The Economics of Cattle Supply

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Abstract

This paper builds a dynamic rational expectations model describing the supply of cattle. The theoretical model improves on existing models by allowing cow-calf operators to make period-by-period investment decisions on both the cow and calf margins, separates the markets for fed and unfed beef, and considers a rich set of exogenous shocks. The model is calibrated and used to simulate arti…cial data that replicates several empirical regularities associated with the cattle cycle.

JEL Codes: C61, Q11 and Q12.

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1 Introduction

Why study the cattle industry? The cattle industry, and agriculture in general, continue to make up a smaller and smaller portion of the total economic activity in the United States. In 1930, the cattle industry accounted for a little over 1% of national economic activity. In 1996, the percentage has plummeted to 0.07%, roughly a 94% decline in cattle's share of overall national output (United States Department of Agriculture, USDA). Nevertheless, there are several reasons why the economics of cattle supply is still an important area of research. First, agricultural issues, including those related to cattle, continue to receive a disproportionately large amount of attention from national policymakers. Issues such as price supports, subsidies, grazing fees on public lands, and international agricultural trade agreements continue to be debated frequently by policymakers. Second, the economies of many Western and Midwestern states, such as Montana, Kansas and Nebraska, are still strongly in‡uenced by agriculture and the cattle industry in particular, where cattle make up anywhere from 3% to 6% of total state product. Third, and …nally, the cattle industry presents some unique and interesting problems from a purely theoretical perspective. Cattle stocks are one of the few, if not only, economic time series to display regular cycles with such long periods. The well-known cycle in the stock of cattle displays amazing regularity with an average duration of approximately 10 years from trough to trough. Furthermore, cattle prices appear to follow a similar cyclic pattern (although much less pronounced than stocks). This cycle in prices raises an interesting economic puzzle. Since ranchers are presumably aware of the cycle in prices, there would appear to be opportunities to pro…t through countercyclical strategies.[1] However, if all ranchers attempted to capitalize on these pro…t opportunities, the incentives and the price cycle should dissipate. The fact that it has not dissipated is puzzling. In order to fully comprehend these and other issues, as well as, make policy recommendations, a thorough, well-articulated model of the cattle industry is required. This paper is meant to be a move in that direction.

A substantial amount of progress has already been made in understanding cattle supply. The seminal article in this area is Jarvis (1974). Jarvis modeled the microeconomics of cattle supply where each cattle producer maximizes a discounted stream of future pro…ts, treating cattle as capital goods. He showed, among other things, that animals of di¤erent age and sex will be treated di¤erently by the producer in response to shocks to the relative price of beef to feed. A particularly interesting result is the potential for an optimal negative short-run supply response by producers. That is, in response to a su¢ciently permanent increase in the relative price of beef, producers will reduce their supply of animals for slaughter, opting to instead retain females as capital goods to take advantage of anticipated higher future prices. As will be shown later, the model presented in this paper makes a prediction in stark contrast to that of Jarvis.

Since Jarvis' article, several other authors have examined extensions of the cattle supply

[1] Livestock and cattle associations have long been reporting the existence of a cycle in the prices of cattle in various publications. Examples include the Western Livestock Journal, Western Farmer Stockman and the National Cattlemens Beef Association.

model.[2] Rucker, Burt and LaFrance (1984) built on Jarvis' work and estimated a dynamic econometric model of cattle inventories. Although not framed within the context of cattle production, Zvi Eckstein (1984, 1985) built a dynamic, rational expectations model of optimal crop rotation, which provided an alternative paradigm to Nerlovian supply with adaptive expectations. Eckstein's work undoubtedly in‡uenced subsequent studies, such as this one, which build on rational expectations. Paarsch (1985) further extended Jarvis' work by modifying some behavioral assumptions and showing that the short-run supply response to an increase in the relative price of beef is instead positive when the rancher manages a succession of herds. Other articles include Trapp's (1986) investigation of optimal herd sizes; Nerlove and Fornari's (1995) model of cattle supply with quasi-rational expectations; Rosen's (1987) and Rosen, Murphy and Scheinkman's (1996) dynamic, rational expectations model of cattle cycles, Mundlak and Huang's (1996) comparison of international cattle cycles; and Marsh's (1999) examination of productivity's e¤ects on the cattle cycle.

This paper clearly builds on the aforementioned work of Rosen, Murphy and Scheinkman (1996), RMS hereafter. Their article was a major contribution to the research on cattle supply and cattle cycles. They show that regular cycles in the stock of cattle emerge as a prediction to a competitive environment where rational, pro…t-maximizing ranchers make economic decisions in an dynamic environment with uncertainty. Based in part on their work, it now appears to be fairly well accepted that the cattle cycle is the result of producers' responses to shocks in their environment, coupled with lengthy biological lags. However, the RMS model is lacking some behavioral features that make some of their results di¢cult to interpret and use for policy recommendations. First, RMS assume that only two-year old adult animals are culled from the stock of cattle. In reality producers make the decision to cull both calves and adult cows. Moreover, the calves (once sent through the …nishing process) and adult cows are sent to essentially two di¤erent markets–one for fed beef and one for unfed beef.[3] Second, they abstract from other characteristics of the cattle supply problem (for example they exclude both productivity and international trade) that are necessary to fully describe the nature of cattle dynamics.

It is interesting to note the resemblance of the model in this paper (and that of RMS) to the equilibrium business-cycle models in macroeconomics, introduced by Long and Plosser (1982) and Kydland and Prescott (1983). In the so-called real business cycle (RBC) models, all agents are assumed to have rational expectations and maximize their respective objective functions subject to various production and market constraints in a competitive, frictionless environment. The ability of …rms to optimally alter investment decisions and workers to alter leisure/labor decisions in response to stochastic changes in their environment is key to explaining the business cycle. The RBC model predicts that aggregate economic activity will ‡uctuates around a long-run steady-state level as workers and …rms respond to changes

[2] This section is not intended to be a comprehensive survey of the research on cattle supply. The research on cattle supply has been quite active in the last couple of decades, and as such, omission of other studies of the cattle industry should not be interpreted as minimizing their importance.

[3] Fed beef refers to meat from primarily young steers and heifers, which have completed a …nishing process (see Section 2.1 for more details on the …nishing process). Unfed beef refers to meat primarily from older cows and bulls, which is of lower quality and not suitable for …nishing.

in the intra and intertemporal rates of substitution between of capital and labor. The cattle supply model presented in this paper is similar. It is set in a competitive environment with no market frictions and fully rational decision makers. Although labor is not explicitly modeled in the cattle problem, optimal investment (or disinvestment) decisions in response to stochastic changes in the producers environment, coupled with biological and market constraints, generate cyclical activity around a long-run steady state. Moreover, the tools used to calculate the solution and test the theory against observation are similar to those in the RBC literature. Like most RBC models, the cattle supply model is …rst calibrated, then used to simulate arti…cial data, which is in turn contrasted with the actual data using standard second-moment criteria and spectral analysis.

The paper is organized as follows. Section 2 presents a simpli…ed description of the US cattle industry. Also, in section 2, I attempt to establish a set of empirical facts for the US cattle industry, which can be treated as benchmarks in this and future research. Section 3 presents the theoretical model describing the problem, as well as, discusses the solution to the model and some of its implications. Section 4 contrasts the actual US data with the arti…cial data generated from the model and discusses some of the more important results. Finally, section 5 concludes by summarizing the paper's most important …ndings and suggesting avenues for further research.

