Disequilibrium Economics and Development

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Abstract

The history of Economic Theory shows in several stages the influence of Thermodynamics. The analogies between economic systems and thermodynamic systems are at the same time quite obvious and misleading. Important thermodynamic notions lack of counterparts in Economics, making the goal of transferring concepts -other than the trivial analogues- from one discipline to the other an epistemological dead end. The transference of formal devices, instead, became a feasible goal. Procedures like comparative statics were imported directly from thermodynamics, making possible to relate equilibria in economic systems to properties of their environments.

Continuing that tradition we claim that it would be profitable for Economics to borrow formalisms from Open Systems Theory, the not yet well-defined set of extensions of Thermodynamics to the analysis of open systems. These methods have been applied in several sciences to study the behavior of systems away from equilibrium. In this paper we discuss briefly the epistemological rationale for this claim and present a model that cannot be analyzed in classical terms as to show how the tools of O.S.T. can be applied in order to represent the relevant economic phenomenon of development.

We postulate that development arises from the non-linear interaction among different sectors of an economy. The spill-over effects make the entire system either grow or fall. After a transient with fluctuations, a sustained development process may arise. In our model we show how this leads to different patterns of development, characterized by either permanent instability or disequilibrium.

I. Introduction

Development Economics is, from all branches of Economics, one of the less mathematized. In stark contrast with the related field of Growth Theory, the theories of economic development have been more a mixture of verbal and statistical analyses than formal schemes detailing the variation of certain aggregate variables in time. This is rather strange, considering that in any common understanding of the terms involved, “development” means change. But instead of considering some well-defined notion like income (which is at least theoretically clear) development theorists considered a variety of equally relevant variables whose variations could account for economic development.

On the other hand, this characterization is fairly common for all the fields of inquiry where the phenomena analyzed are **complex**. That is, where there are numerous non-linear linkages among components of a structure. The tradition in science and in particular in Economics has been to model things using only linear structures. This procedure leaves aside important problems like conflict, instability and fundamentally, the existence of disequilibria.

In the last years a literature that studies properties of non-linear systems and their application to economic modeling has emerged. In the same line of work, this research explores certain features of development using a variant of the spin-glasses (Ising) model. Although spin-glasses are known since the 1940s it was not until the expansion of Opens Systems Theory embraced the entire study of complex phenomena in the physical sciences that they were fully understood. The spin-glasses model is now seen as a general framework for the study of disequilibrium conceived as the result of the interaction of cooperation and competition among agents **[Boldrin 1988]**.

In a not well-known paper of 1959, during the heyday of the so called High Theory of Economic Development, J.H.G. Olivera presented a clear characterization of Economic Development **[Olivera 1959]**. The main point in his definition is the difference between Actual and Potential Outcome. The last notion involves the possibilities of production that may not be achieved because of institutional or social reasons. The degree of development of an economy is then conceived as the relation between the values of both outcomes.

Our model takes a particular characterization of an economy as a structure composed by several sectors linked by relations of demand and supply. We study the evolution of the degree of development of the entire system after an initial random shock. We will run an experiment to show how development is influenced by the peculiarities of the involved sectors. An interesting result presented here is the presence of permanent instability or permanent disequilibrium. This will be shown, explains some features of development that are still under debate.

In section II we will briefly introduce the basic notions of development as arising from the interaction of an actual and a potential output in a n-sector economy. Section III presents the specifics of our model, particularly the dynamic chaining between sectors through the demand exerted on a sector by the other n-1 sectors of the economy. Section IV shows the results of running the model under different scenarios. Finally, section V discusses the validity of those results.

II. Inter-sector Dynamics and Economic Development

The variables and parameters we will consider here are:

- n : number of sectors in the economy
- σ_i : degree of development of sector i
- O^p_i : potential output of sector i
- O^a_i : actual output of sector i
- D_i : demand of products of sector i

These notions require a bit of clarification. As said before, economic development is seen as the process of increase of the ratio actual social output over potential social output. The gross product of the economy usually approximates the former. The later, potential output, is either the highest output that can be obtained with the available resources or the highest output that can be obtained using the resources with which the country is best endowed. This last meaning emphasizes on the **efficient** use of resources (i.e. those with the least opportunity cost). For the purposes of our analysis the last meaning seems preferable.

