The Commodity Storage Model in the Presence of Stockholding by Speculators and Processors

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Abstract

The paper works with an alternative version of the rational expectations commodity storage model, where both speculators and processing firms are stockholders. The model identifies the stocks carried by commodity processors with those stocks described by the supply of storage model, but instead of using the convenience yield approach, processors' demand for stocks is derived from a model for manufacturing inventories (Ramey, 1989). Speculators intervene in the model through enforcing the arbitrage condition. We solve the model numerically to compare the different policy functions implied by each model (speculators, processors and both agents.) Finally, we present estimates of the model based on the same commodity price data used by Deaton and Laroque.

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I. Introduction

The paper works with a version of the rational expectations commodity storage model literature where both speculators (speculative stocks) and processing firms (working or pipeline stocks) are stockholders simultaneously. Recent estimates of the model have focused on solving the storage model for each type of stockholder independently. This is, either for the case of speculative storage (see Deaton and Laroque, 1992, 1995 and 1996), or for the case of working or pipeline stocks, that we identify with the supply of storage model [1] (Miranda and Glauber, 1993 and Miranda and Rui, 1996). Almost no effort has been undertaken to model the interaction of both agents. Two notable exceptions are Weymar, 1968 and Lowry, 1988 works. However, while Weymar specifies similar functions for both agents, Lowry following Brennan, 1958, models merchants instead of processors, as the one who carry stocks under backwardation, this, when the price spread is below to the storage costs.)[2] The interaction of both agents, but modeled separately, is important to model more accurately commodity markets and to represent the relation between stocks and price spreads (i.e., the Working curve, see Working, 1933).

[1] Carter and Revoredo, 2000, we show that the supply of storage cost approach may reflect only the stocks carried by processing firms. This result is obtained when inventories of commodities (raw materials) are modeled as factors of production following Ramey, 1989.
[2] Since both, merchants and speculators, buy and sell the raw material, their difference in the model is that the former perceive convenience yield for carrying the stocks while not

We model explicitly processors storage using a model of manufacturing inventories, instead of using the convenience yield explanation. The advantage of using an explicit structural model for manufacturing inventories is that it allows us to understand what variables are behind the parameters of the so-called supply of storage equation.

The model is solved numerically to compare the different policy functions implied by each model (speculators, processors, and both agents). The model that allows the interaction of both agents combines higher autocorrelation in the simulated price series (since processors always carry inventories) with a non-linear price function, since it imposes the no-negativity of speculative stocks.

Finally, we present estimates of the different versions of the storage model using the same price data used by Deaton and Laroque (1992). The results show the convenience of considering the phenomenon of storage under backwardation in the model, but not of enforcing the arbitrage condition to explain aggregate price data dynamics. However, the arbitrage condition seems appropriate when working with market level data.

II. The model

The existing economics literature considers two main versions of the competitive storage model, each with a different storage cost function. [3] The first model is the Deaton

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the latter. This makes important to explain the origin of the convenience yield, a point that is not addressed by Lowry.

[3] We are only focussing only on those models that imply a non-linear price policy function through imposing the no-negativity constraints of stocks. The reason for this choice is that these models seem to capture better the movements in commodity prices

and Laroque model (1992, 1995, 1996) based on competitive or speculative storage (see Samuelson, 1971; Williams and Wright, 1991). The model is presented in (1):

$$
(1)\qquad
\begin{aligned}
&\frac{1-\gamma}{1+r}E\left[p_{c,t+1}\right]-p_{c,t}=0,\ \text{when } S_t \geq 0\\
&\frac{1-\gamma}{1+r}E\left[p_{c,t+1}\right]-p_{c,t}<0,\ \text{when } S_t = 0\\
&h_t + S_{t-1} = D_t\\
&D_t = C_t + S_t\\
&p_{c,t} = P_c(C_t)
\end{aligned}
$$

Where $p_{c,t}$ is the commodity price in period t, $E[p_{c,t+1}]$ is the conditional expectation of price $p_{c,t+1}$ at period t, S_t is the carryover from period t to t+1, r is the interest rate, h_t is the production in period t (also called “harvest”, see Williams and Wright, 1991), C_t is the consumption of the commodity in the current period, D_t is the total market demand for the commodity, equal to current consumption plus carryover to the next period), γ is the shrinkage coefficient (see Deaton and Laroque, 1992) and P_c (.) is the inverse consumption demand function.

An alternative version of the storage model is found in Miranda and Glauber (1993), and Miranda and Rui (1996). These models use a storage cost function based on the supply of storage (see Working, 1949, Brennan, 1958, and Telser, 1958). The rationality behind the supply of storage is to incorporate the phenomenon of storage under backwardation, a stylized fact extensively studied by Holbrook Working during the 1930s. The Miranda and Rui (1996) model is presented in (2), where the storage cost is given by the function $\theta_0+\theta_1\ln(I_t)$, I_t is the carryover stock, and the other variables have already been defined.

(see Deaton and Laroque, 1992). For an evaluation of other rational expectations commodity models see Gilbert, 1990.

$$
(2) \qquad
\begin{aligned}
&\frac{1}{1+r}E\left[p_{c,t+1}\right]-p_{c,t}=\theta_0+\theta_1\ln(I_t)\\
&h_t+I_{t-1}=D_t\\
&D_t=C_t+I_t\\
&p_{c,t}=P_c(C_t)
\end{aligned}
$$

Two are the main differences implied by models (1) and (2). First, model (2) implies the impossibility of stockouts (i.e, allows for storage under backwardation) and therefore, in the model prices of successive periods are interconnected. This feature is not allowed in model (1). Second, model (1) eliminates arbitrage opportunities through imposing the arbitrage condition when the price spread is above the storage costs while model (2) does not. While it is possible to speculate about the reasons to model in either way a commodity market, it is better to let the stylized facts to guide the modeling.

The main stylized fact for the commodity storage model (and the origin of the commodity storage model) is the relation between price spreads and stocks, originally drawn by Holbrook Working in 1933. The same empirical relationship has been found in other commodity markets (see for instance Gray and Peck, 1981 for wheat; and Gardner and Lopez, 1996 for soybeans). Figure 1 plots the relation of price spreads and stocks for wheat using Holbrook Working's data.