2 Cattle Facts

2.1 A Brief Description of the Cow-Calf Operation

Since the details of the US cattle industry are not universally understood, I will brie‡y outline the environment that is being modeled. In Western and Midwestern states, beef calves are typically born in the Spring.[4] In the …rst six months of life ranchers face few management options. If the calf is male, it is likely to be castrated. Because a mature bull can breed up to 50 cows, the number of males that need to be retained for breeding is small. Moreover, steers (i.e., castrated male calves) are more e¢cient to feed than bulls and are generally easier to handle. Calves are then weaned from their mothers in the fall, at which time, they are typically between six to ten months old. At this point, ranchers face an important management decision for female calves since females are both a consumption and a capital good. Producers decide whether to retain the female calf for addition to the breeding stock (capital good) or send them to slaughter (consumption good). The decision for weaned steers is much simpler as they are only a consumption good and are consequently destined for slaughter.

[4] The timing of the cattle operations in regions other than the West and Midwest vary, although the basic economic problem for the ranchers is the same. For instance, in the South, a substantial number of the cattle operators calve in November and December rather than in the Spring. However, for the US as a whole, the majority of the cattle operations follow the seasonal timing used in the West and Midwest (Gilliam, 1984).

Weaned calves that are sent to slaughter, do not go there immediately. Most will go through a process called …nishing. Finishing typically involves a four to six month period when a weaned calf is maintained on pasture or harvested forage before entering the feedlot. Once this stage is complete, the animal is transferred to a feedlot where it will be fed high-concentrate grains for approximately six months to be fattened for slaughter. By this time, nearly two years have passed since the birth of the calf. The …nishing of young animals is a relatively recent phenomenon. Prior to the 1930s, feeding of high-concentrate grains was atypical. Since then, the practice of …nishing young animals with grains has become commonplace and in more recent times (beginning in the 1960s) the …nishing has been increasingly completed in organized feedlots.[5]

As mentioned above, heifers that are not sold after weaning typically become part of the rancher's breeding stock. Breeding cows can produce at most a single calf per year, have a gestation period of nine months, and can be bred for the …rst time when they are approximately 15 months old. A breeding cow may then be retained and bred in subsequent years until approximately her tenth year. At this point, her reproductive abilities begin to deteriorate. Cows may be culled at any age and are typically culled after pregnancy testing in the fall when the calves are sold. The culled cows will go directly to slaughter as their beef is of lower grade and is not suitable for …nishing.

2.2 The Data

The primary source for data on the cattle industry is collected by the Livestock and Economics Branch of the National Agricultural Statistical Service of the United States Department of Agriculture (USDA). Most of these statistics are reported in their annual publication, Agricultural Statistics. The cattle data in Agricultural Statistics are impressive in their detail and coverage (e.g., the total stock of cattle dates back to 1867). However, there are also several important limitations of the data as well. First, there were abrupt changes in the accounting procedures at various times during the century, and second, several key series do not stretch back to the earlier part of the century. In response to the latter limitation, I begin the sample period in 1930. The sample period ends in 1997, the most recent date for which all the relevant series have been collected and recorded. Using annual data, this generates 68 data points by which to analyze cattle dynamics. While not an overly impressive sample size, this is to my knowledge the only uninterrupted data set available that covers the majority of the 20th century. In the rest of this section, I provide the source and de…nitions for the time series used in this paper, as well as, discuss some of their shortcomings. Unless otherwise stated, the data are taken from Agricultural Statistics.

Since the role of males in this paper is minimal, I focus on three stocks of female animals: calves, heifers and adult cows. Starting with calves, this series is given by the total annual

[5] Nerlove and Fornari (1995) were critical of RMS' use of data that ignored "major structural changes which occurred over the 100+ year period covered by their analysis." In particular Nerlove and Fornari state that "only in the 1930's did "…nishing" with grain....become signi…cant." This study reduces some of the problems associated with Nerlove and Fornari's criticism by beginning the sample in 1930.

calf crop of beef and dairy cattle in the US.[6] The heifer series is the total January 1 stock of yearling heifers. In 1970, the USDA modi…ed the manner in which it classi…ed cattle and calves, changing from an aged-base classi…cation system to a weight-based system. The change makes it di¢cult, if not impossible, to produce an accurate and continuous historical series for individual heifer categories (e.g., beef cow replacements, heifers 1-2 years old, etc.). However, because yearling heifers are almost always over 500 pounds, the series for the total number of heifers is not in‡uenced much by the change in accounting practices. As a result, the heifer series used is the total number of heifers over the entire sample period. The …nal female stock series is the total number of cows and heifers that have calved as of January 1. Figure 1 depicts the time series plot for these three age groups of female cattle. The most prominent feature of the three series is their cycles, which have a period of approximately 10 years. The respective stocks of calves and cows also display a clear upward trend, which peaked in the mid 1970's and has recently fallen back to the levels present in the 1950's.

The total federally inspected slaughter of heifers and cows was recorded as a single series up to 1944. Since then, it has been recorded as two separate series – one for heifers and one for cows. In order to make use of the entire data set back to 1930, I interpolate the individual heifer and cow data by multiplying the total heifer and cow slaughter series prior to 1944 by 0.21 and 0.79 (the fractions of heifer and cow slaughter in 1944) to form the respective series for heifer and cow slaughter between 1930 and 1944. Figure 2 depicts the time series plots for the slaughter of heifers and adult cows. While cow slaughter displayed only a moderate upward trend over the last six decades, heifer slaughter since the mid 1950's has increased rapidly, corresponding to the rise in active …nishing of yearling heifers and steers.

The cost of holding cattle involves both …xed costs (such as equipment, buildings, feeders, fences, etc.) and variable costs (such as feed, labor, vaccines, etc.). However, since the cost of feed is the predominant operating cost for cattle operations, I simplify the analysis by considering only feeding expenses.[7] To measure the price of feed, I use the "prices paid by farmers" feed index (1910-14 = 100). For calf prices, I use the average price received by farmers for calves, which is an average price paid to farmers across the states in a given year. For cows, I use the market price for commercial cows at two di¤erent markets. Prior to 1968, the USDA reports the market price at Chicago. After 1968, the USDA reports the market price paid to farmers at Omaha. For the years 1964 through 1968, both series are reported and produce very similar prices, as the law of one price would predict. All three

[6] For several cattle series, beef and dairy animals are combined. Rather than attempt to separate the two and risk introducing bias, I retain the dairy cattle in the stock and slaughter measures. Retaining dairy cattle also seems reasonable from a theoretical perspective as dairy operators face a similar problem to beef operators. They make period-by-period decisions regarding how many heifer calves to retain for addition to the breeding stock and how many adult cows to send to slaughter. Dairy operators do, however, react to a slightly di¤erent set of variables than beef cattle operators, e.g., the price of milk. When interpreting the empirical results, this needs to be kept in mind.

[7] Gilliam's (1984) survey of the US beef cow-calf industry supports this assumption. Gilliam writes on page 27, "Costs of production or purchasing feedstu¤s frequently comprise more than half of the total direct production cost in cow-calf production."

price series are deflated using the US consumer price index for all goods and services (1967 = 100). Figure 3 shows time series plots of the deflated feed, calf and cow prices. Notice that the real price of feed has gradually fallen over the last 40 years, reaching its low point in 1995. Also note that the prices of calves and cows, as mentioned previously, clearly do not exhibit the same degree of cyclical behavior as do stocks.