The consideration of n sectors of the economy is intended to introduce the inter-sector connections. These links are the sources of spill-over effects: the betterment of one sector affects the state of the related sectors. We assume for simplicity that the connection among sectors is given by the demand of their production. In a certain sense this is reminiscent of the traditional theory of the firm. The key difference here resides in that there are no prices in this economy. Demand of the production of a sector i is defined as just a function of the requirements of productive factors of the other sectors (for them, the production of i is an input for their own production). It is easy to see that unless production uses fixed proportions of factors, this interaction leads to a complicated interrelation that is not amenable of analytical treatment.

According to Olivera the degree of development of a sector and its rate of variation are:

- $\sigma_i O^p_i = O^A_i$
- $\partial \log \sigma_i + \partial \log O^p_i = \partial \log O^A_i$

We assume that in developing countries the expansion of the potential outcome, $\partial \log O^p_i$, depends on the growth of primary factors (say work force) and on technical progress. The rate of development, $\partial \log \sigma_i$, depends on the accumulation of capital and on the conditions that allow this process: existence of a qualified work force, entrepreneurship, institutional framework, etc.

Given a potential outcome based on the full utilization of the resources with which the economy is better endowed (work force and natural resources) we can consider that, *caeteris paribus*, $\partial \log O^p_i$, equals the growth rate of the work force plus the rate of technical progress. Both rates are considered as exogenously given. This is an extreme simplification, considering that technical progress can be endogenous and that the economically active population can fluctuate during the development process.

III. Functional Form of the Model

Now consider an economy where each sector is related to each other sector. Assume, moreover, that there exist external economies causing the increase in the production of one sector as a response to the increase in the actual production of the rest of the economy. Such externalities are consequences of technical, juridical, and other institutional causes. On the other hand, expectations about the future performance of other sectors are important sources of these externalities as well. In the rest of this paper only real effects will be considered, although all production variables will be expressed in terms of their values.

The existence of intra-sector effects is assumed as well: the production of each sector influences its own future growth. If the potential output of a sector i grows at a rate τ_i (which means that technical progress in the sector as well as the growth of the work force allocated to the sector may differ among sectors) we have the following relation:

- $\partial \log \sigma = \partial \log O^{a}_{i} - \tau_i$

Production plans of each sector i react with delay to the demand of its products:

- $O_i^{at} = F_i (D_i^{t-1})$

Variations of demand of a sector i are a result of variations in the production of the other sectors j, weighted up by the requirement of products of i in the production of j (k_{ij}) :

- $D_i^t - D_i^{t-1} = \sum_{i \neq j} k_{ij} \; (O_j^{at} - O_j^{a(t-1)}) + H_i^t$

where H_i is a component of exogenous variation.

The functions $\{F_i\}$ are the key elements in this model. We will make two strong assumptions about these response functions. Namely, that they are functionally identical for all sectors in the economy, i.e. that all the F_is coincide, except for the values of their functional constants. This means that -as the responses are generated by expectations and productive capabilities- all sectors respond in a similar fashion to the demands of the rest of the economy. This, in a certain sense, indicates that the technical constraints as well the expectation-formation processes in the entire economy are rather similar. Furthermore, we will assume that inventories are not accumulated since the accurate short-range forecasts of economic performance allow the sectors to meet the needs of their buyers.

The second strong assumption about the response functions is that the underlying costs are not convex when aggregate demands are small. This may obey to indivisibilities which at large demands become smoothed out. As a consequence, in low ranges, responses will be non-linear. At larger demands, instead, the smoothing out of indivisibilities induces monotone responses.

In order to represent these characteristics we have to introduce a functional schema that represents the assumed properties of the F_is. We choose to give the generic function F_i the following characterization, exhibiting the features discussed above:

$$F_i(x) = \begin{cases} O_{pi} & \text{if } x > O_{pi} / \alpha_i \\ \alpha_i . x & \text{if } x \in [\ \delta_i . O_{pi} / \alpha_i\ ,\ O_{pi} / \alpha_i] \\ \beta . \sin(x) + \mu & \text{otherwise} \end{cases}$$

δ_i is a threshold value defining the limit between the regions of non-concave and concave relations. α_i is a factor of "sustained growth", it determines how fast will the sector grow once it surpassed the initial stage of low demand of its products. Finally, β and μ are arbitrary parameters that define the amplitude and a baseline for the initial non-linearity.