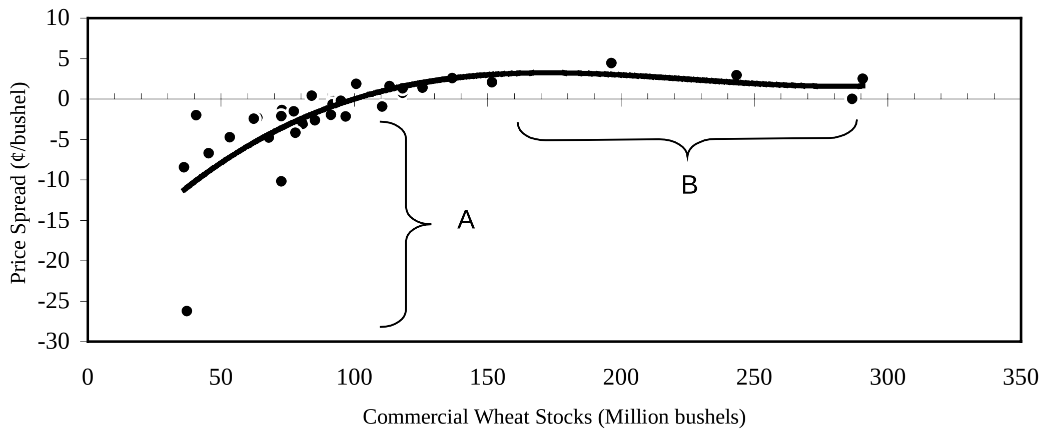


Figure 1: September-July Wheat Price Spread and July 1st Total U.S. Commercial Stocks 1896-1932

Source: Table VI, Working, 1933.
***Note* : The regression line has been generated fitting a cubic polynomial.**

There are two interesting facts shown in figure 1 worth noting. First, the portion denoted as A indicates the presence of storage under backwardation, and favors the use of model (1), or any other model that predicts storage under backwardation. Second, the observed relation between the price spread and stocks is not increasing when the spread is more positive (portion B in figure 5). Instead, the relationship is a flat line, indicating the effect of competitive forces driving extraordinary profits to zero. This portion of the curve is evidence that favors the introduction of the arbitrage constraint.

The evidence presented in figure 1, allows us to consider a third type of model that incorporates models (1) and (2) in just one model, since sections A and B of figure 1 suggest the use of a combination of the speculative storage model and the supply of storage model. Instead of using the supply of storage cost, we model those inventories explicitly using a model for inventories carried by commodity processors. Two reasons motivate this choice: first, the criticism to the supply of storage model for not considering a microeconomic explanation about the origin of the convenience yield term (see Deaton and Laroque, 1995 and Brennan, Williams and Wright, 1997). In fact, that empirical approach has consisted on regressing price spreads on commercial stocks. Such equation is a reduced form, and its parameters may be functions of policy variables such as storage fee, interest rate, prices of other factors of production, etc. This generates problems for using the estimated equation for commodity policy evaluation, which has been its main role.

The second reason, as pointed out in the literature (see Working, 1949), is that processing firms carry an important part of raw material stocks during the year as part of their production process, therefore their stocks represent an important part of the stocks observed while prices are in backwardation. The stocks carried by processors are well documented in the economic literature, (see Abramovitz, 1950). In Carter and Revoredo (2000), we show that a model for processors inventories can encompass a model based on the supply of storage cost and the convenience yield explanation, such as the model used by Miranda and Rui (1996). This result is derived from modeling raw material inventories as factors of production such as in Ramey (1989). The model that incorporates processors and speculators is presented in (3), and the derivation of processor's demand for inventories is given in the appendix.

$$
(3) \qquad
\begin{aligned}
&P_t^P = P(Q_t) \\
&Q_t = \exp\left\{ \beta_1 \left[\frac{P_t}{\beta_0} - \left\{ (1+r)(p_{c,t} + ko) - E[p_{c,t+1}] \right\} \right] \right\} \\
&I_t = \beta_0 Q_t \\
&\frac{1}{1+r} E(p_{c,t+1}) - P_c(A_t - S_t - I_t) = ko, \text{ if } S_t \geq 0 \\
&\frac{1}{1+r} E(p_{c,t+1}) - P_c(A_t - S_t - I_t) < ko, \text{ if } S_t = 0
\end{aligned}
$$

A_t is the current period availability (current production plus past carryover). P_t^P is the price of the processed good (say flour in the case of wheat). Processors' carryover (I_t) has been derived for a particular specification of the production function as shown in the appendix.[4]

III. Solution of the model

The solution of the model can be better understood by means of the diagram presented in figure 1. This figure resembles Deaton and Laroque's, 1992, figure 2, and allows us to compare all the policy functions in a schematic way.

[4] The functional form (quasi-fixed proportions production function) was selected because of its algebraic tractability, but the result is robust to the use of other functional forms, since those inventories are not more than a derived demand. Also in (3) we have chosen an specification that allows to encompass the Miranda and Rui , 1996 supply of storage equation.

