end of the period $t = 1$. The state of nature ω at the second date is uncertain. The number of possible states are limited and can be described by $\omega = 1, ..., \Omega$. We assume that there are more then two states of nature, $\Omega > 2$. There exists a objective probability distribution F^I under an information set I at the beginning of the period, where π_ω^I is the probability of the occurrence of state ω under the information set I. The knowledge of the objective probability distribution F^I is common to all agents. For simplicity we drop the index I for the rest of the paper.

The good is produced by a representative firm. The output of this firm $Y(\omega)$ varies at the end of the period according to the state of nature w. For simplicity the commodity is taken as a numeraire.

The financial market is composed of two divisible assets. This means that the market is incomplete by the assumption $\Omega > 2$. On the one hand we have a primary asset which is the stock of the representative firm. The stock gives the owner a claim of a fraction $y(\omega)$ of the representative firms profits $Y(\omega)$. Thus, the payoff $y(\omega) = (y(1), ..., y(\Omega))$ is a vector in $\mathbb{R}^{\Omega}_{++}$. The stocks are in positive net supply. Each agent is endowed with one share at the beginning of the period.

On the other hand, we have a second asset, a call option written on the stock, which is in zero net supply. The option is a security, that gives the holder the right, but not the obligation to purchase a share of the underlying stock at a previous fixed strike price k on the maturity date of the option. In our case the maturity date is the end of the period ($t = 1$) and the payoff of the option is:

$$g(\omega) = max(y(\omega) - k; 0) \tag{1}$$

Thus $g(\omega) = (g(1), ..., g(\Omega))$ is a vector in the positive orthant of $\mathbb{R}^{\omega}$. The financial market opens at the beginning of the period, when the state of nature has not yet been revealed. The agents are able to take long or short positions in the option and the stock. Thus, each agent n possesses at $t = 1$ a portfolio x^n composed of the amount shares $x^{n,s}$ and options $x^{n,o}$ he has obtained at $t = 0$.

At the end of the period at date 1 the payoffs of all assets are consumed. The prices of the stocks and the options at $t = 0$ are $p = (p^s, p^o)$. The agent has under the available information I a density function $F(p^1(\omega); I)$ about the distribution of the asset prices $p^1(\omega)$ at date $t = 1$ which equals the payoffs $y(\omega)$ and $g(\omega)$.

We assume that the agents in the economy have a utility function depending on the consumption of the only available good. Consumption takes place only once per period, so that the end of period payoff of agent´s portfolio $x^n = (x^{s,n}, x^{o,n})$ is his consumption c^n.

$$c^n(\omega) = y(\omega)x^{s,n} + g(\omega)x^{o,n} \tag{2}$$

Because of the net zero supply of the options, we require for an equilibrium:

$$\sum_{n=1}^{N} c^n(\omega) = Y(\omega) \tag{3}$$

In the following the index n will be dropped for simplicity. In each state of nature the agents have a utility function $v(W(\omega))$ depending on the payoff of their portfolio, with the following properties:

$$v'(c(\omega)) > 0 \quad v''(c(\omega)) < 0 \quad \lim_{c \to 0} = \infty \tag{4}$$

Hence, the agent´s expected utility $E[v]$ for the end of the period is given by:

$$E[v(c)] = \sum_{\omega=1}^{\Omega} \pi_\omega v(y(\omega)x^s + g(\omega)x^o) \tag{5}$$

The agent tries to maximize his expected utility by choosing a portfolio at the beginning of the period. He faces the budget constraint, which describes the possibility of trading the initial endowment of one share at date $t = 0$. It is allowed to buy or sell the divisible shares and options at the market prices $p = (p^s, p^o)$.

Thus, the maximization problem can be written as follows:

$$\max_{x^s, x^o} E[v(c)] = \max_{x^s, x^o} \sum_{\omega=1}^{\Omega} \pi_\omega v(y(\omega)x^s + g(\omega)x^o) \tag{6}$$

subject to the budget restriction:

$$0 = p^s(x^s - 1) + p^o x^o \tag{7}$$

The first-order conditions for this maximization problem are:

$$\sum_{\omega=1}^{\Omega} \pi_\omega v'(y(\omega)x^s + g(\omega)x^o)y(\omega) = \lambda p^s \tag{8}$$

$$\sum_{\omega=1}^{\Omega} \pi_\omega v'(y(\omega)x^s + g(\omega)x^o)g(\omega) = \lambda p^o \tag{9}$$

$$p^s(x^s - 1) + p^o x^o = 0 \tag{10}$$

This leads to the risk-bearing theorem for asset markets:

$$\frac{\sum_{\omega=1}^{\Omega} \pi_\omega v'(y(\omega)x^s + g(\omega)x^o)y(\omega)}{p^s} = \frac{\sum_{\omega=1}^{\Omega} \pi_\omega v'(y(\omega)x^s + g(\omega)x^o)g(\omega)}{p^o} \tag{11}$$

