Competitive Dynamics of Web Sites

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Abstract

We present a dynamical model of web site growth in order to explore the effects of competition among web sites and to determine how they affect the nature of markets. We show that under general conditions, as the competition between sites increases, the model exhibits a sudden transition from a regime in which many sites thrive simultaneously, to a "winner take all market" in which a few sites grab almost all the users, while most other sites go nearly extinct. This prediction is in agreement with recent measurements on the nature of electronic markets.

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1 Introduction

The emergence of an information era mediated by the Internet brings about a number of novel and interesting economic problems. Chiefly among them is the realization that ever decreasing costs in communication and computation are making the marginal cost of transmitting and disseminating information essentially zero. As a result, the standard formulation of the competitive equilibrium theory is inapplicable to the Internet economy. This is because the theory of competitive equilibrium focuses on the dynamics of price adjustments in situations where both the aggregate supply and demand are a function of the current prices of the commodities [7, 2]. Since on the Internet the price of a web page is essentially zero, supply will always match demand, and the only variable quantity that one needs to consider is the aggregate demand, i.e. the number of customers willing to visit a site or download information or software. As we will show, this aggregate demand can evolve in ways that are quite different from those of price adjustments.

A particular instance of this different formulation of competitive dynamics is provided by the proliferation of web sites that compete for the attention and resources of millions of consumers, often at immense marketing and development costs. As a result, the number of visitors alone has become a proxy for the success of a web site, the more so in the case of advertising based business models, where a well defined price is placed on every single page view. In this case, most customers do not pay a real price for visiting a web site. The only cost a visitor incurs is the time spent viewing an adbanner, but this cost is very low and practically constant. Equally interesting, visits to a web site are such that there is non-rival consumption in the sense that one's access to a site does not depend on other users viewing the same site. This can be easily understood in terms of Internet economics: once the fixed development cost of setting up a web site has been paid, it is relatively inexpensive to increase the capacity the site needs to meet increased demand. Thus, the supply of served web pages will always track the demand for web pages (neglecting network congestion issues) and will be offered at essentially zero cost.

The economics of information goods such as the electronic delivery of web pages has recently been reviewed by Smith et. al. [13]. They show that when the marginal reproduction cost approaches zero, new strategies and behaviors appear, in particular with respect to bundling [3], price dispersion [4], value pricing versus cost pricing [14], versioning [15], and complicated price schedules [9].

Since supply matches demand when the price become negligibly small, the only variable quantity that we will consider in our model is the aggregate demand, i.e. the number of customers willing to visit a site. This is the quantity for which we study the dynamics as a function of the growth and capacity of web sites, as well as the competition between them. In particular, we explore the effects that competitive pressures among web sites have on their ability to attract a sizeable fraction of visitors who can in principle visit a number of equivalent sites. This is of interest in light of results obtained by Adamic and Huberman [1], who showed that the economics of the Internet are such that the distribution of visitors per site follows a power-law characteristic of winner-take-all markets. They also proposed a growth model of the Internet to account for this behavior which invokes either the continuous appearance of new web sites or different growth rates for sites.

While such a theory accounts for the dynamics of visits to sites, it does not take into account actions that sites might take to make potential visitors to several similar sites favor one over the other. As we show, when such mechanisms are allowed, the phenomenon of winner-take-all markets emerges in a rather surprising way, and persists even in situations where no new sites are continously created.

Our work also explains results obtained from computer simulation of competition between web sites by Oğuş et. al. [11]. Their experiments show that brand loyalty and network effects together result in a form of winner-take-all market, in which only a few sites survive. This is consistent with the predictions of our theory.

In section 2 we present the model and illustrate its main predictions by solving the equations in their simplest instance in section 3. In section 4 we show that the transition from fair market share to winner-take-all persists in the general case of competition between two sites and in section 5 we extend our results to very many sites. We also show the appearence of complicated cycles and chaotic outcomes when the values of the competitive parameters are close to the transition point. A concluding section summarizes our results and discusses their implications to electronic commerce.