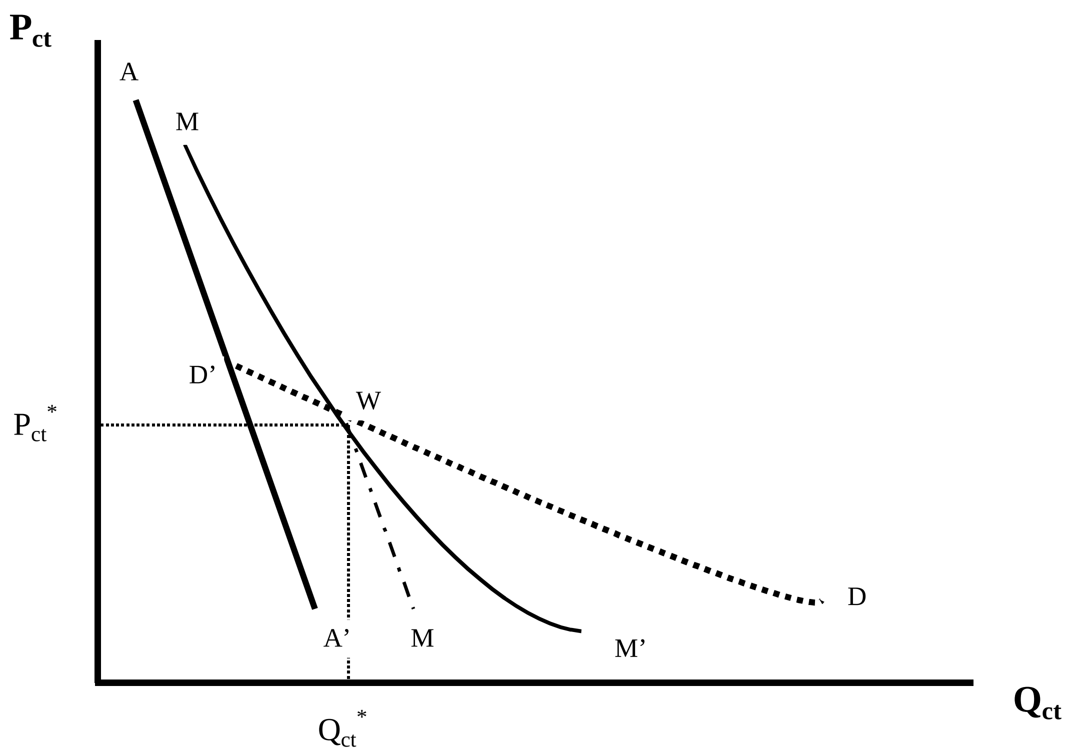


Figure 2. Policy Function of Different Commodity Market Models

The curve A-A’ represents the current consumption line (i.e., for exports or other current utilization of the commodity). The model used by Deaton and Laroque (1992, 1995, 1996) is represented by the curve A-D’-D. It represents the aggregate demand when there are no processors of the commodity in the market (e.g., sugar may be a good example since it is already a processed good, and its trade may allow for more presence of speculation). The aggregate demand implied by the Miranda and Glauber (1993), and Miranda and Rui (1996) models is represented by the curve M-M’, representing the case when there is no speculation in the raw commodity. When all the agents (processors and speculators) are considered together, the aggregate demand curve is given by M-W-D, where speculators start demanding commodity inventories when the availability is above to Q_{ct}^*. When the availability is below Q_{ct}^*, processors are the only ones who are carrying inventories (their inventories are measured by the horizontal distance in between A-A' and M-M'). Above Q_{ct}^{*} both processors and speculators are carrying inventories (processors' inventories are measured by the horizontal distance between A-A' and W-M; speculators inventories are given by the horizontal distance in between W-M and W-D). When the availability is above Q_{ct}^{*} processors are carrying the maximum amount of inventories they can demand given their storage capacity and/or the conditions in the processed good market, since the rental price for their inventories is zero. For the particular case of model (3), the maximum storage carried by processors is given by the solution of the system (4). Where p_{ct} is the price of the commodity consistent with the level of total inventories that eliminates all the arbitrage possibilities, therefore, it can be considered constant in the solution of (4), and the system be solved for P_t^P and I_t.

$$\begin{aligned} & P_t^P = P\left(\frac{I_t}{\beta_0}\right) \\ (4) \qquad & \\ & I_t = \beta_0 \exp\left\{\beta_1\left[\frac{P_t^P - p_{ct}}{\beta_0}\right]\right\} \end{aligned}$$

The key step in the algorithm is to discriminate between the existence of arbitrage possibilities. If there are arbitrage possibilities, then speculators are going to carry inventories to the next period and forcing prices (i.e., current and expected) to satisfy exactly the arbitrage condition. In that case the rental cost of inventories for the processing firms is equal to zero (i.e., $(1+r)(p_{c,t}+ko)-E\lfloor p_{c,t+1}\rfloor = 0$), and they reach their maximum storage according to the system (4).

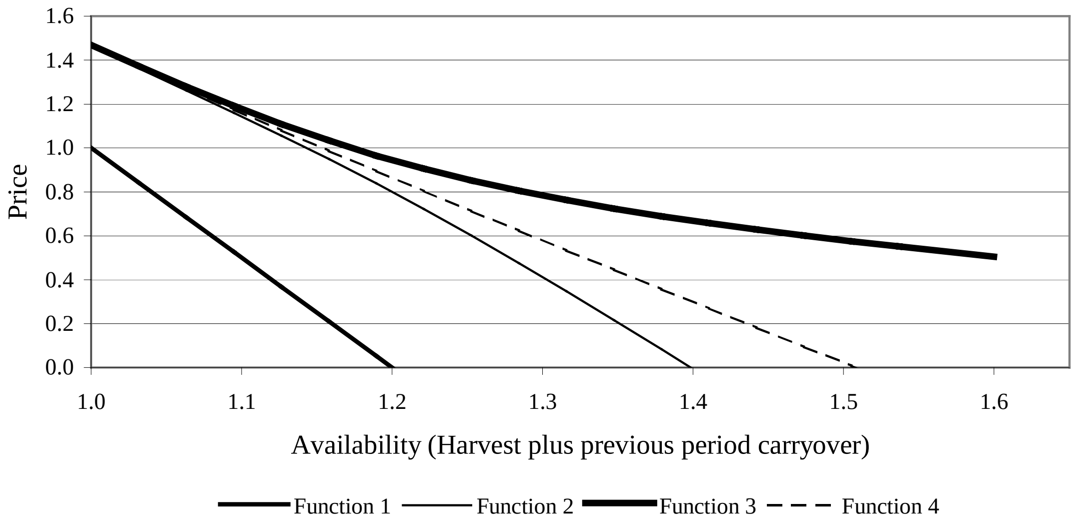
The algorithm starts assuming (given the parameters of the model and given the level of availability), that processors are the only ones that are carrying stocks in the market (i.e., solve the model without the presence of speculators). Once the level of working stocks is computed, the routine tests the presence of arbitrage possibilities. If there are no arbitrage possibilities, the total carryover in the market is equal to processor's carryover. On the other hand, if there are arbitrage possibility, the routine finds the level of total inventories that eliminates all the arbitrage possibilities, and the commodity price consistent with that level of inventories. Given the commodity price and the satisfaction of the arbitrage condition, the routine finds the corresponding level of processor's inventories by solving (4), and by residual, the level of speculative inventories. Figure 1.a in the appendix presents a detailed flowchart of the algorithm for the case of the price inelastic commodity supply.