This model, a system of coupled partial difference equations (**[Brock 1991]**), cannot be handled analytically. Therefore it has to be simulated by giving values to its parameters, obtaining the desired trajectories through a very simple numerical integration. As usual with simulation models, genericity cannot be ascertained: experiments can be performed for a finite number of possible values of the parameters. Despite this limitation, the usefulness of working with toy models as ours is obvious **[Ruelle 1988]**.

We have to remark here the strong analogies between our model and the spin-glasses model of statistical mechanics. The later model was proposed to provide a framework for the analysis of the propagation of qualitative changes in disordered media (e.g. magnetization or percolation). The idea is that in such systems, heterogeneous substructures coexist interacting in a mixture of conflict and cooperation, yielding global outcomes out of equilibrium. This is highly reminiscent of what happens with the phenomenon of development: the concurrent forces of complementarities among sectors, increasing returns and competition lead to similar outcomes as those of spin-glasses.[1]

IV. Behavior of the System

For the model given before we choose to fix the values of the following parameters and variables (the data for t = -2 and t = -1 are just border conditions for the simulation):

- n : the number of sectors
- $\{\alpha_i\}_i, \beta, \{\delta_i\}_i, \{\tau_i\}_i, \mu$
- $\{k_{ij}\}_{i,j} = 1..n$
- $O_{Pi}^{0}, D_i^{-2}, O_{Ai}^{-2}, O_{Ai}^{-1}, H_i^{-1}$

[1] See Chapter 5 of **[Ray 1998]**.

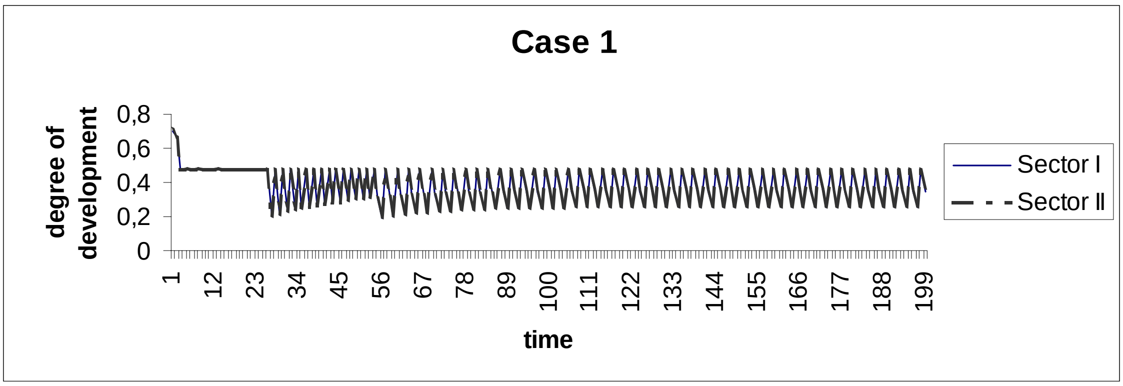
With these data at hand the evolution of $\{\sigma_i^t\}_{i=1..5}$ can be followed for t = 0..T.

Elsewhere (**[London-Tohmé 1998]**) we presented the results of running several simulations. We obtained interesting intuitions about the properties of the system proposed here. A remarkable property is that the results are independent of the number of sectors considered. This allows us to concentrate on the case with two sectors. Moreover, a greater number of cases call for the imposition of certain uniformity among sectors to facilitate the analysis. With two sectors this is no longer necessary. Therefore, our selection of values is as follows:

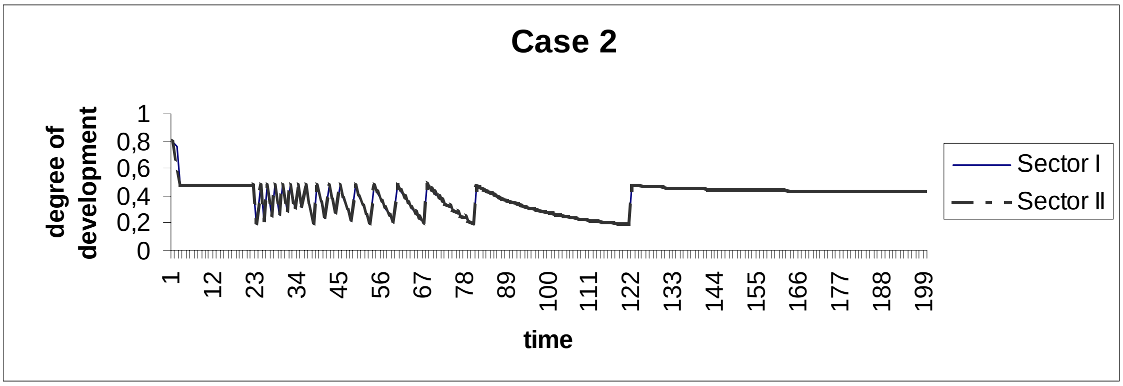
- n = 2 (sectors will be called i and j)
- $\delta_i = \delta_j$
- β = random in [0,1] (kept constant for each run)
- $\mu = 0.5. \max [O_{Pi}, O_{Pj}]$
- $||k_{ij}||$: matrix of numbers in [0,1], normalized by rows and columns (the total of weights of demands to sector i as well as i's total weights of demands to other sectors must sum up 1. The same is true for j.).

The second, third and fourth constraints indicate that the nonlinear part of the response functions is not only similar for both sectors, i and j, but also that its shape and size is proportional. The last constraint implies that $k_{ij} = k_{ji}$. Under these constraints we can find families of interesting analogous behaviors. The more remarkable cases we found are the following:

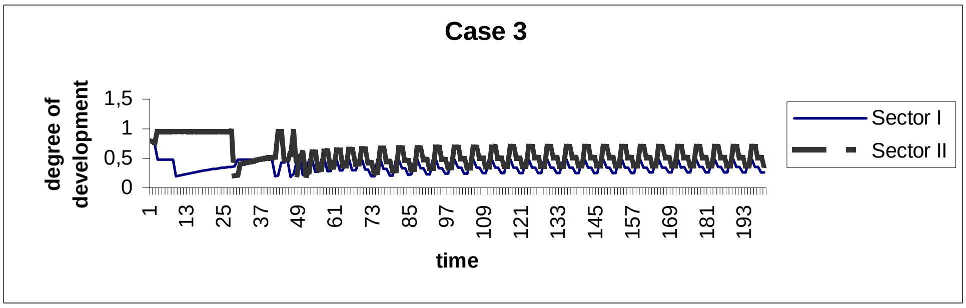
1. $O_{Ai}^{-1} = O_{Ai}^{-2} = O_{Aj}^{-1} = O_{Aj}^{-2} = 70$; $D_i^{-2} = D_j^{-2} = 70$; $O_{Pi}^{-2} = O_{Pj}^{-2} = 100$; $H_i^{-1} = 0$; $\alpha_i = \alpha_j = 1.1$; $\tau_i = \tau_j = 0.05$



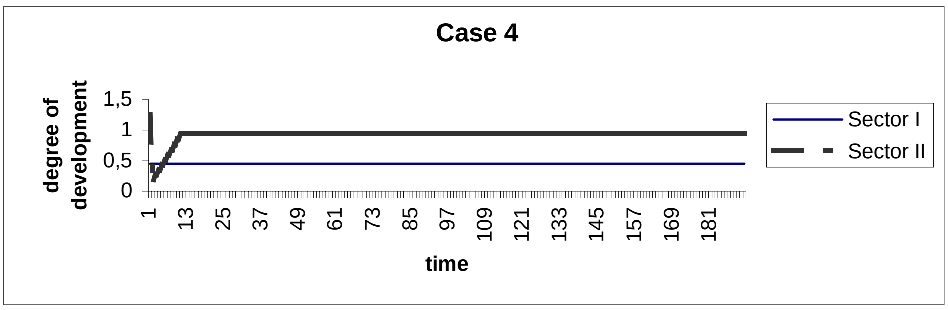
2. $O_{Ai}^{-1} = O_{Ai}^{-2} = O_{Aj}^{-1} = O_{Aj}^{-2} = 80$; $D_i^{-2} = D_j^{-2} = 80$; $O_{Pi}^{-2} = O_{Pj}^{-2} = 100$; $H_i^{-1} = 1$; $\alpha_i = \alpha_j = 1.5$; $\tau_i = \tau_j = 0.05$