2 The Model

Consider n web sites offering similar services and competing for the same population of users, which we'll take to be much larger than the number of sites. Each site engages in policies, from advertising to prize reductions, that try to increase their share of the customer base f_i. Note that while f_i is the fraction of the population that is a customer of web site i, it can be more generally taken to be the fraction of the population aware of the site's existence. This could be measured by considering the number of people who bookmark a particular site.

The time evolution of the customer fraction f_i at a given site i is determined by two main factors. If there is no competition with any other sites, the user base initially grows exponentially fast, at a rate α_i, and then saturates at a value β_i. These values are determined by the site's capacity to handle a given number of visitors per unit time. If, on the other hand, other sites offer competing services, the strength of the competition determines whether the user will be likely to visit several competing sites (low competition levels) or whether having visited a given site reduces the probability of visiting another (high competition level).

Specifically, the competition term can be understood as follows: if fractions f_i and f_j of the people use sites i and j, respectively, then assuming that the probability of using one site is independent of using another, a fraction $f_i f_j$ will be using both sites. However, if both sites provide similar services, then some of these users will stop using one or the other site. The rate at which they will stop using site i is given by $\gamma_{ij} f_i f_j$, and the rate at which they abandon site j is given by $\gamma_{ji} f_i f_j$ (note that γ_{ij} is not necessarily equal to γ_{ji}).

Mathematically the dynamics can thus be expressed as

$$\frac{\mathrm{d}f_i}{\mathrm{d}t} \;=\; \alpha_i f_i(\beta_i - f_i) - \sum_{i \neq j} \gamma_{ij} f_i f_j, \tag{1}$$

where α_i is the growth rate of individual sites, β_i denotes their capacity to service a fraction of the customer base and γ_{ij} is the strength of the competition. The parameter values are such that $\alpha_i \geq 0$, $0 \leq \beta_i \leq 1$ and $\gamma_{ij} \geq 0$.

The system of equations (1), which determines the nonlinear dynamics of user visits to web sites, possesses a number of attractors whose stability properties we will explore in detail *. In particular, we will show that as a function of the competition level, the solutions can undergo bifurcations which render a particular equilibrium unstable and lead to the appearance of new equilibria. The most striking result among them is the sudden appearance of a winner-take-all site which captures most of the visitors, a phenomenon that has been empirically observed in a study of markets in the web [1].

Since the complexity of the equations may obscure some the salient features of the solutions, we will first concentrate on the simplest case exhibiting a sharp transition from fair market share to a winner-take-all site, and then

*The equations are functionally similar to those describing the competition between modes in a laser [6], and to those describing prey-predator equations in ecology [10].

consider more complicated examples.

3 Fair Market Share to Winner-Take-All

Let us first consider one of the simplest instances of the problem described above, in which two web sites have the same growth rates $\alpha_1 = \alpha_2 = 1$, the same capacities $\beta_1 = \beta_2 = 1$ and symmetric competion $\gamma_{12} = \gamma_{21} = \gamma$. In this case the equations take the form

$$\begin{aligned} \frac{\mathrm{d}f_1}{\mathrm{d}t} &= f_1(1 - f_1 - \gamma f_2) \\ \frac{\mathrm{d}f_2}{\mathrm{d}t} &= f_2(1 - f_2 - \gamma f_1) \end{aligned}$$

The four fixed points of this equation, which determine the possible equilibria, are given by

$$(f_1, f_2) \in \{(0,0), (1,0), (0,1), (\frac{1}{1+\gamma}, \frac{1}{1+\gamma})\}$$

Since not all of these equilibria are stable under small perturbations, we need to determine their time evolution when subjected to a sudden small change in the fraction of visitors to any site. To do this, we need to compute the eigenvalues of the Jacobian evaluated at each of the four fixed points. The Jacobian is

$$\mathbf{J} = \begin{pmatrix} 1 - 2f_1^0 - \gamma f_2^0 & -\gamma f_1^0 \\ -\gamma f_2^0 & 1 - 2f_2^0 - \gamma f_1^0 \end{pmatrix}$$

and the eigenvalues at each of the fixed points are given in the following table:

equilibrium	eigenvalues
$(0,0)$	1 (twice)
$(\frac{1}{1+\gamma}, \frac{1}{1+\gamma})$	-1 and $\frac{\gamma-1}{1+\gamma}$
$(1,0)$ or $(0,1)$	$\frac{1}{2}(-\gamma \pm \sqrt{(2-\gamma)^2})$