The routine with a price responsive commodity production (i.e., harvest) follows Williams and Wright's (1991) algorithm. It requires the creation of a vector of planned commodity production, which is improved upon in a special loop after the equilibrium storage (for speculators and processors) has been found for all states of nature. It should be noted that for the case of multiplicative disturbances, the supply depends on the price incentive function (i.e., the expected marginal revenue) instead of expected commodity prices (see, Wright, 1978). The algorithm for the elastic supply case is presented in figure 1.b in the appendix.

IV. Model simulation

Figure 3 presents the simulated policy function for model (3), and it corresponds to the thick line shown in figure 3.

Figure 3: Market Demand Functions



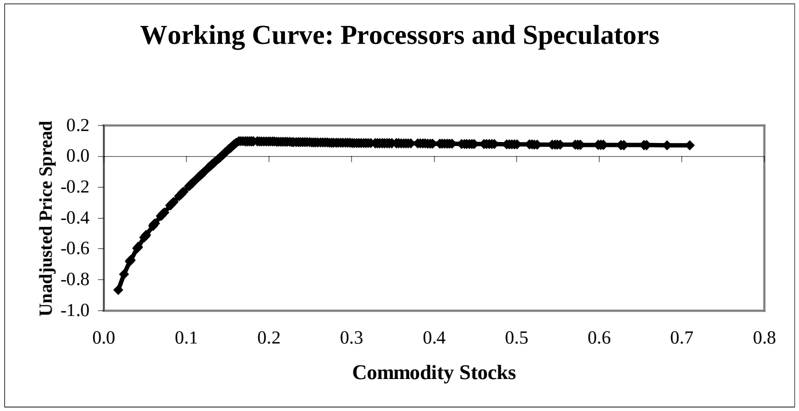
Notes:
Function 1 = Expected price without carryover
Function 2 = Expected price considering only processors' stocks (but from solution including speculators).
Function 3 = Expected price when only processors carry stocks.
Function 4 = Expected price when processors and speculators carry stocks.

It is clear from the commodity price literature (Deaton and Laroque, 1992 and Miranda and Rui, 1996) that a model like (3), preserves many characteristics of the other two models (supply of storage and speculative commodity storage model). The no-negativity constraint of speculative stocks generates a non-linear response in prices that seem to be an important market characteristic for commodity prices (see, Gilbert, 1990). Also, the model predicts higher autocorrelation (i.e., such as in the supply of storage model) since total stockout never occurs because processors are carrying stocks in all periods. On the other hand, speculators carry the commodity when they expect capital gains.

In addition, it is worthwhile to generate for comparison purposes (see figures 4a to 4c) the Working curve (graph that relates price spreads to stocks) implied by each model solved independently (i.e., speculators only, processors only, and both stockholders). The differences among each of the generated Working curve are striking. The curve that considers only processors' storage (4b) presents an increasing pattern over the whole range of stocks. On the other hand, the curve considering only speculators' storage (4c) does not allow for the existence of storage under backwardation, and implies a flat line for positive values of the inter-temporal price spread.[5] Finally the curve that consider both types of stockholders (4a) combines both types of pattern (storage under backwardation and a flat portion for positive values of the spread).

Figure 4d repeats figure 1 to ease the comparison. We have over imposed a cubic regression line, instead of the probable quadratic or logarithmic curve used originally by Holbrook Working (unfortunately not reported). The Working curve was the empirical basis for his supply of storage theory (see Working, 1949).

[5] In order to make the models comparable, I have substituted in the original shrinkage coefficient used by Deaton and Laroque (1992), which results in a decreasing marginal storage cost, for a constant storage cost. That explains the flat portion shown by the Working curve for the region of positive price spreads.



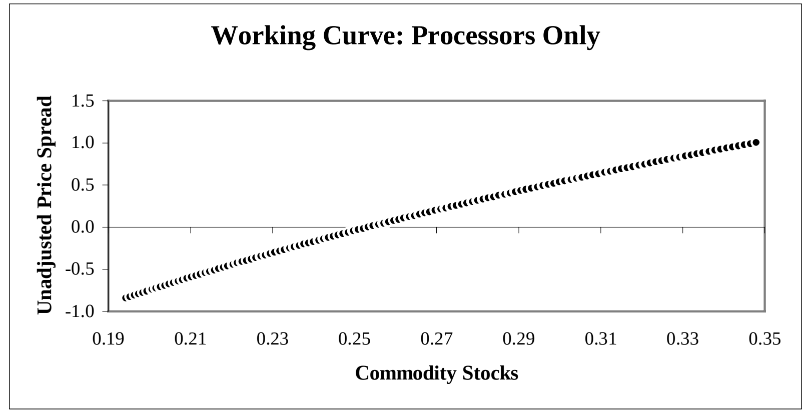
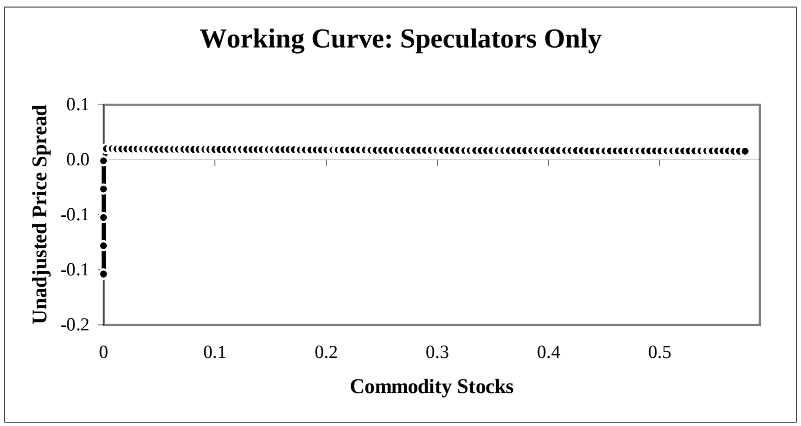