To solve for the asset prices, we have to introduce the prices p_ω for Arrow securities. These securities pay only one unit of good if and only if a certain state of nature ω occurs. Thus, (11) becomes:

$$\frac{\sum_{\omega=1}^{\Omega} \pi_\omega v'(y(\omega)x^s + g(\omega)x^o)y(\omega)}{\sum_{\omega=1}^{\Omega} p_\omega y(\omega)} = \frac{\sum_{\omega=1}^{\Omega} \pi_\omega v'(y(\omega)x^s + g(\omega)x^o)g(\omega)}{\sum_{\omega=1}^{\Omega} p_\omega g(\omega)p^o} \tag{12}$$

(12) is met if the first fundamental theorem of risk-bearing[2] is met for each pair of Arrow securities. This theorem states that at the investors risk-bearing optimum the coefficient of the marginal utility of the payoffs in different states of nature weighted with the probability of occurrence π_s is equal to the coefficient of the Arrow securities in the corresponding states:

$$\frac{\pi_{\omega+1} v'(c(\omega+1))}{\pi_\omega v'(c(\omega))} = \frac{p_{\omega+1}}{p_\omega} \tag{13}$$

Because of (3), the amount of consumption in each state is determined by the payoff of the share, and due to the assumption of all agents being homogeneous in their initial endowment and their preferences, the price of the stock is the same as the expected payoff:

$$p^s = E[y(\omega)] \tag{14}$$

Hence, it follows that we loose a degree of freedom, because we can determine the price of the share.

[2] see for example Hirshleifer/ Riley (1992) p. 46

For simplicity we restrict ourself to the case of three states of nature. Thus, the following system can be solved [3]:

$$[p_1 \quad p_2 \quad p_\Omega] \begin{bmatrix} y(1) & y(2) & y(\Omega) \\ 1 & -\frac{\pi_1(v'(W(1))}{\pi_2(v'(W(2))} & 0 \\ 0 & 1 & -\frac{\pi_{\Omega-1}(v'(W(\Omega-1))}{\pi_\Omega(v'(W(\Omega))} \end{bmatrix} = \begin{bmatrix} p^s \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

The prices of the assets are built by summing up the prices of the contingent claims according to the payoff in the different states of nature:

$$p^o = \sum_{\omega=1}^{\Omega} p_\omega y(\omega) \tag{16}$$

$$p^s = \sum_{\omega=1}^{\Omega} p_\omega \max[y(\omega) - k; 0] \tag{17}$$

To investigate the influence of preferences on the asset prices $p = (p^s, p^o)$, we have specify the utility function. As a benchmark, we choose a utility function of the CRRA type:

$$v = \begin{cases} \frac{c^{1-\rho}}{1-\rho} & \rho \neq 1 \\ \ln(c) & \rho = 1 \end{cases} \tag{18}$$

ρ is the measure of the risk aversion. $\rho = 0$ is defined as risk neutrality. For $\rho = 1$ we have the case of logarithmic utility.

So that the agents expected end-of-period utility is given by:

$$E[v] = \sum_{\omega=1}^{\Omega} \pi_\omega \frac{[y(\omega)x^s + g(\omega)x^o]^{1-\rho}}{1-\rho} \tag{19}$$

Numerical Simulation We find now by numerical simulations, that if the market is incomplete and there exist two assets only, all the agents have the same CRRA utility function and the wealth is determined by the payoff of the stock then:

$$\frac{\partial p^o}{\partial \rho} < 0 \qquad \frac{\partial p^s}{\partial \rho} = 0 \tag{20}$$

[3] See appendix A for the pricing of the contingent claims in extensive form.