From this it follows that the fixed point $(0,0)$ is never stable. On the other hand, the equilibrium at $(\frac{1}{1+\gamma}, \frac{1}{1+\gamma})$ is stable provided that $\gamma < 1$. And the fixed points $(0,1)$ and $(1,0)$ are both stable if $\gamma > 1$. From these results, we can plot the equilibrium size of the customer population as a function of the competition γ between the two competitors. As Figure 1 shows, there is a sudden, discontinuous transition at $\gamma = 1$. For low competition, the only stable configuration has both competitors sharing the market equally. For high competition, the market transitions into a "Winner-Take-All Market" [5], in which one competitor grabs all the market share, whereas the other gets nothing.

As we will show below, this sudden transition persists under extremely general conditions, for two competitors as well as for n competitors. The significant feature is that a very small change in the parameters can radically affect the qualitative nature of the equilibrium.

Another feature is that near the transition, the largest eigenvalue of the stable state is very close to zero (but negative). This means that the time of convergence to equilibrium diverges. In more complicated systems, this

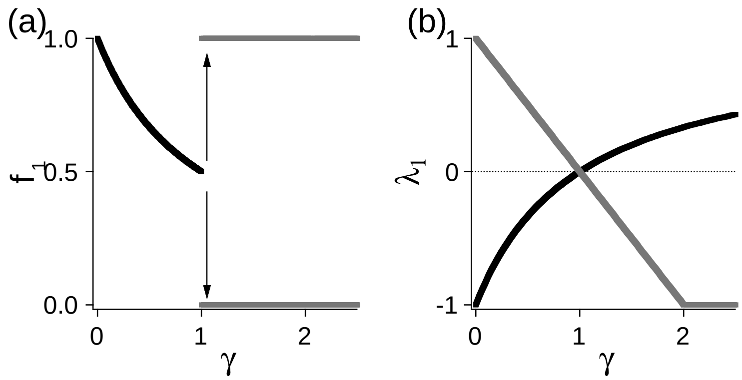

Figure 1: (a) Equilibrium values for f_1 and (b) largest eigenvalues of the Jacobian for the $(\frac{1}{1+\gamma}, \frac{1}{1+\gamma})$ fixed point (dark) and the $(1, 0)$ fixed points (light), as a function of the competition value γ.

may make it extremely difficult to predict which equilibrium the system will converge to in the long term.

4 Competition between two sites

We now analyse the two site model in its most general form, without restricting the values of the parameters to be the same for the two sites. The system of equations is

$$\frac{\mathrm{d}f_1}{\mathrm{d}t} = f_1(\alpha_1(\beta_1 - f_1) - \gamma_{12} f_2)$$
$$\frac{\mathrm{d}f_2}{\mathrm{d}t} = f_2(\alpha_2(\beta_2 - f_2) - \gamma_{21} f_1)$$

As in the previous section these equations possess four fixed points at:

$$
\begin{aligned}
(f_1^0, f_2^0) &= (0, 0) \\
(f_1^0, f_2^0) &= (\beta_1, 0) \\
(f_1^0, f_2^0) &= (0, \beta_2) \\
(f_1^0, f_2^0) &= (\frac{\alpha_2(\alpha_1\beta_1 - \beta_2\gamma_{12})}{\alpha_1\alpha_2 - \gamma_{12}\gamma_{21}}, \frac{\alpha_1(\alpha_2\beta_2 - \beta_1\gamma_{21})}{\alpha_1\alpha_2 - \gamma_{12}\gamma_{21}})
\end{aligned}
$$