2.3 Empirical Facts about the Cattle Industry

In his famous 1986 article on business cycle measurement, Edward Prescott wrote the following about business cycles: "I ... do not refer to business cycles, but rather business cycle phenomena, which are nothing more nor less than a certain set of statistical properties of a certain set of important aggregate time series. The question I and others have considered is, Do the stochastic difference equations that are the equilibrium laws of motion for the stochastic growth [model] display the business cycle phenomena?" Following Prescott's lead, I attempt to establish a set of statistical properties for the cattle industry that characterize the cattle cycle phenomena. In subsequent sections, I then examine if the equilibrium laws of motion for the theoretical cattle model display the cattle cycle phenomena.

Several authors have conducted empirical studies of the cattle industry (e.g., Mundlak and Huang (1996), Rucker et al. (1984), and Jarvis (1982)), and although there are some widely agreed upon regularities such as the existence of cycles in the aggregate stocks, there are inconsistencies in other areas. Most notably, the presence (or lack thereof) of cycles in prices and consumption. In response, I attempt to establish a consistent set of empirical facts using well-recognized aggregate time series for the cattle industry. The intent is to create a set of empirical regularities (based on standard deviations, cross correlations and spectral decompositions), which can be used as benchmarks to discuss the properties of the cattle cycle phenomena and to assess the performance of models describing the industry.

2.3.1 Standard Deviations

The first statistical property of interest for the cattle industry is the relative volatility of various time series, as measured by their standard deviations. Table 1 presents the standard deviations for the growth rates in the US cattle data over the period 1930-1997.[8] Three key features of Table 1 stand out as key empirical regularities. First, note that the standard deviations of calves, heifers, cows and the total stock of cattle are all approximately equal at between 3% and 3.5% per annum. The stock of cows vary slightly less than calves and

[8] An alternative method for detrending the data is the Hodrick-Prescott (HP) filter, which is commonly used in real business cycle studies. The HP filter is a flexible method for extracting the trend from a stationary time series. Let y_t^c be the cyclical component and y_t^g be the growth component of a time series y_t. The HP filter is then given by choosing the cyclical and growth components to minimize $\sum_{t=1}^{T}(y_t^c)^2 + \lambda \sum_{t=1}^{T}(y_{t+1}^g - 2y_t^g + y_{t-1}^g)^2$. As $\lambda \to 1$, the growth component becomes a linear trend. As $\lambda \to 0$, the growth component becomes the series itself (Cooley and Prescott, 1995). Lambda is commonly set equal to 1600 in quarterly studies, but using annual data, I set $\lambda = 6.25$ as argued in Ravn and Uhlig (1997). The HP filter, however, is sensitive to the scale of the data and is therefore not used when presenting standard deviations of the detrended data.

heifers and the total stock of cattle vary slightly less than that of cows. Second, slaughter numbers are as much as four times as volatile as stocks – cow slaughter being 25% more volatile than heifer slaughter. Third, and …nally, the prices of calves and cows are even more volatile than slaughter, with standard deviations around 17% per annum. This is suggestive of substantial year-to-year uncertainty in the prices of cattle for ranchers.

2.3.2 Contemporaneous Cross Correlations

Another important feature of the cattle cycle is the pairwise correlation between respective time series. Table 2 presents the contemporaneous cross correlations between the nine US cattle series mentioned above – panel A and B employing growth rate and HP …lter data respectively. Statistics for the US data are in the lower triangular matrix of each panel. The cross correlation matrix for the HP …ltered data di¤ers slightly in magnitude from the growth rate data; however, the relative orderings are generally invariant to the detrending method. For simplicity, I therefore focus solely on the growth rate data in panel A. There are …ve prominent features of the contemporaneous cross correlations for the detrended US data that I wish to highlight.

First, notice that calves, heifers and cows stocks are all highly correlated (i.e., correlation coe¢cient of 0.69 or better). This is indicative of strong persistence in the (detrended) breeding stock. Above average numbers of calves, heifers and cows in period t, also imply above average numbers for the breeding stock in periods $t - 1$ and $t - 2$ due to the intergenerational laws of motion that link the respective cohorts of cattle together. That is, a female calf that survives through the …rst year becomes a heifer in the next year, and a heifer that survives through the second year and has a calf becomes a cow in her third year.

Second, cow slaughter and heifer slaughter are positively correlated with the stock of calves, heifers and cows – cow slaughter displaying stronger cross correlation with stocks than heifer slaughter does. Furthermore, there is a relatively strong positive correlation between cow and heifer slaughter. This suggests that common factors are in‡uencing the demand and/or supply for fed and unfed beef.

Third, the price of feed is positively correlated with the various stock measures. Although other factors are not being controlled for, this is suggestive of an upward sloping marginal cost curve for the industry, at least in terms of the costs of feed.

Fourth, calf and cows prices are negatively correlated with the stock and slaughter series. This is consistent with equilibrium prices for beef that are dominated by annual shifts in the supply for beef, as compared to demand.

Fifth, and …nally, there is strong positive correlation between the real prices for live calves and cows. In other words, periods of abnormally high (low) prices for calves are also associated with abnormally high (low) prices in the market for cows. This is most likely due to common shocks to the demand for fed and unfed beef, as well as, arbitrage conditions that keep the di¤erences between calf and cow prices from growing too large.

2.3.3 Spectral Density Functions

The third measure used to characterize the cattle cycle phenomena is the spectral density function. Spectral density functions decompose stationary time series into a weighted sum of periodic functions. By decomposing a time series in this fashion, it is possible to attribute the variance of a time series to cycles of di¤ering frequencies. This measure is especially well-suited to the cattle cycle phenomena given the regular nature of the cycles in the stocks of the various series.[9]

I use the modi…ed Bartlett kernel estimator of the spectral density function; see Hamilton (1995, pp. 330-332):

$$s_y(!) = \frac{1}{2¼}\left[\hat{}_0 + 2\sum_{j=1}^{h}(1 - \frac{j}{h+1})\hat{}_j \cos(j!)\right]; \quad (1)$$

where ! is the frequency parameter and h is set equal to eight. Since a cycle of frequency ! has a period of 2¼=!, peaks in the estimated spectral density function at frequency ! indicate that cycles of periodicity 2¼=! are contributing a disproportionately large amount to the variance of time series $\{Y_t\}_{t=1}^{T}$.

Figures 4, 5 and 6 display the estimated spectral density functions for stocks, slaughter and prices. Beginning with Figure 4, each of the three stock series have strong peaks in their spectral density functions at around the 0.67-0.70 frequency, which translates into a period of approximately 9 to 9.5 years. The estimated length of the cycle is slightly shorter than other studies (see for example, Mathews et al. (1999), Mundlak and Yair (1997) and Beale et al. (1983)) and is robust to whether the data are measured in growth rates or passed through the HP …lter.