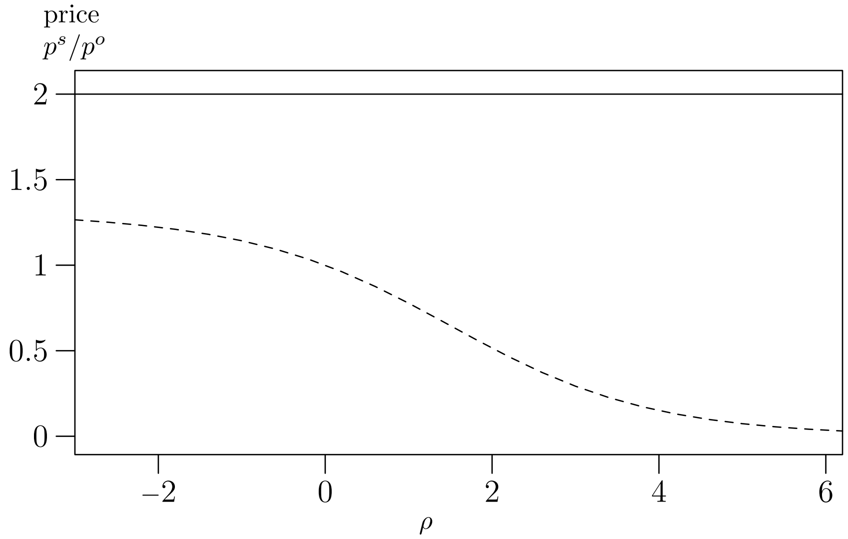

Figure 1: The relationship of the share price (solid line) and option price (broken line) by varying the relative risk aversions measure ρ. The payoffs in the three states are: $y(\omega) = 1; 2; 3 \quad g(\omega) = 0; 1; 2$.

Because of the assumption that the stock is the only asset in positive supply, it is the only source of aggregate income in the period. Thus, in figure 1 the price of the stock is determined by the expected payoff at the end of the period. In case of homogeneous agents, which is assumed here, there is a fair price of the option. Since this asset is in zero net supply, its price must be such that the agents are indifferent between choosing the stock or the option. This is the only possible equilibrium price. A derivation of this price would indicate an immediate entrance of all agents in one side of the market. Thus, in equilibrium no trading takes place.

Figure 1 also shows, that the price of the option falls with increasing risk aversion. In the case of risk-neutrality the option price is the expected payoff. In the area of risk-lovers $\rho < 0$ the price is higher as the expected payoff, because the agents value the high payoffs more than the less payoffs. If the agents are risk-avers $\rho > 0$, they value the payoffs in unfavorable states of nature higher than the payoffs in pleasant states.

Trading can only be induced by agents who differ in certain aspects. We will concentrate on differences in the shape of the utility-function. Differences in expectations or endowment will not be studied in this paper.

3 The model with heterogeneous agents

We now investigate the relationship between the constellation of the prices and different levels of heterogeneity of the agents in the model. Because of the non-linearity of the problem (15) it is not possible to solve it analytically. For that reason we perform numerical computation.

As it was seen in the previous section, the price relations between the option and the share varies with the degree of risk aversion of the representative agent. Now, we assume the existence of two different types of agents, who have different attitudes towards risk. This is a plausible assumption because the agent with relatively low risk aversion can be seen as institutional investors, whereas the agents with relatively high risk aversion might be private investors. Institutional investors are often less risk averse because of the big number of investments with possible negative correlation.

According to the solution of the maximization problem (6), each type of agent has a fair price of the option in relation to the stock price, where he is indifferent between purchasing a share or an option. If M is the number of less risk averse agent and $N-M$ is the number of more risk averse agents, such that:

$$-\frac{v''_M(c(\omega))c(\omega)}{v'_M(c(\omega))} < -\frac{v''_{N-M}(c(\omega))c(\omega)}{v'_{N-M}(c(\omega))} \qquad \forall\ \omega = 1, ..., \Omega \tag{21}$$

Where $-\frac{v''(c(\omega))c(\omega)}{v'(c(\omega))}$ is the Arrow-Pratt index of relative risk aversion, which is in our case ρ because of the CRRA utility function.

Now the restriction has to be introduced, that for each agent in any state bankruptcy is excluded[4]:

$$c^n(\omega) > 0 \qquad \forall\ \ n = 1, ..., N \quad \ \omega = 1, .., \Omega \tag{22}$$

Then we can show, that the more risk averse agents sell options, where as the less risk averse agents buy them. Because of the different degrees of risk aversion the two types of agents have different fair prices. $p^o_{\rho^h}$ is the price where the relative less risk averse agents are indifferent between purchasing the stock or the option. $p^o_{\rho^l}$ is the price where the relatively more risk averse agents are indifferent between purchasing the stock or the option. Hence it follows:

$$p^o_{\rho^h} > p^o_{\rho^l} \qquad \forall\ \ \rho^h < \rho^l \tag{23}$$

[4]In the former this restriction is not required, because nobody of the homogeneous agents sell options short in equilibrium no bankruptcy occurs.