Let's analyze the stability of each fixed point. This is done by evaluating the Jacobian

$$
\mathbf{J} = \begin{pmatrix} \alpha_1\beta_1 - 2\alpha_1 f_1^0 - \gamma_{12} f_2^0 & -\gamma_{12} f_1^0 \\ -\gamma_{21} f_2^0 & \alpha_2\beta_2 - 2\alpha_2 f_2^0 - \gamma_{21} f_1^0 \end{pmatrix}
$$

and computing its two eigenvalues. Each fixed point will be stable only if the real parts of both eigenvalues are negative. The first trivial fixed point is always unstable, since the eigenvalues $\alpha_1\beta_1$ and $\alpha_2\beta_2$ are both positive quantities. The second equilibrium, with $(f_1, f_2) = (\beta_1, 0)$ is stable provided $\frac{\gamma_{12}}{\alpha_1} > \frac{\beta_1}{\beta_2}$. The third equilibrium $(f_1, f_2) = (0, \beta_2)$ is similarly stable only if $\frac{\gamma_{21}}{\alpha_2} > \frac{\beta_2}{\beta_1}$. The final case is the most complicated one, and is stable in three distinct regimes. However, two of the stable solutions have a negative f_1 or f_2 and can never be reached from an initial condition with both populations positive. The only remaining equilibrium is stable whenever the other two aren't. In order to summarize these results in the table below, it is convenient to define the following parameters:

$$
\alpha = \frac{\alpha_1}{\alpha_2} \qquad \beta = \frac{\beta_1}{\beta_2} \qquad \gamma_1 = \frac{\gamma_{12}}{\alpha_1} \qquad \gamma_2 = \frac{\gamma_{21}}{\alpha_2}.
$$

equilibrium	stable if
(0, 0)	never
$(\beta_1, 0)$	$\beta > \frac{1}{\gamma_2}$
$(0, \beta_2)$	$\beta < \gamma_1$
$(\frac{\beta_1-\beta_2\gamma_1}{1-\gamma_1\gamma_2}, \frac{\beta_2-\beta_1\gamma_2}{1-\gamma_1\gamma_2})$	$\beta > \gamma_1 \quad \beta < \frac{1}{\gamma_2}$

Note that in the last row, $\beta > \gamma_1$ and $\beta < \frac{1}{\gamma_2}$ imply that $\gamma_1\gamma_2 < 1$. As a result, for fixed γ_1 and γ_2, there are two different regimes. Either $\gamma_1\gamma_2 < 1$, in which case intermediate values of β ($\gamma_1 < \beta < \frac{1}{\gamma_2}$) lead to a "fair" equilibrium in which both sites get a non zero f_i, or $\gamma_1\gamma_2 < 1$, in which case intermediate values of β ($\frac{1}{\gamma_2} < \beta < \gamma_1$) lead to a situation in which either of the two "winner-take-all" equilibria is stable. In this latter case, hysteresis occurs if β slowly changes with time. This is illustrated in Figure 2. Starting from a low value of β, the only stable equilibrium is $(0, \beta_2)$. If we slowly increase the ratio β, this equilibrium remains stable, until $\beta > \gamma_1$. At that point, the equilibrium $(0, \beta_2)$ becomes unstable and the system relaxes to the only new equilibrium $(\beta_1, 0)$. If we now reverse the process, and decrease the value of β, the new solution remains stable as long as $\beta > \frac{1}{\gamma_1}$.

Thus, there always is at least one stable equilibrium, but there never are more than two. If there are two stable equilibria, then the initial conditions determine into which of the two the system will fall. Which of the equilibria are stable depends on a total of three parameters (down from the five parameters that are required to fully describe the system once the time variable is rescaled).

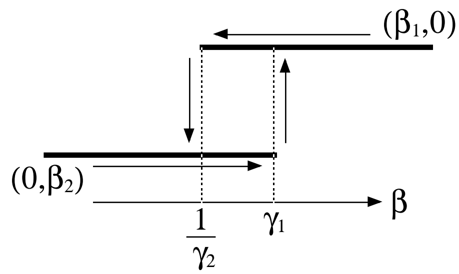

Figure 2: Hysteresis behavior as β is changed with γ_1 and γ_2 fixed.

It is interesting to ask which of the two sites will win as a function of a set of fixed parameters and as a function of the starting point. To do this, we plot the motion of (f_1, f_2) as a vector field in Figure 3, for a particular set of parameters. As can be seen, the space of initial conditions is divided into two distinct regions, each of which leads to a different equilibrium.