Figure 4.a

Figure 4.b



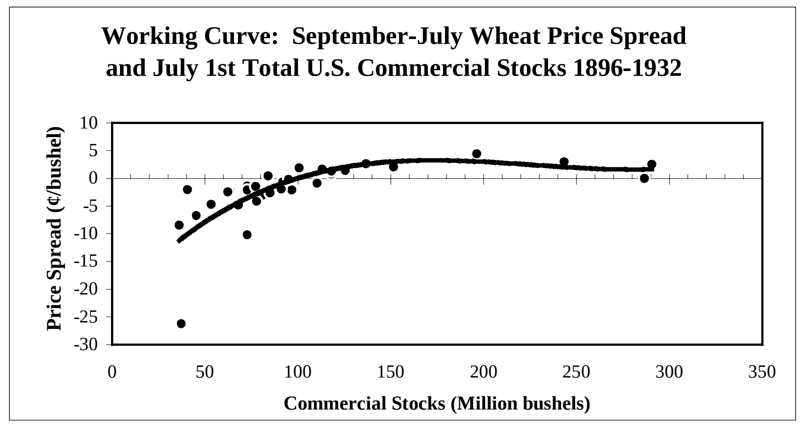


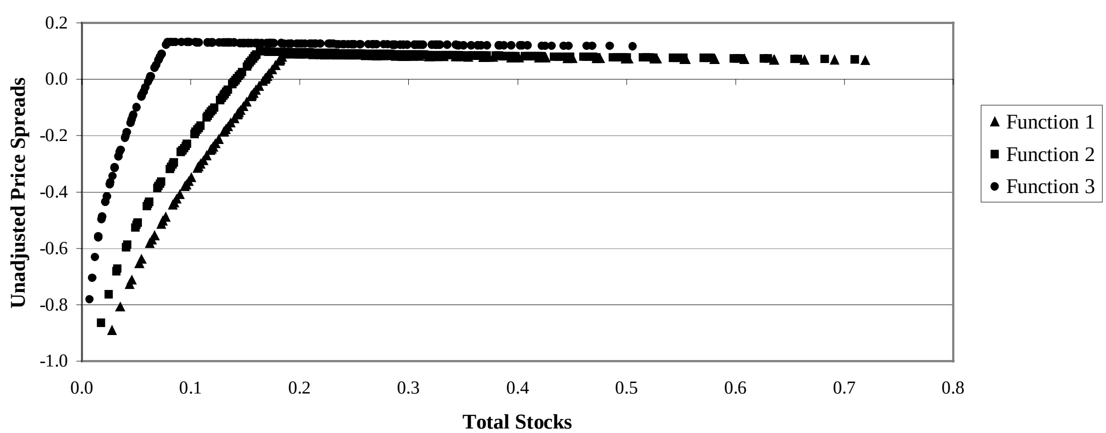
Figure 4.c

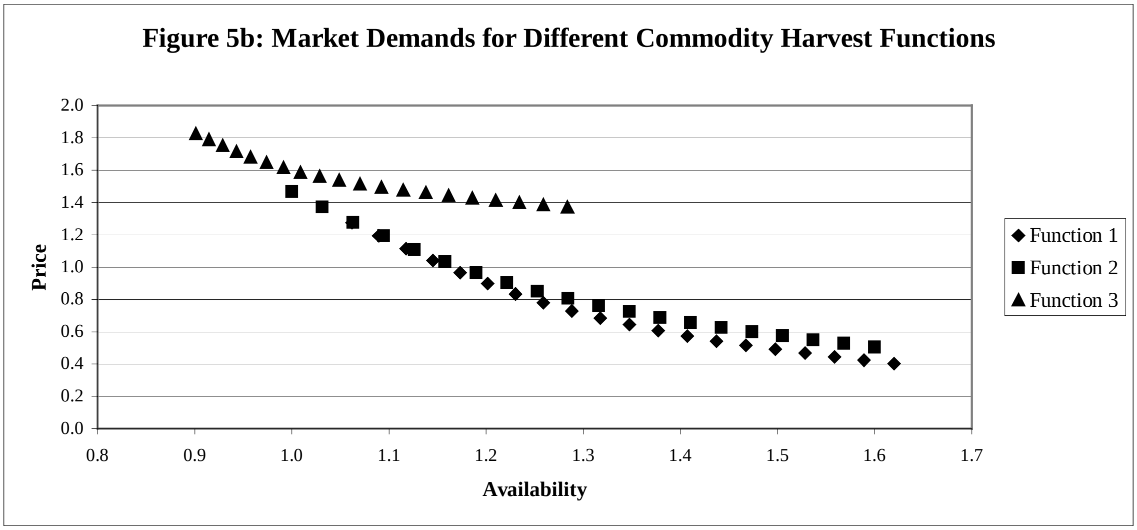
Figure 4.d

Source : Figure 4.a to 4.c were computed using the following parameters (d_{12} =6, d_{22} =-5, λ =00 9, w=02 , d_{11} =2, d_{21} =-0.5). Figure 4.d is based on Table VI, Working, 1933. The re gression line in figure 4.d was generated by fitting a cubic polynomial.

Figures 5a and 5b answer the question of how important for the resulting policy function is to assume a price responsive harvest or not. This question is interesting since recent empirical models aimed to explain price distributions have assumed that the commodity production (i.e., harvest) is inelastic (see Deaton and Laroque, 1992; Miranda and Rui, 1996; Ng, 1996). In figure 5a and 5b we have simulated the Working curve and the market demand for three different harvest specifications. Two price responsive harvests (with and without intercept) and a price inelastic harvest. As observed in the figures the assumption about the functional form of the harvest and production is not trivial since it implies important changes in the resulting market demand functions.