Unlike RMS, I …nd evidence of cycles in consumption and prices. Similar to stocks, the spectra for HP …ltered heifer and cow slaughter in Figure 5 are dominated by a single peak. Peaks at ! = 0:89 and ! = 0:78 imply that heifer and cow slaughter display shorter cycles of approximately 7 and 8 years respectively. The spectra for the growth rate heifer and cow slaughter data display primary peaks at similar frequencies, but unlike the HP …ltered data, also display a secondary peak at a frequency of approximately ! = 2:25 (a second peak was also noted in Mundlak and Yair (1996)). This implies a secondary cycle of approximately 2.75 years in both heifer and cow slaughter. There is some theoretical support for the dual peaks in heifer and cow slaughter spectra. Slaughter numbers in the aggregate are given by the total stock of animals (heifers or cows) times the rate at which they are culled from the stock. Since the spectrum for the sum of two stationary series is the sum of their spectra (Hamilton, 1995), if the aggregate cull rates were cyclical with a shorter period than stocks, we would expect to see dual peaks in (linearized) slaughter spectra similar to those observed. Unfortunately, I am unaware of any independent measures of aggregate cull rates for which to test this hypothesis.

Lastly, Figure 6 shows the estimated spectral density function for feed, calf and cow

[9] Other authors have used spectral analysis to investigate the cattle cycle, including RMS (1994) and Mundlak and Huang (1996).

prices. In all three series there are two peaks, a primary peak at approximately ! = 0:90 and a secondary peak at approximately ! = 1:8, corresponding to cycles with periods of approximately 7 and 3.5 years respectively.

In sum, there is evidence of strong cycles in cattle stocks, consumption, and prices. Cattle stocks display a strong cycle with period of between 9 to 9.5 years, while consumption and prices contain two cycles (less prominent than stocks) – a primary one with a period of approximately 7 to 8 years and a secondary one with a period of approximately 3 years.

3 Theoretical Model

The theoretical model is set in discrete time with decision intervals one year in length. It is assumed that once a year, cow-calf operators make decisions regarding how many heifer calves to retain and adult cows to cull. Similar to RMS (1994), I minimize the role that males play in the model. All males are destined to become either steers, which subsequently go through a one-year …nishing process, or are kept as bulls for breeding purposes. Operators are assumed to be forward-looking, rational agents that maximize a discounted expected future stream of pro…ts subject to biological and market constraints. All operators are assumed identical and make decisions in competitive input and output markets.

3.1 Biological Constraints

Perhaps the feature that distinguishes the cattle industry the most from other industries is the long biological lags, which cause the time between breeding decisions and consumption to be measured in years rather than months. In this section, the laws governing stock dynamics are modeled. Begin with the stock of retained yearling heifers at time t, $\tilde{K}_t^{(1)}$, which depends on last period's stock of female calves, $\tilde{K}_{t-1}^{(0)}$, the fraction of female calves sent to market in period t (i.e., the cull rate for heifer calves), $\delta_t^{(0)}$, and the death rate for calves, $\pm_0$:[10] Using these items, we can write the law of motion for the stock of yearling heifers as

$$\tilde{K}_{t+1}^{(1)} = (1 - \pm_0)(1 - \delta_t^{(0)})\tilde{K}_t^{(0)}. \tag{2}$$

In other words, the stock of retained yearling heifers available in period t + 1 is equal to the number of heifer calves in period t which did not either die or get sent to market (i.e., culled from the stock). Once a female calf becomes a yearling heifer, her fate for the next year is entirely predetermined. If she was culled from the calf stock, she then enters the …nishing process for the next period on her way to slaughter. If she was retained for addition to the

[10] For stocks, I di¤erentiate between the number of animals and the total weight of the animals. Variables with tildes (~) above refer to the total number of animals and are measured in animal units while those without tildes refer to the total weight of the stock of animals and are measured in pounds.

breeding stock, she will be bred approximately three months after her …rst birthday and will produce her …rst calf at age two.

Rather than keep track of the entire age distribution of adult females, all ages of adult females are aggregated into a single measure, $\mathbf{b}_t$. Net investment into the stock of breeding cows can take one of two forms. First, positive investment into the breeding stock occurs as last period's retained yearling heifers mature into animals of breeding age. Negative investment or disinvestment into the breeding stock occurs as mature cows die or are culled from the herd. The law of motion for the stock of adult breeding cows is thus

$$\mathbf{b}_{t+1} = (1 - \pm_1)\mathbf{k}_t^{(1)} + (1 - \mathbb{R}_t^{(b)})(1 - \pm_b)\mathbf{b}_t, \tag{3}$$

where $\pm_1$ and $\pm_b$ are the death rates for yearling heifers and adult cows and $\mathbb{R}_t^{(b)}$ is the cull rate for adult cows. I abstract from the possibility of purchasing heifers and adult cows to add to the breeding stock because nearly all increases in the adult cow stock takes the form of heifer retention (Gilliam, 1984).

The number of females calves in any period is taken to be proportional to the number of breeding cows in the previous period. The factor of proportionality is $0{:}5\mu$, where 0.5 indicates that half the calves born in each period are female and μ is the successful birthing rate. Therefore, the stock of female calves evolves according to

$$\mathbf{k}_t^{(0)} = 0{:}5\mu\mathbf{b}_{t-1}. \tag{4}$$

In addition to the cyclical variation in the breeding stock, there has also been substantial growth in its productivity over the last 50 or so years; see also (Marsh, 1999). To account for this growth in productivity, I introduce a stochastic productivity term, A_t, which is assumed to follow (in logs) a random walk process with drift:

$$A_t = \frac{1}{2}A_{t-1}\exp("_{A;t}), \tag{5}$$

where $"_{A;t}$ follows a white noise process with variance $¾_A^2$. Rather than have productivity directly a¤ect the number of animals, I introduce three conversion factors, $¹_0$, $¹_1$, and $¹_b$, which convert calves, yearlings and adult cows into units of pounds per animal. This conversion is convenient because prices can then be measured in dollars per pound rather than dollars per animal. The conversion equations take the form

$$\begin{aligned} k_t^{(j)} &= ¹_j A_t \mathbf{k}_t^{(j)} \qquad j = 0;1 \\ b_t &= ¹_b A_t \mathbf{b}_t. \end{aligned} \tag{6}$$

3.2 Market Constraints

In reality, there are several distinct markets involved in the process of supplying beef to consumers: an input market, feeder cattle market, fed cattle market, retail market, etc. Building demand and supply relationships for all these markets directly from microeconomic

fundamentals (i.e., individual optimizing behavior subject to the appropriate constraints) would be a daunting task. Instead, I specify ad hoc demand and markup equations which attempt to capture in a crude fashion the interaction between these di¤erent markets.

I begin by assuming that the input market is perfectly competitive so that individual ranchers treat the price of inputs as given. Each individual operator considers herself to be too small to in‡uence the market price, but when forecasting future input prices, recognizes that shifts in the industry-wide demand and supply will in‡uence future prices. There are numerous operating expenses for a cattle producer – feed, labor, vaccines, vehicles, corrals, etc. These costs are given by single term, $!_t$, which represents per animal costs. The unit cost function for the industry is assumed to follow

$$!_t = \tilde{A}_0 \mathbf{e}_t^{\tilde{A}_1} \exp(\grave{A}_{!;t}) \tag{7}$$

where $\mathbf{e}_t = \mathbf{R}_t^{(1)} + \mathbf{e}_t$ and $\grave{A}_{!;t}$ follows the …rst-order autoregressive, AR(1), process $\grave{A}_{!;t} = ½_! \grave{A}_{!;t\,i\,1} + "_{!;t}$ with $0 \quad ½_! \quad 1$ and $"_{!;t} \sim iid(0; ¾^2_!)$.