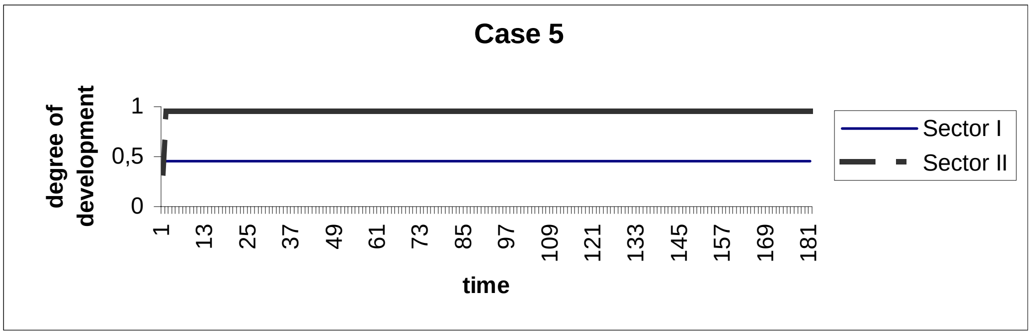
3. $O_{Ai}^{-1} = O_{Ai}^{-2} = 80$; $O_{Aj}^{-1} = O_{Aj}^{-2} = 40$; $D_i^{-2} = 70$; $D_j^{-2} = 20$; $O_{Pi}^{-2} = 100$; $O_{Pj}^{-2} = 50$; $H_i^{-1} = 0$; $\alpha_i = 1.5$; $\alpha_j = 1.1$; $\tau_i = \tau_j = 0.05$



4. $O_{Ai}^{-1} = O_{Ai}^{-2} = O_{Aj}^{-1} = O_{Aj}^{-2} = 70$; $D_i^{-2} = D_j^{-2} = 70$; $O_{Pi}^{-2} = O_{Pj}^{-2} = 100$; $H_i^{-1} = 0$; $\alpha_i = \alpha_j = 1.1$; $\tau_i = 0.1$; $\tau_j = 0.05$

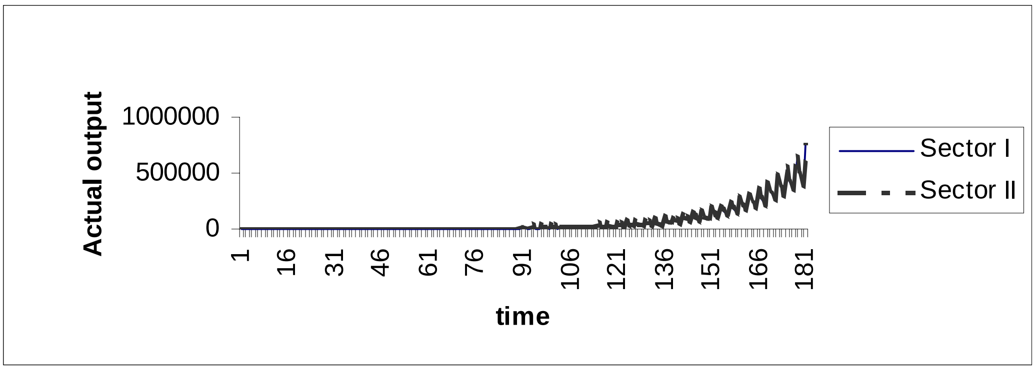


5. $O_{Ai}^{-1} = O_{Ai}^{-2} = 80$; $O_{Aj}^{-1} = O_{Aj}^{-2} = 40$; $D_i^{-2} = 70$; $D_j^{-2} = 20$; $O_{Pi}^{-2} = 100$; $O_{Pj}^{-2} = 50$; $H_i^{-1} = 1$; $\alpha_i = 1.5$; $\alpha_j = 1.1$; $\tau_i = 0.1$; $\tau_j = 0.05$