Figure 5a: Working Curve for Different Commodity Harvest Functions





Assumptions:

Function 1: $h_t=1+0.05\ P^R_t$
Function 2: $h_t=1$
Function 3: $h_t=0.5P^R_t$

V. Econometric Estimates

Table 1 compares estimates of four versions of the commodity storage model. All the models have been estimated using the pseudo maximum likelihood estimator described by Deaton and Laroque (1995). Even if prices in the presence of speculators are not normally distributed, the estimator is consistent.

In all the cases the harvest has been assumed to be i.i.d. normally distributed with mean zero and variance one, and approximated by the same discrete distribution used by Deaton and Laroque (1995). Furthermore, in all the case we have assumed an interest rate equal to 5 percent and the consumption functions linear with parameters (a, b)

Model I, was taken directly from Deaton and Laroque (1995) and corresponds to the commodity storage model where the cost of storage is given by the interest rate and the shrinkage coefficient (γ). Model II corresponds to Miranda and Rui (1996) model, which was re-estimated using the original data (instead of using prices deflated by the historical means as in Miranda and Rui). The parameters of the supply of storage costs are (θ_1 and θ_2). Model III, is a variation of model I, that substitute the shrinkage coefficient for a fixed storage cost, ko, (such as in Williams and Wright, 1991). Model IV, with processors and speculators, includes the parameters of models II and III.

The econometric results, comparing the pseudo likelihood values of models II and IV with models I and III, confirm the importance of including a component that takes into account the phenomenon of storage under backwardation. It improves the commodity storage model performance, as pointed out by Miranda and Rui, 1996. On the other hand, comparison of models I and III makes clear that the shrinkage coefficient imposes a too high cost compare with the fixed storage cost, reducing the stocks carried by speculators and increasing the percentage of stockouts.

The comparison between models II and IV indicates that the enforcement of the arbitrage condition is not necessary to explain the dynamics of most of the commodity prices. The reason is in the nature of the data, aggregate average prices presents what may be considered non-existent arbitrage opportunities, this has already been proved in the literature (see Working, 1961 and Gilbert, 1981). Furthermore, the use of aggregate data may explain the poor results obtained with model I, since the model that assumes an homogeneous commodity.

Table 1 - Parameter estimates and pseudo likelihood (PLE) of different commodity storage models 1/

	Commodities											
	Cocoa	Coffee	Copper	Cotton	Jute	Maize	Palm Oil	Rice	Sugar	Tea	Tin	Wheat
Model I - Deaton and Laroque (1992, 1995)												
a	0.160	0.260	0.540	0.640	0.570	0.630	0.460	0.600	0.640	0.480	0.260	0.720
	0.010	0.020	0.040	0.040	0.030	0.040	0.050	0.030	0.050	0.020	0.040	0.040
b	-0.220	-0.160	-0.330	-0.310	-0.360	-0.640	-0.430	-0.340	-0.630	-0.210	-0.170	-0.390
	0.030	0.030	0.050	0.040	0.060	0.150	0.060	0.030	0.060	0.020	0.050	0.030
γ	0.120	0.140	0.070	0.170	0.100	0.060	0.060	0.150	0.180	0.120	0.150	0.130
	0.040	0.020	0.020	0.030	0.050	0.030	0.030	0.040	0.030	0.030	0.050	0.030
PLE	125.2	111.0	73.9	29.8	44.8	32.1	22.2	26.0	-10.7	69.3	108.9	24.6
Model II - Miranda and Rui (1996) 2/												
a	0.185	0.224	0.647	0.600	0.575	0.743	0.452	0.603	0.449	0.494	0.217	0.381
	0.033	0.025	0.048	0.097	0.049	0.134	0.139	0.071	0.117	0.029	0.030	0.047
b	-0.330	-0.217	-0.654	-0.939	-0.427	-1.413	-1.155	-0.611	-0.923	-0.244	-0.289	-0.697
	0.070	0.049	0.142	0.357	0.077	1.173	0.305	0.112	0.170	0.025	0.036	0.057
θ_1	-0.029	-0.018	-0.041	-0.113	-0.038	-0.073	-0.087	-0.165	5/	5/	-0.029	-0.011
	0.013	0.027	0.007	0.051	0.030	0.026	0.014	0.050			0.009	0.003
θ_2	0.023	0.025	0.028	0.058	0.051	0.043	0.047	0.122	0.013	0.022	0.019	0.002
	0.010	0.022	0.003	0.027	0.023	0.026	0.007	0.031	0.010	0.003	0.005	0.001
PLE	134.4	132.3	97.4	79.9	59.2	48.1	74.4	64.7	-2.1	78.3	160.5	33.6
Model III 3/												
a	0.142	0.255	0.552	0.681	0.634	0.690	0.595	0.768	0.545	0.510	0.397	0.757
	0.027	0.031	0.048	0.063	0.039	0.100	0.608	0.064	0.118	0.019	0.036	0.042
b	-0.234	-0.240	-0.376	-0.351	-0.348	-0.602	-0.686	-0.392	-0.758	-0.196	-0.430	-0.348
	0.036	0.052	0.066	0.051	0.053	0.107	0.245	0.077	0.116	0.023	0.116	0.032
ko	0.003	0.008	0.011	0.030	0.033	0.017	0.013	0.048	0.029	0.041	0.010	0.045
	0.002	0.003	0.004	0.008	0.012	0.008	0.036	0.022	0.016	0.010	0.003	0.007
PLE	132.5	132.1	92.3	46.3	52.9	37.7	54.7	36.1	-4.2	75.9	144.0	26.2
Model IV 4/												
a	0.152	0.222	0.463	0.758	0.575	0.743	0.478	0.523	0.469	0.519	0.212	0.629
	0.024	0.036	0.071	0.078	0.047	0.135	0.212	0.157	0.239	0.024	0.028	0.138
b	-0.249	-0.315	-0.594	-0.977	-0.429	-1.413	-1.017	-1.048	-0.923	-0.208	-0.262	-1.274
	0.037	0.072	0.135	0.250	0.083	1.197	0.471	0.312	0.158	0.020	0.030	0.277
θ_1	-0.011	-0.024	-0.153	-0.098	-0.039	-0.073	-0.076	-0.241	5/	5/	-0.028	-0.133
	0.001	0.020	0.033	0.020	0.031	0.026	0.027	0.148			0.009	0.017
θ_2	0.029	0.018	0.118	0.064	0.051	0.043	0.044	0.167	0.016	0.052	0.019	0.072
	0.004	0.010	0.024	0.016	0.022	0.026	0.009	0.096	0.040	0.004	0.005	0.000
ko	0.003	0.009	0.005	0.024	0.051	0.056	0.011	0.011	0.020	0.057	0.015	0.067
	0.002	0.007	0.005	0.009	0.158	0.127	0.004	0.026	0.019	0.024	0.025	0.016
PLE	135.8	135.4	101.4	77.3	59.2	48.1	73.9	64.7	-2.0	85.5	160.2	57.2