5 Many sites

5.1 Analytic treatment

We now show that the sharp transition to a winner-take-all market that we found in the two site case is also present when many sites are in competition. In order to do so, we first examine the case where the parameters are the same for all sites, i, so that Equation (1) can be rewritten as

$$\frac{\mathrm{d}f_i}{\mathrm{d}t} = f_i(\alpha\beta - \alpha f_i - \gamma \sum_{i \neq j} f_j).$$

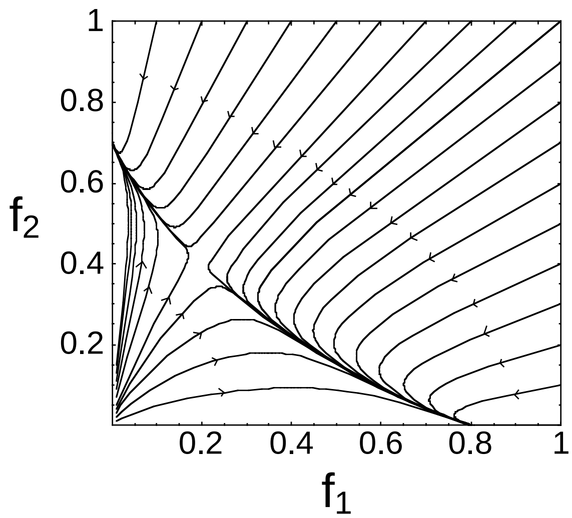

Figure 3: Basins of attraction for the solutions to a winner-take-all market as a function of the initial market share, for $\alpha_1 = 1$, $\alpha_2 = 0.8$, $\beta_1 = 0.8$, $\beta_2 = 0.7$, $\gamma_{12} = 1.5$ and $\gamma_{21} = 1.2$

where $i = 1, \ldots, n$ and n is the number of sites. For n equations, there are 2^n different vectors $(f_1, \ldots, f_n)$ for which all the time derivatives are zero, since for each equation, either $f_i = 0$ or $\alpha\beta - \alpha f_i - \sum_{i \neq j} \gamma f_j = 0$ at equilibrium. Without loss of generality, we can relabel the f_i such that the first k of them are non-zero, while the remaining n are zero.

At equilibrium, the value of the f_i with $1 \leq i \leq k$ will be given by the solution of

$$\begin{pmatrix} \alpha & \gamma & \dots & \gamma \\ \gamma & \alpha & \dots & \gamma \\ \vdots & \vdots & \ddots & \vdots \\ \gamma & \gamma & \dots & \alpha \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_k \end{pmatrix} = \begin{pmatrix} \alpha\beta \\ \alpha\beta \\ \vdots \\ \alpha\beta \end{pmatrix}$$

Except for the degenerate cases, the matrix on the left hand side is invertible, so that

$$f_i = \begin{cases} \frac{\alpha\beta}{\alpha+(k-1)\gamma} & \text{if } 1 \leq i \leq k \\ 0 & \text{if } k+1 \leq i \leq n \end{cases}$$

We are now ready to compute the Jacobian about this equilibrium. It takes the form

$$\mathbf{J} = \begin{pmatrix} X & Y & \dots & Y & 0 & \dots & 0 \\ Y & X & \dots & Y & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots \\ Y & Y & \dots & X & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & Z & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & Z \end{pmatrix}$$

where

$$\begin{aligned}
X &= \alpha\beta - (2\alpha + (k-1)\gamma)\frac{\alpha\beta}{\alpha + (k-1)\gamma} \\
Y &= -\gamma\frac{\alpha\beta}{\alpha + (k-1)\gamma} \\
Z &= \alpha\beta - k\gamma\frac{\alpha\beta}{\alpha + (k-1)\gamma}
\end{aligned}$$

The eigenvalues of the Jacobian are Z (with multiplicity $n-k$), $X-Y$ (with multiplicity $k-1$) and $X+(k-1)Y$ (with multiplicity 1). Note that the last two eigenvalues are absent if $k=0$. Thus there are four distinct cases to check: $k=0$, $k=1$, $1<k<n$ and $k=n$. In the first case, the only eigenvalues are $Z=\alpha\beta>0$, so this solution is always unstable.