We assume a Walrasian auctioneer, who determines the equilibrium price $p^{o.*}$. This price will be between the two fair prices of both types of agents:

$$p^o_{\rho^h} > p^{o,*} > p^o_{\rho^l} \tag{24}$$

In order to compute the price and to determine the amount of shares which will be traded, we assume the specific case of two types of agents. The less risk averse group of agents has a CRRA utility function with $\rho = 0.5$. The more risk averse group has the same class of utility function with $\rho = 4$. The number of agents in both groups should be the same. The expected utility of both groups is:

$$E[v_M] = \sum_{\omega=1}^{\Omega} \pi_\omega \frac{[y(\omega)x^s_M + g(\omega)x^o_M]^{1-0.5}}{1-0.5} \tag{25}$$

$$E[v_{N-M}] = \sum_{\omega=1}^{\Omega} \pi_\omega \frac{[y(\omega)x^s_{N-M} + g(\omega)x^o_{N-M}]^{1-4}}{1-4} \tag{26}$$

An equilibrium in the financial market exists when:

$$\sum_{n=1}^{N} x^s_n = N \tag{27}$$

and

$$\sum_{n=1}^{N} x^o_n = 0 \tag{28}$$

In equilibrium the number of shares and options sold and bought are equal. To get corresponding equilibrium price of the option $p^{o,*}$, we have computed it in different steps[5]:

We begin with one type of agent with ρ^l and determine his demand or supply of options. The demand or supply of the share results from the budget constraint (7). The agent maximizes his expected utility (6) taking the asset prices p in $t = 0$ as given. First, we fix this prices. Then, we vary the quantity of the share and the option in the portfolio according to the budget constraint (7). The expected utility of each portfolio is determined by (25) and it is checked whether the no-bankruptcy condition (22) is satisfied in each state.

[5]Because of the complexity and the non-linear influence of the measure of risk aversion on the quantity each agent wants to trade, we have to compute it numerically.

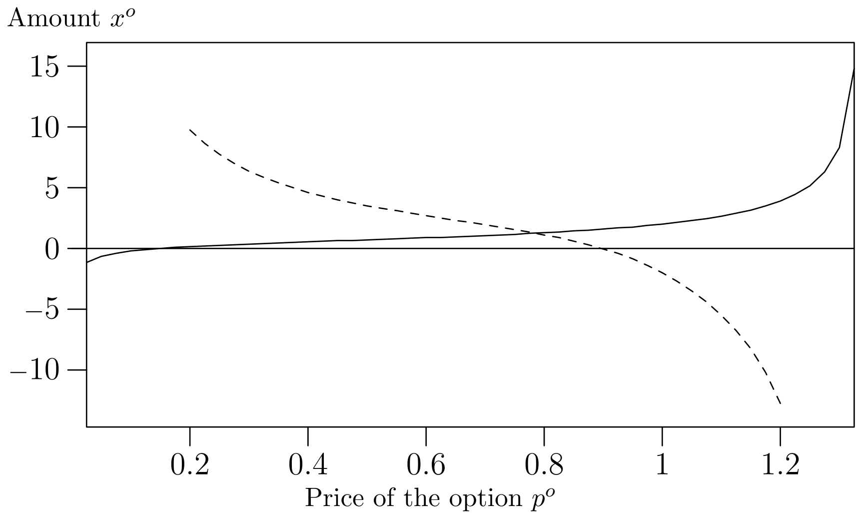

Figure 2: Amount of options which are demanded by the less risk averse agents ($\rho = 0,5$) is plotted as the broken line and supply of options by the more risk averse agents ($\rho = 4$) is plotted as the solid line. The intersection is the market equilibrium by $p^{o,*} = 0.784$ and $x^{o,*} = 1,252$.

Then, the maximum possible utility is selected and the corresponding price of the option and the quantity of option traded is plotted.

In a next step, the price of the option is varied and the procedure is repeated. Thus, we get a function $f^{\rho^l}(p^o)$ which describes the relationship of assets an agent is willing to trade depending on the price of the option.