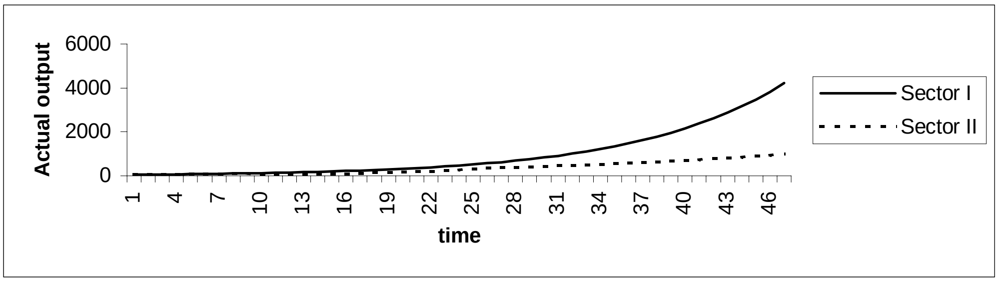


Case 1 shows that when we assume full homogeneity among the sectors, in absence of shocks, the degree of development oscillates around a middle value (less than 0.5). Case 2 shows that an initially higher degree of development and a higher factor of sustained growth (α) common to both sectors, plus an initial shock leads, after a transient, to a steady state, approximately the average of the extremes reached by the oscillation of Case 1. In Case 3, the oscillation reappears, when disparities between the sectors are present. That is, if the degree of development is the same but due to different amounts of output and the factors of sustained growth differ, the degree of development of both sectors oscillates around the same middle value as that of the previous cases. Notice, also, that in these three cases the interaction is harmful for development.

An explanation of why this happens is straightforward: a high amount of uniformity among the sectors dampens the generation dampens the demands (and therefore the steady increase of actual outputs), while the exogenous growth of the potential output makes it independent from the actual performance of the sectors. In fact, the corresponding time path for the actual output in Case 1 shows that it grows with oscillations:



Cases 4 and 5 show, instead, the existence of permanent disequilibria. If both sectors differ at least in their potential growth rates, one of the sectors reaches the highest degree of development (the one with the least rate of growth of the potential output), while the other stays at a medium degree of development. Again, this discrepancy is due to the fact that the potential output grows independently of the actual output. In fact, the sector with higher degree of development is not the one that grows faster:

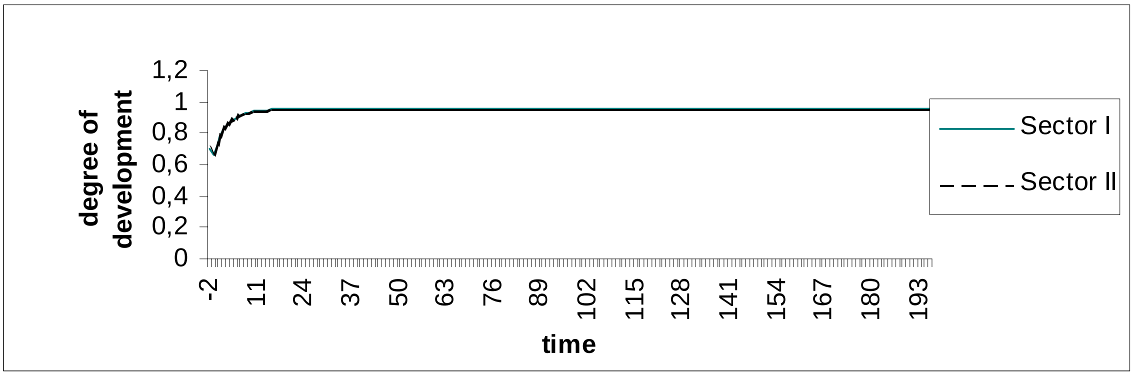


V. Discussion

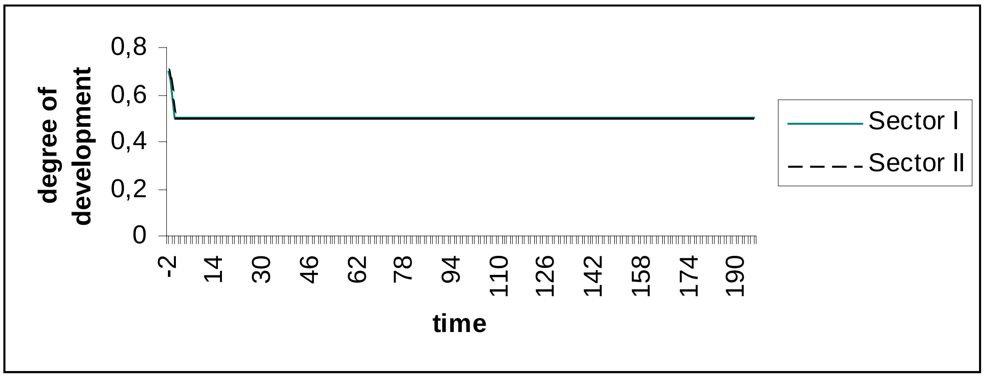
Analytical methods developed in Open Systems Theory can be useful as modeling tools in economics, allowing to incorporate in an elegant way the notion of economic equilibrium. This is most remarkable in the case of economic development. In the process an economy undergoes it may show notorious disparities among its sectors. Sometimes the lead in development is taken by one in detriment of the others, becoming the top sector in the economy. But another possibility is that the competition among sectors is so tough that neither one gets the lead, being a source of permanent instability. Another case, also possible, is that all the sectors attain the same degree of development, not the highest, but just a mediocre level shared by all of them.

As already said, there are numerous features in common in economies and physical systems with a great number of components. The analogy works better in the case of open systems, with their ever changing and unstable behaviors. Of course, substantial differences exist and the analogy should not be pushed beyond certain limits. The generality of O.S.T. makes it easier for the economist (as it has been for other scientists) to apply its tools to the analysis of phenomena not directly related to thermodynamics.

In this paper we took the idea of applying a well-known model of O.S.T., the spin-glasses (or Ising) model, to the analysis of economic development. As usual with spin-glasses, two classes of solutions arise: permanent oscillations and disequilibria. The first case is shown dramatically in Case 1, where twin sectors oscillate permanently. On the other hand, in certain circumstances, illustrated by Case 2, an external shock may eliminate the oscillation and forcing the system into a steady state. These two cases show good approximations for the pattern of development in which sectors not enough differentiated fight for the lead in development, being each one both a source of growth as well as a constraint to the other. This is because the growth of each sector increases the demand of the other while at the same time the rigidities of the reaction function may induce a *reduction* of the production. Only initial shocks may stabilize the situation, although the fact that the sectors are assumed similar makes them substitutes instead of complementary. Therefore they cannot attain the highest degree of development since there is no market for a huge expansion of their production. Curiously enough, in the situation of Case 1, even with much higher scope of the initial non-linear part ($\delta_i = 0.7$), a really strong initial demand shock ($H_i^{-1} = 70$) leads to a steady situation of high development. This is consistent with the hypothesis of the Big Push (**[Higgins 1959]**): once the economy surpasses a critical threshold it attains a high degree of development. The following graph illustrates this variant of Case 1:



The existence of disequilibria is shown in Cases 4 and 5. The first one exhibits clearly that initial symmetries may be broken by slightly different dynamic properties. On the other hand, in all these case, the disparities that arise (due to the existence of different growth rates for the respective potential outputs) have a direct impact not only on the degree of development (which is trivial) but also, through the action of the “production functions” F_i, on the actual output. But before blaming the exogeneity of the growth of the potential output for the results found here, we have to note that this is not that much a severe constraint. Maybe it is more much serious to assume that it will grow steadily. But, this is again an assumption often used in growth theory. On the other hand, even if we fix the potential output, the results may not be very different. Just consider what happens with Case 1 with a fixed potential output:



Perhaps more interesting to note is that the cases of disequilibrium are a good representation of what happened in many countries during several decades. A sector is highly developed while the rest just thrive. In fact, this was the core of the proposal of the so-called hypothesis of disequilibrium growth **[Hirschman 1975]**. Of course, it was assumed that in the long run the less favored sectors would finally benefit from the more advanced, getting also to a higher degree of development.

In general, we could say that the functional form of the F_is is the main suspect for many strange results reported here. But without doubt their real world counterparts have to exhibit some sort of non-linearity. Otherwise, the degree of development at any period t could be easily determined as a function of the degree of development at t=0. This violates the intuition that the phenomenon of economic development behaves in rather unstable fashion.

Of course, this is a very preliminary research aimed to the detection of the peculiar characteristics of an encompassing non-analytic formal model of economic development. Much deeper and exhaustive experiments have to be performed in order to settle (if possible) this question.

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