With $k=1$, the eigenvalues are

$$\begin{aligned}
X &= -\alpha\beta \\
Z &= (\alpha-\gamma)\beta
\end{aligned}$$

That is, an equilibrium with one out of n winners is stable provided $\alpha<\gamma$. Note that this is the same condition we obtained for two competitors.

With $1<k<n$ we have the eigenvalues

$$\begin{aligned}
Z &= \alpha\beta\frac{\alpha-\gamma}{\alpha+(k-1)\gamma} \\
X-Y &= \alpha\beta\frac{\gamma-\alpha}{\alpha+(k-1)\gamma} \\
X+(k-1)Y &= -\alpha\beta
\end{aligned}$$

The first and second eigenvalues above can not be negative simultaneously, thus there are no stable solutions with $1 < k < n$.

Finally, for $k = n$ we have the same eigenvalues as for $1 < k < n$, except that the eigenvalue Z is now inexistent. As a result, the solution with $k = n$ is stable provided that $\gamma < \alpha$.

To summarize, the only stable solutions are

$$f_i = \begin{cases} \frac{\alpha\beta}{\alpha+(n-1)\gamma} & \text{if } \gamma < 1 \\ \beta\delta_{ij} & \text{if } \gamma > 1 \end{cases}$$

That is, the winner-take-all dynamics observed for two sites persists independently of the number of competitors involved, at least in an idealized symmetric configuration. In the next section, we consider the dynamics for large systems in which the parameter value are drawn from a random distribution.

5.2 Critical dynamics

In the most general case, the dynamical change in the fraction of visitors to web sites can be determined by numerically solving the general equations of our model. In addition, provided the number of sites n remains small one can check each of the 2^n candidate equilibria for stability and verify whether the numerical simulation converged to the only equilibrium or missed an existing but hard to reach equilibrium.

In Figure 4, we show the time evolution of f_i for sixteen web sites, ob-

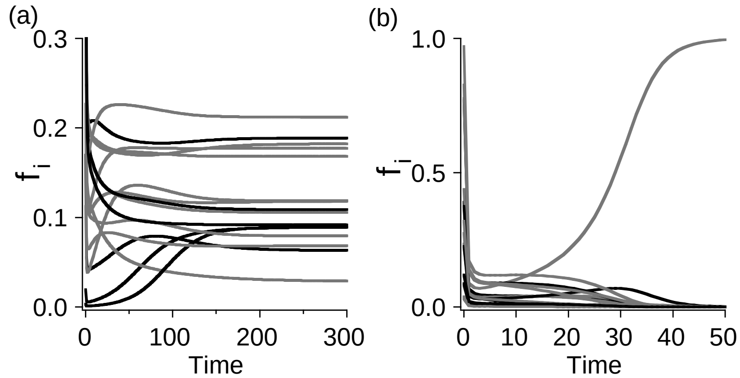

Figure 4: Solutions obtained by numerical integration for two different distributions of parameters. In (a) we are below the transition to winner-take-all, with $\bar{\gamma} = 0.5$, while in (b) we are above the transition to winner-take-all, with $\bar{\gamma} = 1.5$. In both cases $\alpha_i = \beta_i = 1$ for all i.

tained by numerically integrating the equations using a Runge-Kutta scheme. The parameters defining the competitive strength between sites, γ_{ij}, were randomly chosen from a Gaussian distribution with a standard deviation of 0.1, and a fixed mean $\bar{\gamma}$. On the left panel we exhibit a solution for $\bar{\gamma} = 0.5$, far below the transition point. On the right panel, $\bar{\gamma} = 1.5$ places us well above the transition, and we observe the evolution towards a winner-take-all market. Whereas below the transition the equilibrium has all sixteen competitors sharing the market, above it one web site takes all visitors.

Given the fixed set of parameters for the model, it is possible to diagonalize the Jacobian, evaluated at each of the $2^n = 2^{16}$ fixed points. As in the case of the symmetric case, or for the general two site case, only one equilibrium is stable when $\bar{\gamma} \ll 1$. In addition, the values of the f_i at the single stable equilibrium found in this manner match the values that the numerical simulation converges to.

When $\bar{\gamma} \gg 1$, numerically diagonalizing the 2^n Jacobians shows that the n equilibria of the form $f_i = \delta_{ik}$, and no others, are stable. Thus the transition to a winner-take-all market subsists even when the parameters come from a randomized distribution.