Then, the procedure is repeated with the ρ^h type of agent. There are two functions $f^{\rho^l}(p^o)$ and $f^{\rho^h}(p^o)$ each of them determines the demand or the supply, depending on their sign. A negative sign means a supply and a positive sign a demand. To determine the equilibrium in the market the function according to the more risk averse group is depicted at the x-axis. The intersection of the two resulting functions is the equilibrium $(p^{o,*}; x^{o,*})$ (See figure 2).

$$f^{\rho^l}(p^o) = -f^{\rho^h}(p^o) \tag{29}$$

As we can see in figure 3, the heterogeneity with respect to the measure of risk aversion of the two groups of agents leads to a different equilibrium price of the option $p^{o,*}$ and a different trading volume $x^{o,*}$.

In order to illustrate the consequences of heterogeneity, we simulate the amount of trading and the option prices by varying the spread of risk aversion. We start with homogeneous agents, who have a coefficient of relative risk

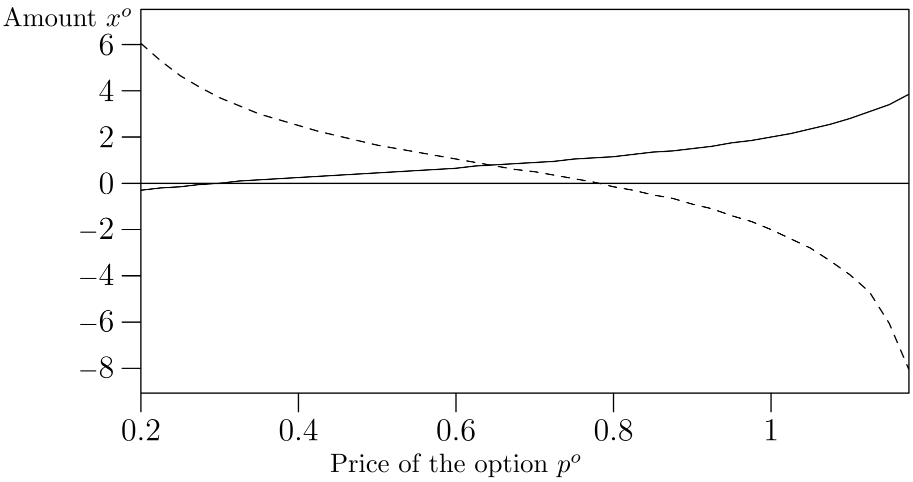

Figure 3: Demand and supply of the option if the less risk averse agents have $\rho = 0,99$ (broken line) and the more risk averse agents have ($\rho = 3$) (solid line). The market equilibrium is at $p^{o,*} = 0.644; x^{o,*} = 0,79$. The payoffs of the share and the option are always assumed to be $y(\omega) = (1;2;3)$ and $g(\omega) = (0;1;2)$.

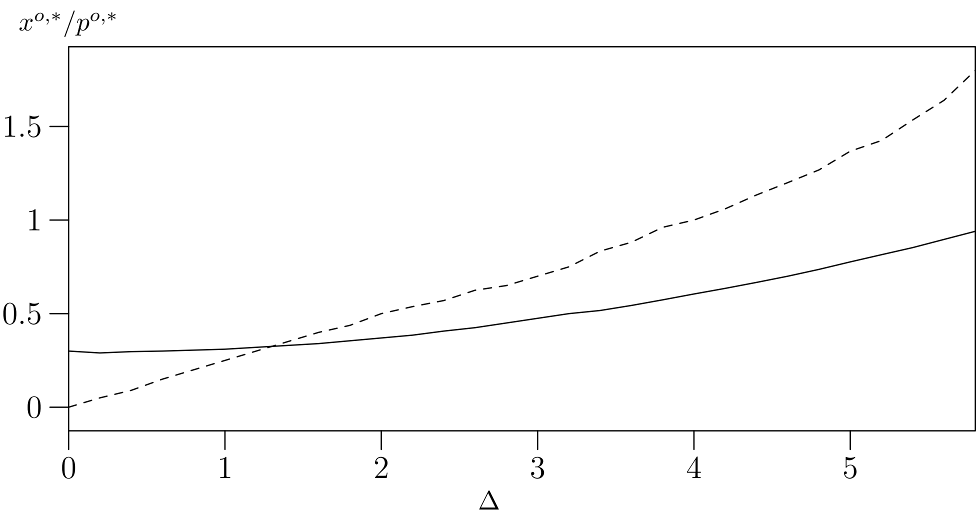

Figure 4: The graph shows the amount of trading $x^{o,*}$ (broken line) depending on the span of heterogeneity measured in the difference of the Arrow-Pratt measure of relative risk aversion Δ and the equilibrium price $p^{o,*}$ build in the market (solid line).