After a rancher sells his animal and the animal completes the …nishing process, it is typically purchased by a packing plant, slaughtered, and then processed for retail sale. Each of these steps adds value to the …nal product. To capture the added value, I specify the following linear markup equations that relate the live cattle price to the retail price of beef:

$$p_t^{(k)} = Á_k E_t rp_{t+1}^{(k)} \tag{8}$$

$$p_t^{(b)} = Á_b rp_t^{(b)} \tag{9}$$

where $p_t^{(j)}$ is the live price the rancher receives for an animal of type $j \in (b; k)$ at time t, $rp_t^{(j)}$ is the retail price of beef for an animal of type $j \in (b; k)$ at time t, and E_t is the mathematical expectation operator conditional on all information dated t and earlier. Equation (8) states that the price a rancher receives for his calves in period t, $p_t^{(k)}$, is proportional to the conditional expectation of the retail price he will receive for his …nished beef one period hence, $E_t rp_{t+1}^{(k)}$. Since adult cows do not go through the …nishing process, (9) is a contemporaneous markup equation, such that the live price of cows is simply proportional to retail price of unfed beef in the same period.

Following RMS (1994) and Nerlove and Fornari (1995), I assume that the demand for retail beef is (log) linear and depends upon the price of chicken and pork, national income, and an unobserved stochastic term. Inverse demand for retail beef is given by

$$rp_t^{(k)} = ¸_0 (c_t^{(k)}/A_t)^{¸_1} I_t^{¸_2} pc_t^{¸_3} pp_t^{¸_4} \exp(°_{k;t}) \tag{10}$$

$$rp_t^{(b)} = ¼_0 (c_t^{(b)}/A_t)^{¼_1} I_t^{¼_2} pc_t^{¼_3} pp_t^{¼_4} \exp(°_{b;t}) \tag{11}$$

where $°_{k;t}$ and $°_{b;t}$ follow mean-zero AR(1) processes:

$$\omega_{j,t} = \rho_j \omega_{j,t-1} + \varepsilon_{j,t}$$

and $\varepsilon_{j,t} \sim iid(0, \sigma_j^2)$ for $j \in (k, b)$.

Total domestic consumption or slaughter in the respective markets for fed and unfed beef is given by

$$c_t^{(k)} = \theta_{t-1}^{(0)}(1-\delta_0)(1-\delta_1)k_{t-1}^{(0)} - NX_t^{(k)} \tag{12}$$

$$c_t^{(b)} = \theta_t^{(b)}(1-\delta_b)b_t - NX_t^{(b)} \tag{13}$$

where $NX_t^{(k)}$ and $NX_t^{(b)}$ are net exports of fed and unfed beef respectively. In other words, total domestic consumption of fed beef at time t, $c_t^{(k)}$, is given by the total weight of calves that were sent to market in period $t-1$ less the net exports of fed beef in period t. Likewise, total domestic consumption of unfed beef, $c_t^{(b)}$, is given as the total weight of cows sent to slaughter less net exports of unfed beef.

3.3 The Rancher's Problem

All ranchers are assumed to maximizes the discounted lifetime value of their operation subject to (2) - (13); the initial stocks, $k_0^{(1)}$ and b_0; and an initial productivity term, A_0. The objective function is

$$E_t \sum_{s=0}^{\infty} \beta^s \pi_{t+s} \tag{14}$$

where

$$\pi_t = p_t^{(k)}\theta_t^{(0)}(1-\delta_0)k_t^{(0)} + p_t^{(b)}\theta_t^{(b)}(1-\delta_b)b_t - \tau_t(k_t^{(1)} + b_t).$$

The rancher then chooses a sequence of cull rates $\left\{\theta_t^{(0)}, \theta_t^{(b)}\right\}_{t=0}^{1}$ to maximize (14) subject to the relevant constraints.

The necessary first-order conditions (assuming an interior solution and ignoring productivity shocks for the moment) are

$$p_t^{(k)} = \beta E_t\left[(1-\delta_1)\frac{1_b}{1_0}p_{t+1}^{(b)} - \frac{1_1}{1_0}\tau_{t+1}\right] \tag{15}$$

and

$$p_t^{(b)} = \beta E_t\left[p_{t+1}^{(b)}(1-\delta_b) - \tau_{t+1}\right] + (\beta)^2 E_t\left[p_{t+2}^{(k)}(1-\delta_0)0.5\mu\frac{1_0}{1_b}\right]. \tag{16}$$

The intuition behind (15) and (16) is clear. Profit maximization requires that the returns

from either culling or retaining an animal are equivalent at the margin. Beginning with equation (16), it states that the market value of an adult female in the current period must equal the expected discounted net market value of the same animal in the next period plus the expected discounted market value of her calf two periods from now. Equation (15) states that the market of value of a female calf must be equal to the discounted, expected net value of a cow next period. Moving (16) forward one period and substituting it into the right-hand side of (15) then states that the market value of this female calf must equal the discounted, expected net value when she becomes a cow two periods hence plus the discounted, expected value of her calf three periods hence.

3.4 Equilibrium and Solution Technique

An equilibrium for this problem is a sequence of prices, cull rates, and stocks which solve the rancher's problem and clear the respective markets in each period. Since all ranchers are identical and there are constant returns to scale in the production function, the equilibrium values of the variables will be the same for all ranchers and it is notationally simpler to treat the problem as if there is only a single representative rancher.

The system of equations to be solved is (2) - (13), (15), (16) and the initial values $k_0^{(1)}$; b_0 and A_0. This is a second-order system of nonlinear equations under rational expectations. The technique used to solve this system begins by …rst transforming the nonstationary variables so the system will settle down to a stationary long-run steady state. This involves dividing the nonstationary variables by the productivity trend variable A_t. The steady state is then calculated, the variables are written in terms of percentage deviations from their respective steady-state values, linearized around that steady state and solved for the unique equilibrium paths of the variables using the Blanchard-Kahn (1970) method. A similar solution technique was employed in King, Plosser and Rebelo (1988).

Denoting percent deviations from the steady-state values with carets (^) above the variables, the solution takes the form

$$\hat{x}_t = B\hat{x}_{t-1} + C\epsilon_{t-1} \tag{17}$$

for $t = 1; ...; T$, where the predetermined (and exogenous) variables are

$$\hat{x}_t = (\hat{k}_{t+1}^{(0)} \; \hat{k}_t^{(0)} \; \hat{k}_{t-1}^{(0)} \; \hat{k}_t^{(1)} \; \hat{b}_{t-1}^{(0)} \; \hat{b}_{t-1}^{(b)} \; \hat{b}_{k;t-1} \; \hat{b}_{b;t-1} \; \hat{b}_{l;t-1} \; \widehat{NX}_{t-1}^{(k)} \; \widehat{NX}_{t-1}^{(b)} \; \hat{I}_{t-1} \; \hat{pc}_{t-1} \; \hat{pp}_{t-1})' \tag{18}$$

and

$$\epsilon_t = (\epsilon_{k;t} \quad \epsilon_{b;t} \quad \epsilon_{l;t} \quad \epsilon_{nxk;t} \quad \epsilon_{nxb;t} \quad \epsilon_{I;t} \quad \epsilon_{pc;t} \quad \epsilon_{pp;t})' \tag{19}$$

is a vector of white-noise disturbances.[11] The non-predetermined variables can then be