1/ All the models assume an interest rate equal to 5 percent. Standard errors are below each parameter.
2/ Miranda and Rui model estimated by pseudo maximum likelihood as in Deaton and Laroque (1995).
3/ Model with only arbitrageurs and substituting the shrinkage coefficient of Model I by a constant storage cost.
4/ Model with manufacturers, arbitrageurs and constant storage cost.
5/ Model without intercept.

The results of model IV are mixed. Despite of a possible problem of identification of the 5 parameters involved in the model [6] it is interesting to note that it seems to perform well for cocoa, coffee, cotton and wheat prices. However, the model is expected to perform better with market level data such as the one used in Working's empirical work.

Final Remarks

The purpose of this paper has been to extend of the rational expectations commodity storage model, to the case where both speculators and processing firms are stockholders. Instead of using the convenience yield explanation, we have model processors inventory demand using a model for manufacturing inventories. This allows to incorporate the phenomenon of storage under backwardation (obtaining a better description of the stylized facts observed in commodity markets) and present a structural model that can be used for policy purposes.

The model is numerically solved to simulate the policy functions implied by different versions of the commodity storage model. The combination of processors and speculators allows us to obtain a model that combines the two characteristics of the supply of storage and speculative storage models, which are a higher autocorrelation together with non-linear price response to change in quantities.

Econometric results using aggregate price data show the convenience of including a component that takes into account the phenomenon of storage under backwardation to explain commodity price dynamics. On the other hand, the enforcement of the arbitrage condition is not necessary to explain the dynamics of most of the commodity prices. The reason is probably in the aggregate nature of the data, even if it seems useful when working with market level data.

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[6] The possible identification problem can be solved expanding the model incorporating a

demand for the processed good and a responsive supply of the commodity

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Appendix

Derivation of processor's demand for inventories

Let us assume that the output (Q_t, measured as the shipments of the processed good, see Ramey, 1989) of a competitive processing industry is represented by a quasi fixed proportions production function (i.e., $Q_t = \min\{\frac{I_t}{\lambda}, f(K_t)\}$).[7] Where λ is a parameter of the production function (i.e., turnover inventory). K_t is a composite index of other factors of production, and f (.) is an increasing function that relates other factors with output. Under this assumption, risk neutral processors that maximizes the expected profits ($E[\pi]$) at period t solve the following problem:

$$(1) \qquad \underset{I_t, K_t}{\text{Max}}\ E[\pi] = E[P_t \min(\frac{I_t}{\lambda}, f(K_t)) - m_t I_t - w_t K_t]$$

Where P_t is the processed product price (i.e., P_t^P) net of the raw material price p_{ct}, m_t is the price of raw material inventories (this has been defined slightly different than in Ramey (1989), and is defined as $m_t = (1 + r)(p_{ct} + ko) - p_{ct+1}$). Where r is the interest rate, ko is the storage cost, w_t is the price of the composite factors of production. Cost minimization implies that $Q_t = \frac{I_t}{\lambda}$ and $Q_t = f(K_t)$. Then the expected cost function is given by:

$$(2) \qquad E[C(m_t, w_t, Q_t)] = E[m_t]\lambda Q_t + w_t f^{-1}(Q_t)$$

[7] The derivation does not necessarily require that specific production function. The quasi fixed proportions production functions was chosen because of its tractability and because it is bounded when the price of the factor of production is equal to zero.

Let us assume (for the purpose of obtaining the supply of storage function developed below) that $f^{-1}(Q_t) = Q_t(\ln(Q_t)-1)$. Thus, replacing the expected cost function into the profit function (1) and maximizing with respect to the output. We obtain expressions for the output (Q_t) and processors' raw material inventories of processors (I_t) that depends on the expected price for the commodity.

$$Q_t = \exp\left\{\frac{P_t - \lambda\{(1+r)(p_{ct} + ko) - E[p_{ct+1}]\}}{w_t}\right\}$$

(3)

$$I_t = \lambda \exp\left\{\frac{P_t - \lambda\{(1+r)(p_{ct} + ko) - E[p_{ct+1}]\}}{w_t}\right\}$$

Derivation of the supply of storage equation

Let us start from the storage cost function used by Glauber and Miranda, 1993, and Miranda and Rui, 1996. The function is written in (4) such as:

$$(4) \qquad \frac{E[p_{c,t+1}]}{(1+r)} - p_{c,t} = \theta_0 + \theta_1 \ln(I_t)$$

Equation (4) is a reduced form for particular parameters of the processors' demand for inventories (I_t), for a quasi fixed proportions production function. To show this, let us write the processors' demand for inventories (3) as in (5), and let us call $P_t^* = \dfrac{P_t}{\lambda}$.

$$(5) \qquad I_t = \lambda \exp\left[\frac{\lambda}{w_t}\left[P_t^* - \{(1+r)(p_{c,t} + ko) - E[p_{c,t+1}]\}\right]\right]$$