A more interesting situation is posed by the dynamics of competition when the competitive strength approaches the critical value, $\bar{\gamma} \approx 1$. Since near the transition point the largest eigenvalue has an absolute value very close to zero the transients to equilibrium are very long. Moreover, the nature of the transients is such that many sites alternate in their market dominance for long periods of time.

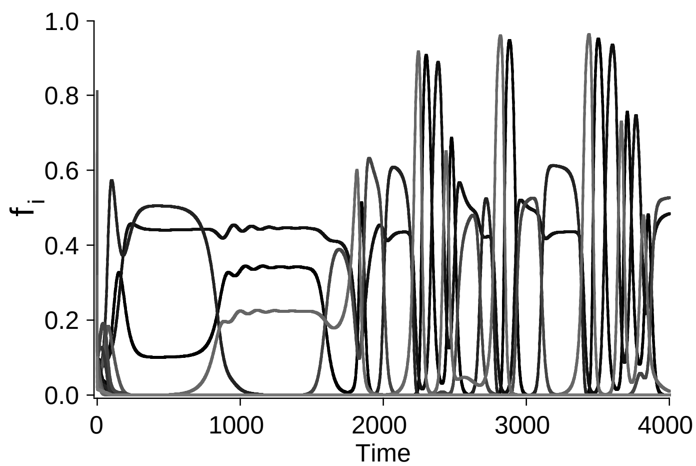

Figure 5: Dynamics near the transition to winner-take-all, for the same parameters as in Figure 4, but with $\bar{\gamma} = 1.0$.

Numerical diagonalization of all the possible Jacobians near criticality shows that frequently there are several stable equilibria, in which some sites have non-zero f_i, and some sites don't. However, these solutions typically are not reached in a finite amount of time (if at all) when numerically integrating the equations. Furthermore, numerical integration, as shown in Figure 5, suggests that for this range of parameters the dynamics are chaotic (small differences in the initial conditions lead to diverging trajectories). For some initial conditions the system may converge to limit cycles, rather that to static equilibria. Thus, when the parameter values of γ_{ij} are drawn from a distribution, the transition is not sudden in $\bar{\gamma}$. There is a range of values of $\bar{\gamma}$ for which the dynamics are more complicated.

For much larger n, it is no longer possible to verify every single candidate fixed point for stability. However, it is still possible to numerically integrate the equations. Either way, as the example in Figure 5 shows, the question of existence and stability of the equilibria is irrelevant if the stable equilibria are never reached, or only reached after an unreasonable amount of time.

6 Conclusion

In this paper we have shown that under general conditions, as the competition between web sites increases, there is a sudden transition from a regime in which many sites thrive simultaneously, to a "winner take all market" in which a few sites grab almost all the users, while most other sites go nearly extinct, in agreement with the observed nature of electronic markets. This transition is the result of a nonlinear interaction among sites which effectively reduces the growth rate of a given site due to competitive pressures from the others. Without the interaction term, web sites would grow exponentially fast to a saturation level that depends on their characteristic properties.

Moreover, we have shown that the transition into a winner-take-all market occurs under very general conditions and for very many sites. In the limiting case of two sites, the phenomenon is reminiscent of the "Principle of Mutual Exclusion" in ecology [10, 8, 12], in which two predators of the same prey can not coexist in equilibrium when competitive predation is very strong.

Smith et. al. [13, 4] attribute the price dispertion of goods sold online to several features of web sites: differences in branding and trust, in the appearance and in the quality of the search tools, switching costs between sites, and last but not least retailer awareness. A winner-take-all economy may thus have strong consequences for price dispertion, since a few sites can charge more by virtue of dominating the mind share of their customers.

It is interesting to speculate about the applicability of this model to different markets. We motivated the model for a massless Internet economy, in which demand can be instantly satisfied by supply at a negligible cost to the supplier, and in which competition does not occur on the basis of cost, but rather on advertising and differentiation in the services provided by the web sites. However, since winner-take-all markets are being observed in a much broader range of markets, it might well be the case that the sudden transition to winner-take-all behavior might also be a feature of these markets as well.

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