[11] The exogenous variables $NX_t^{(k)}$, $NX_t^{(b)}$, I_t, pp_t, and pc_t are assumed to follow AR(1) processes with similar notation to calf and cow price disturbances.

written as a contemporaneous function of the predetermined variables and the disturbances,

$$\mathbf{b}_t = D\mathbf{b}_t + F\,{}^2{}_t \tag{20}$$

where

$$\mathbf{b}_t = (\mathbf{b}_t \quad \mathbf{b}^{(k)}_{t+1} \quad \mathbf{b}^{(k)}_t \quad \mathbf{b}^{(b)}_t)'.$$

3.5 Calibration

In order to generate artificial data from the system, it is first necessary to assign values for the parameters. To begin, the discount factor is assigned the same value used by RMS (1994), $\bar{} = 0{:}909$. Next, consider the parameter values associated with the physiology of cattle (i.e., death rates, birth rates, weights and productivity trends). These parameters are set to the following values:

$$\begin{aligned}(\pm_0 \quad \pm_1 \quad \pm_b \quad \mu \quad {}^1{}_0 \quad {}^1{}_1 \quad {}^1{}_b \quad \tfrac{1}{2} \quad \tfrac{3}{4}_A) &= \\ (0{:}07 \quad 0{:}01 \quad 0{:}04 \quad 0{:}88 \quad 337 \quad 888 \quad 744 \quad 1{:}0031 \quad 0{:}016)&: \end{aligned} \tag{21}$$

The death rates (i.e., $\pm_0$, $\pm_1$ and $\pm_b$) are calculated using the death loss figures from Agricultural Statistics. Death loss figures are published for two categories: cattle and calves. To obtain the natural death rate for calves, I use the historical (1930-1997) average of the ratio of calf death losses to the total calf crop. The death rates for yearling heifers and adult cows are more difficult to obtain because the death loss figures for the cattle series include both yearlings and adults. Although historical data are not available, the "average loss rates of weaned calves and yearlings from all causes on beef cow-calf farms and ranches" for 1980 is reported in Gilliam (1984). The reported rates were slightly less than 1%. The rates for adult cows contains an additional problem in that the measured natural death rate is certainly an underestimate of the true natural death rate because older, less healthy cows are typically culled rather than allowed to die of natural causes. Notwithstanding this point, the measured "average losses of cows and replacement heifers from all causes on beef cow-calf farms and ranches" in 1980 was approximately 2%. To account for the measurement problem discussed above, I double this figure and use a natural death rate of 4% for adult cows.[12]

The birthing rate is set at 88%, which is calculated using the 1930-1997 historical average of the ratio of the calf crop to the total number of cows (USDA). This is near to the 85% value used in RMS. As for the head-to-weight conversion factors (i.e., ${}^1{}_0$, ${}^1{}_1$ and ${}^1{}_b$), they are calculated by extrapolating back to 1930 the "average dressed weight of federally inspected calves, heifers and cows" (USDA). Collection of data on the dressed weights for these categories began in 1974. Using the growth rate between 1930 and 1974 in the average live weight of federally inspected slaughter cattle and the historical conversion rate

[12] I also used values of 2% and 10% for the death rates of cows. The results do not appear to be sensitive to moderate changes in the death rates.

of 0.6 from live-to-dressed weight, gives the above estimates for the initial weights of calves, heifers and cows.[13]

The last two parameters in (21) are calculated by …tting (in logs) a random walk model with drift to the "average live weight of federally inspected slaughter cattle." The estimated drift and standard error of the estimate from this model over the period 1930 to 1997 are given in (21).

The remaining parameters are comprised of price and income elasticities, autoregressive coe¢cients, and standard deviations for the disturbance processes.[14] The …rst set of parameters are as follows:

$$(\gamma_1 \quad \gamma_2 \quad \gamma_3 \quad \gamma_4 \quad \eta_1 \quad \eta_2 \quad \eta_3 \quad \eta_4 \quad \tilde{A}_1 \quad \acute{A}) =$$
$$(-1.0 \quad 0.8 \quad 0.0 \quad 0.1 \quad -1.0 \quad -0.1 \quad 0.4 \quad 0.1 \quad 1.0 \quad 0.6): \qquad (22)$$

Obtaining accurate estimates of the above parameters, particularly the elasticities, is an important step in properly calibrating the cattle model. Fortunately, there is a wealth of empirical information on retail market responses for fed (i.e., prime, choice and select) beef and unfed (i.e., hamburger and canned) beef. Several sources report estimated elasticities for either the fed and unfed retail beef markets. The sources include, but are not limited to, Capps et al. (1994), Lesser (1993), Marsh (1991), Smallwood et al. (1989), and Wholgenant (1989). The …rst eight parameters in (22) were selected as approximate midpoints to the estimated elasticities in these studies. Although, the reported elasticities vary from study to study depending on di¤erences in the sample period, data employed, functional forms, control factors, etc., the numbers in (22) appear to be a reasonable set of baseline values. In particular, there is strong evidence that retail demand for beef is downward sloping, nonfed beef is an inferior good, fed beef is a normal good, and pork and chicken are substitutes for beef at the retail level.

Next consider the elasticity of the cost of feed with respect to the total stock of heifers and cows, $\tilde{A}_1$. As far as I know, there are no studies that directly estimate the e¤ect of the total stock of cattle on feed prices. Presumably, an increase in the total stock of cattle should, all else equal, raise the demand for feed and therefore its price. Since I could not …nd any reported estimates of the elasticity, $\tilde{A}_1$, I set the value equal to one. This turns out to be almost the exact estimated elasticity when estimating (7) with an autocorrelation correction.

I also do not know of any empirical evidence for the individual markup parameters, $\acute{A}_k$ and $\acute{A}_b$. This is largely due to the lack of a reliable retail price index for unfed beef. In response, I assume that there is but a single markup parameter $\acute{A} = \acute{A}_k = \acute{A}_b$. Mathews

[13] Notice that the average weight for heifers is approximately 150 pounds greater than more mature cows. This is due to the …nishing process whereby yearling heifers are fattened with grain and consequently outweigh their adult counterparts at slaughter.

[14] Actually, since the retail demand functions are in their inverse forms with price as the dependent variable, the γ's and η's are often labeled as own-price and income ‡exibilities rather than elasticities. I continue to use the term elasticities rather than ‡exibilities, but the inverse form of the demand functions needs to be kept in mind.

et al. (1999) provide time series evidence of the spread between farm level and retail level beef, including a weighted average of both choice beef and hamburger. The spread between the two has been growing in recent decades (a trend that has prompted a large amount of literature regarding the competitiveness of the beef-packing industry), however for simplicity I abstract from the time-varying nature of this parameter and use the historical average which is approximately Á = 0:6.

Lastly, values for the autoregressive coe¢cients and standard deviations for the price and cost disturbances (i.e., $½_k$, $½_b$, $½_!$, $¾_k$, $¾_b$ and $¾_!$) were calculated by estimating (7), (10) and (11) with an autocorrelation correction and the data discussed above. The results are

$$(½_k \quad ½_b \quad ½_! \quad ¾_k \quad ¾_b \quad ¾_!) = (0{:}785 \quad 0{:}803 \quad 0{:}959 \quad 0{:}154 \quad 0{:}115 \quad 0{:}105): \qquad (23)$$

3.6 Impulse Response Functions

As a precursor to a full-‡edged simulation of the cattle model, I calculate and graph the responses of certain variables to one-time unit shocks in the disturbances. These graphs are useful in helping to understand the economics behind stock, slaughter and price dynamics. To highlight the propagation methods of the model, unless otherwise noted, I temporarily set all the autoregressive parameters (i.e., the ½'s) for the disturbance processes equal to zero.