aversion of $\rho_1^h = \rho_1^l = 3$. The amount of trading and the corresponding option price is plotted. Now, again two groups of the same number of agents are formed. It is assumed that they have a risk aversions coefficient differing from the previous, in a way, that the one group is more and the other is less risk averse than the starting group. The derivation $|1/2\Delta|$ of the Arrow-Pratt risk measure of the starting group is the same in both groups:

$$\rho_2^h = \rho_1^h + 1/2\Delta \qquad \rho_2^l = \rho_1^l - 1/2\Delta \tag{30}$$

The derivation $1/2\Delta$ is added respectively deducted from both group in each step which is simulated. Figure 4 shows the impact of the heterogeneity on the amount of trading and on the price relation of the assets.

Growing differences in the heterogeneity of the agents leads to a growing trading volume. But the equilibrium price of the option grows too, with increasing differences. Hence, the heterogeneity of the agents has a significant influence on the equilibrium asset price built in the market.

4 Conclusion

This paper studies the influence of preferences on the valuation of European-style options and the underlying asset in a simple one period equilibrium asset pricing model with an incomplete financial market. It shows, that trading of options only takes place, when the investors are different. We have investigated the impact of differences in their attitude towards risk. The more risk averse investors will wish to supply the options, whereas the less risk averse investors will wish to obtain them. It was shown by numerical computations that the span of difference of the agents has a major impact on the option price and the volume of trading. Both grow with increasing differences of agents.

A Appendix

From the system (15) we get:

$$y(1)p_1 + y(2)p_2 + y(3)p_3 = p^s$$

$$p_1 - \frac{\pi_1 v'(W(1))}{\pi_2 v'(W(2))} p_2 = 0$$

$$p_2 - \frac{\pi_2 v'(W(2))}{\pi_3 v'(W(3))} p_3 = 0$$

This leads to prices for the contingent claims :

$$p_1 = \frac{p^s}{y(1)}$$

$$-\frac{y(2)}{y(1)} \left(\frac{p^s/y(1)}{\frac{y(2)}{y(1)} + \frac{\pi_1 v'(W(1))}{\pi_2 v'(W(2))}} - \frac{p^s/y(1)}{\frac{y(3)}{y(1)} + \left(\frac{y(2)}{y(1)} + \frac{\pi_1 v'(W(1))}{\pi_2 v'(W(2))}\right) \frac{\pi_2 v'(W(2))}{\pi_3 v'(W(3))}} \frac{y(3)/y(1)}{\frac{y(2)}{y(1)} + \frac{\pi_1 v'(W(1))}{\pi_2 v'(W(2))}} \right)$$

$$- \frac{y(3)}{y(1)} \frac{p^s/y(1)}{\frac{y(3)}{y(1)} + \left(\frac{y(2)}{y(1)} + \frac{\pi_1 v'(W(1))}{\pi_2 v'(W(2))}\right) \frac{\pi_2 v'(W(2))}{\pi_3 v'(W(3))}}$$

$$p_2 = \frac{p^s/y(1)}{\frac{y(2)}{y(1)} + \frac{\pi_1 v'(W(1))}{\pi_2 v'(W(2))}} - \frac{p^s/y(1)}{\frac{y(3)}{y(1)} + \left(\frac{y(2)}{y(1)} + \frac{\pi_1 v'(W(1))}{\pi_2 v'(W(2))}\right) \frac{\pi_2 v'(W(2))}{\pi_3 v'(W(3))}} \frac{y(3)/y(1)}{\frac{y(2)}{y(1)} + \frac{\pi_1 v'(W(1))}{\pi_2 v'(W(2))}}$$

$$p_3 = \frac{p^s/y(1)}{\frac{y(3)}{y(1)} + \left(\frac{y(2)}{y(1)} + \frac{\pi_1 v'(W(1))}{\pi_2 v'(W(2))}\right) \frac{\pi_2 v'(W(2))}{\pi_3 v'(W(3))}}$$

This was written in an extensive form to elucidate the impact of the marginal utility on the contingent claim prices.

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