Let us simplify expression (4) by introducing parameters $ß_0$ and $ß_1$.

$$
(6) \qquad
\begin{aligned}
&\beta_0 = \lambda \\
&\beta_1 = \frac{\lambda}{w_t} \\
&I_t = \beta_0 \exp\left\{\beta_1 \left[P_t^* - \left\{(1+r)\left(p_{c,t} + ko\right) - E\left[p_{c,t+1}\right]\right\}\right]\right\}
\end{aligned}
$$

Taking natural logarithms to both sides and factoring terms we get (7)

$$
(7) \qquad \frac{E\left[p_{c,t+1}\right]}{(1+r)} - p_{c,t} = ko - \frac{P_t^*}{(1+r)} - \frac{1}{(1+r)\beta_1}\ln\left(\beta_0\right) + \frac{1}{(1+r)\beta_1}\ln\left(I_t\right)
$$

Re-writing (7) in terms of the parameters θ_0 and θ_1 we obtain the storage cost function (i.e., expression (4)). The values for the parameters θ_0 and θ_1 are given in (8):

$$
(8) \qquad
\begin{aligned}
&\theta_0 \equiv ko - \frac{P_t^*}{(1+r)} - \frac{1}{(1+r)\beta_1}\ln\left(\beta_0\right) \equiv ko - \theta_1\left(\frac{P_t}{w_t} + \ln\left(\beta_0\right)\right) \\
&\theta_1 \equiv \frac{1}{(1+r)\beta_1}
\end{aligned}
$$

Therefore, the commodity model presented by Miranda and Glauber, 1993, and Miranda and Rui, 1996, represents only the inventories carried by processors, excluding speculative stockholding. The requirements for considering θ_0 and θ_1 as parameters are: (1) the relative price of the processed good (net of the price of the raw material) with respect to the price of other factors of production must be constant; (2) the prices of other factors of production must be constant, and (3) the period considered in the model has be relatively short to preclude of technological change.

Figure 1.a – Flowchart for the Price Inelastic Supply Case

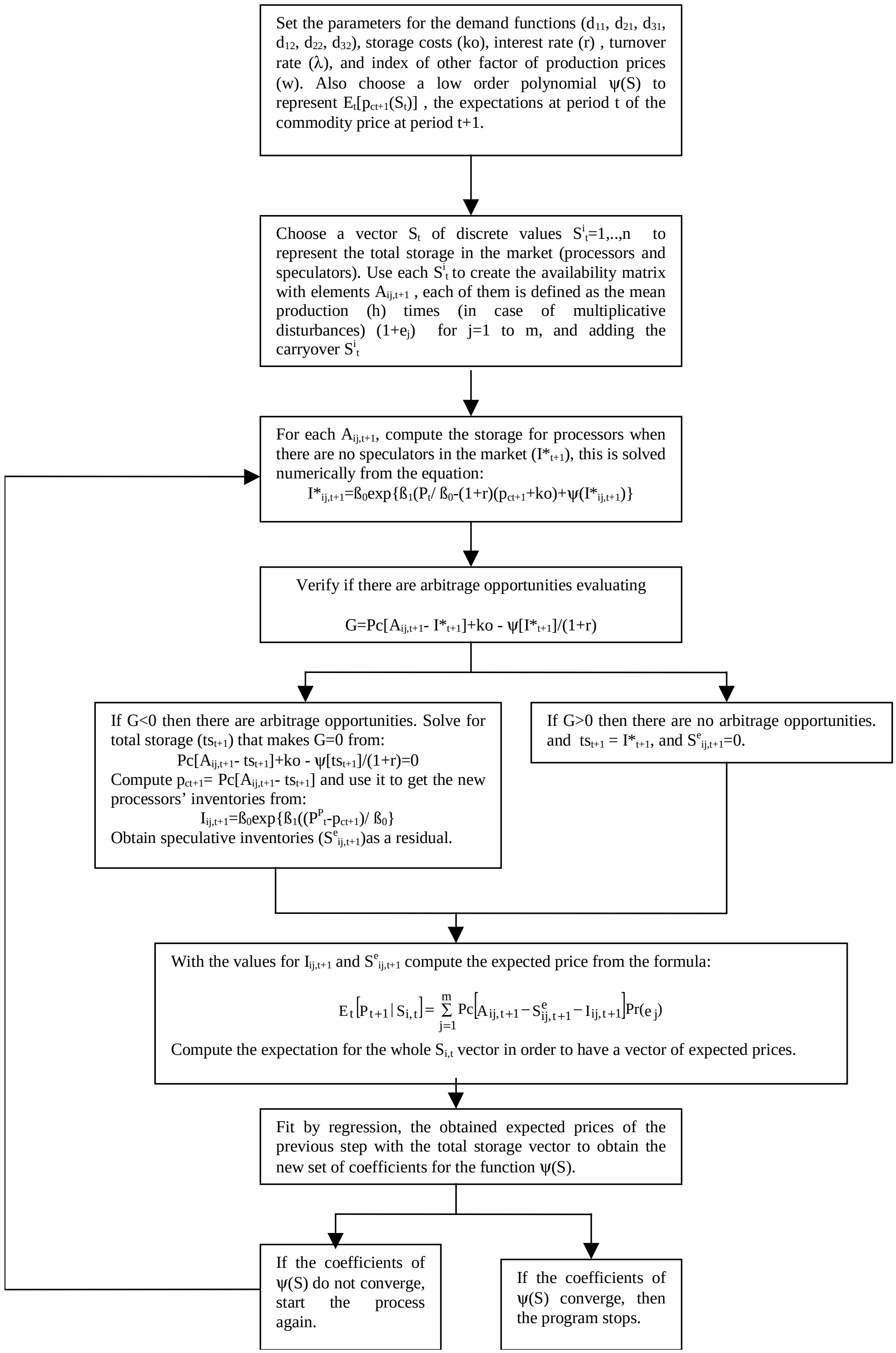

Figure 1.b – Flowchart for the Price Elastic Supply Case