Begin by considering a one-time unit shock to the demand for fed beef under two different scenarios[15]: $½_k$ equal to 0.5 and 1.0. The responses are shown in Figures 7 and 8 respectively. In Figure 7, the impulse to the price of fed beef causes an immediate increase in the price of calves because agents rationally anticipate a higher retail price for fed beef in the following period. The increase in the price of calves in period 1 induces the rancher to contemporaneously cull more calves and fewer adult cows. This is an intuitive optimal response on the part of the ranchers as the relative return to calves is now higher than in the steady state. Since fewer cows are now being sent to slaughter, the price of cows in period 1 also increases as we move up the demand curve for unfed beef (11).

In period 2, as a result of the change in cull rates, the stock of retained yearling heifers goes down and the breeding stock goes up. The calf stock is una¤ected in period 2 because it is predetermined by the number of cows in period 1 – movements in the calf stock always lag the breeding stock by one period. Also, in period 2, the rancher begins to cull calves again at a lower rate (although still higher than in the steady state) but continues to retain more cows in order to compensate for future rami…cations on the breeding stock of selling an inordinately high number of female calves in period 1. The calf cull rate returns (approximately) to the steady state two periods after the shock while the cow cull rate gradually returns to its steady-state level over a period of approximately 15 years. In period

[15] When $½_k$ equals zero, a one-time shock to the demand for retail fed beef has no impact on stocks, consumption or farm-level prices in any time periods. Although $²^k_t$ a¤ects rp^k_t directly, through the markup equations $p^{(k)}_t$ is only in‡uenced by time t expectations of rp^k_{t+1}, which in turn is not a¤ected by $²^k_t$ because the demand shock is strictly transitory. Similar arguments apply for feed costs and net exports of fed beef.

3, the stock of retained yearling heifers increases as the calf cull rate fell in the previous period. At the same time, the breeding stock decreases due to both the contemporaneous increase in the cow cull rate and the fall in the stock of retained yearling heifers in the previous period.

Also, notice how the stock of retained yearling heifers oscillates on its path back to the steady state. These oscillations are caused by initial changes in the stock of retained heifers, which in turn reverberate through the breeding stock and back to the retained stock of yearlings. A similar phenomenon is mentioned in RMS (1994). The cyclical dynamics of retained yearlings is related to the age distribution of the breeding stock. Although I do not keep track of the age distribution of the breeding stock, under the assumption that cows are culled from oldest to youngest, the following measure can be used to glean information regarding the changing age distribution of the breeding stock:

$$AI_t = -[(1 - \pi_1)(K_t^{(1)} - K_{t-1}^{(1)}) + (1 - \pi_b)(\theta_t^{(b)} b_t - \theta_{t-1}^{(b)} b_{t-1})]: \tag{24}$$

Equation (24) measures the (negative) sum of the change in the inflow and outflow of the breeding stock between periods t and t + 1. Since the inflow of yearling heifers into the breeding stock and the outflow of old cows to the unfed beef market both tend to decrease the age of the breeding stock, AI_t is positively related to the average age of the breeding stock.

Figure 9 depicts the response of AI_t to a unit increase in the price of calves. The cyclical nature of AI_t helps describe the dynamics of the system off the steady state. Initially, the age of the breeding stock increases as fewer old cows are culled and more heifer calves are sent to market. Shortly thereafter, however, the average age of the breeding stock begins to fall as ranchers move to build their breeding stocks back up to sustainable levels by culling more old cows and fewer female calves. The oscillations in the age index (as well as the stock of yearlings and cows) occur because past culling decisions influence stocks in subsequent periods resulting in the "echo effects" mentioned in RMS.

Next, consider the response of the system to a unit shock in the demand for unfed beef presented in Figure 10. As expected, the calf and cow cull responses are mirror images of the case of a demand increase in the fed beef market. In period 1, the rancher optimally sends more cows to market and begins to retain more heifer calves to compensate for the initial reduction in the breeding stock. The cow price jumps up initially in response to the shock as does the calf price. The calf price increases because agents rationally anticipate that next period's retail price for fed beef will increase due to the reduction in calves sent to start the finishing process. The breeding and calf stocks fall with a lag of one and two periods to the initial price shock and then oscillate back to their steady-state values. The stock of retained yearlings increases in the periods following the shock because more calves were optimally retained in the previous periods, but falls shortly thereafter and oscillates back to it steady-state position. The dynamics of the total stock of cattle is similar for both transitory increases to the demand for fed and unfed beef.

Notice in Figures 7 through 10 that increases in the price of calves or cows induce positive short-run own supply responses and negative short-run cross supply responses. For

example, in response to a one-time unit increase in the price of calves, ranchers optimally choose to increase the supply of calves sent to market and reduce the numbers cows sent to market, even when the impulse is permanent as in Figure 8. This response is in stark contrast to Jarvis' (1982) prediction of a perverse supply response. The absence of a negative supply response in this paper is due to the fact that ranchers are allowed to make culling decisions on both the calf and cow margins. As a result, ranchers send more calves (cows) to market in response to a relative price increase in calves (cows) and use the cow (calf) margin to compensate for the negative future impact on the breeding stock.

The third set of impulse responses are with respect to the cost of feed and are presented in Figure 11. A unit increase in the cost of feed ($½_{!}$ = 0:5) induces the rancher to sell more calves and cows as the cost of retaining yearlings and cows increases relative to their market values. The increases in the number of cows and calves going to slaughter initially lowers the price of cows and the price of calves. However, as the ranchers begin to reduce their cull rates again back to their steady-state levels, the prices begin to rise, overshooting their steady-state values, and then fall back to the steady-state. As a result of the higher than average cull rates, all the respective stock variables fall over time and then gradually increase back to their steady-state levels.

The fourth experiment involves a unit decrease in net exports of fed beef ($½_{nxk}$ = 0:5). One example of such a shock would be the implementation of a trade agreement, which increases the amount of prime beef shipped from, say, Canada to the US. As shown in Figure 12, a shock of this type would cause ranchers to decrease the cull rate for calves as the increase in domestic consumption of fed beef will cause its price to fall. The increase in the relative price of cows induces ranchers to cull more cows, which in turn causes the price of cows to fall. The prices of both cows and calves remain below their steady-state values until the e¤ects of the shock wear out. The total stock of animals increases in the period after the shock, remains around its peak for a few periods and then over a period of 20 or so years falls back to its steady-state level.

The …fth shock is to the price of chickens – calibrated as a substitute for unfed beef and independent of fed beef at the retail level. In Figure 13, the responses to a unit decrease in the price of chicken are presented. The fall in the price of chicken tends to decrease the retail price of unfed beef and the price of cows as people substitute away from unfed beef and toward chicken. The fall in the price of cows (relative to calves) causes ranchers to cull more calves and thus reduces the price of calves. This causes increases in the breeding and total stocks and decreases in the stock of retained yearlings with a one period lag. The stocks of calves, heifers and cows, as discussed above, then oscillate back to their steady-state values.

Finally, consider a positive productivity shock. Since the stochastic productivity term, $"_{A;t}$, does not directly enter any of the …rst-order conditions or biological laws of motion, it has no impact on the transition dynamics or the steady-state variables. However, since the steady state for the stock weight and consumption series, is de…ned only after dividing through by the productivity variable, A_t, the stock weights and consumption series will experience a discrete jump in the period of the shock and then continue to grow at rate $\ln(½)$.

4 Contrasting the Actual and Simulated Data

In this section, I contrast a fully simulated version of the theoretical cattle model with actual observations on key cattle time series. The artificial data sets are generated using actual observation on the exogenous variables, realizations for ϵ_t drawn from independent Gaussian distributions, and the equilibrium laws of motion (17) and (20).

To make sure the results are not influenced by an abnormal draw of ϵ_t, I simulate 500 artificial data sets of the same length as the actual data, calculate the ensemble average of the various statistics, and then contrast these ensemble averages with the actual data. The variation within the 500 simulations is used to calculate generalized Wald statistics, which can in turn be used to test whether the difference between the actual and simulated moments can be ascribed to sampling variation from the model. The generalized Wald statistics take the form

$$W = (\gamma - \mu)' (\widehat{Var}(\mu))^{-1} (\gamma - \mu) \tag{25}$$

where γ is an $(n \times 1)$ vector of statistics from the actual data, μ is the $(n \times 1)$ vector of associated ensemble average of statistics from the model, and W is asymptotically distributed chi-square with n degrees of freedom (Cogley and Nason, 1995a). The estimated variance of the ensemble-average statistics is

$$\widehat{Var}(\mu) = \frac{1}{500} \sum_{i=1}^{500} (\gamma_i - \mu)(\gamma_i - \mu)'.$$

Looking forward, the model appears to do a good job of capturing several key statistical regularities present in the actual data. However, it is less successful in replicating other features of the actual data. The successes and shortcomings of the model with respect to standard deviations, cross correlations and spectral decompositions are detailed below.

4.1 Standard Deviations

Alongside the standard deviations of the US cattle time series in Table 1, I present the associated ensemble averages of the standard deviations from the model. There are two primary observations to note. First, the model does a good job of matching the relative volatility of the stocks, slaughter and feed price series. The standard deviation in the growth of the artificial stock variables are approximately equal as in the actual data, with the total stock varying less than the individual components.[16] Growth in the artificial slaughter series vary approximately four to five times that of the artificial stocks, with cow slaughter varying slightly more than heifer slaughter. Finally, the estimate of the standard deviation for artificial feed prices is approximately one and a half times that in the actual data, but cannot be distinguished statistically once sampling variation is taken into account.

[16] Recall that the artificial calf stock is scaled to have the same standard deviation as the actual calf stock.

The second observation is that artificial calf and cow prices tend to understate the volatility in the actual prices. The calf and cow price series are approximately five times as volatile as the actual stocks, while in the model, calf and cow prices are only three times as volatile as stocks. This difference between theory and observation indicates that there may be some additional stochastic terms in the retail demands for beef that are not accounted for in the model.

4.2 Contemporaneous Cross Correlations

Previously, I noted five prominent features of the correlations between actual US cattle time series: (i) a strong positive correlation amongst different stocks; (ii) a positive correlation between slaughter and stocks; (iii) a positive correlation between the price of feed and stocks; (iv) a negative correlation between the price of calves and cows and the stock or slaughter measures; and (v) a strong positive correlation between calf and cow prices. The model does a good job in replicating most of these empirical facts.

Beginning with the stocks in Table 2 (the bold statistics in the upper right portion of the matrices refer to the simulated data), the contemporaneous correlations amongst the different stock series in the artificial data are generally positive as in the actual data. However, the artificial correlations are substantially lower than in the actual data. This problem is especially acute for calf, heifer and cow correlations but less so for total stocks. The weaker contemporaneous correlations between artificial stocks are suggestive of a lack of persistence in the model. This lack of persistence in turn is related to the ability of the model to propagate the shocks through time and/or the degree of persistence in the shock processes themselves.

Second, the artificial stock - slaughter correlations are generally of the same sign as those in the actual data. The generalized Wald tests indicate that in approximately half the cases we cannot reject the null hypothesis of equal cross correlations between the model and US data. A key shortcoming, however, is with respect to the cross correlation between heifer and cow slaughter. While the US slaughter series have strong positive correlation (i.e., 0.637 for growth-rate data and 0.561 for HP filtered data), the artificial data display a weak positive correlation (i.e., 0.067 and 0.022 respectively). Again, this indicates a lack of persistence in the model. For instance, a common positive shock to the retail demand for beef that persisted years into the future would cause heifer and cow slaughter in the same period to be higher than average. Another possible explanation for the difference is related to the length of the finishing process. In reality, the finishing process for some animals can take less than one year. To the extent that the finishing process is completed within the same year as the culling decision, it will tend to increase the contemporaneous correlation between heifers and cow slaughter.

Third, the correlation between the price of feed and stocks is replicated well in the simulated data. The generalized Wald tests indicate that the correlations between the price of feed and the various stock measures from the model are all statistically indistinguishable from those in the US data.

Fourth, the US correlations between calf and cow prices and stocks or slaughter are

uniformly negative, indicating that year-to-year shifts in cattle supply play an important role in the market for cattle (and beef). The same is true in the arti…cial data, but the correlations are generally of a smaller magnitude. This suggests that the model may be overstating the magnitude of annual shifts in the demand for beef relative to supply.

The …fth and …nal correlation is that between calf and cow prices. The US correlation is strong and positive at 0.897 and 0.880 for growth rate and HP …ltered data respectively. The model also predicts a strong positive correlation between these two prices (i.e., 0.994 and 0.997), but tends to overstate the correlation. This near perfect correlation in the model is the result of an arbitrage condition between the fed and unfed markets for beef. If there is a fed-beef speci…c shock to demand, then it will pass through directly to the price of calves in the same period and will alter the relative returns for holding calves and cows. The higher relative prices for calves induce ranchers to sell more calves and fewer cows, which in turn causes the price of cows to increase. If the relative prices did not return to their previous levels, ranchers would have an incentive to continue to sell their calves and retain their cows, eventually driving the breeding stock to zero. By assuming an interior solution, this type of behavior is ruled out and prices will necessarily be highly correlated. Much of the di¤erence between these two correlations may be due to the fashion in which US prices are …gured. Since US prices for calves are averages across states, it seems reasonable that arbitrage conditions between calves and cows which only hold regionally due to transportation costs will be diminished once calf prices are averaged across regions.

4.3 Spectral Density Functions

The most celebrated feature of US aggregate cattle data is their cycles. As mentioned above, the various US cattle stock series display strong and regular cycles with a period of approximately 9-9.5 years. The US slaughter and price series have dual cycles – a primary cycle of approximately 7-8 years and a secondary cycle of approximately 3 years. Clearly, any model attempting to explain the long-run behavior of the cattle industry will need to produce cyclical dynamics in these series.

The model in this paper does indeed produce cyclical behavior in stocks, with mixed evidence regarding slaughter and prices. Figures 14, 15 and 16 present the spectra estimated from the simulated data and can be thought of as the theoretical counterparts to Figures 4, 5 and 6 for the US cattle industry. Unfortunately, the spectra from the simulated data appear to, at times, be sensitive to the whether the data are detrended using the HP …lter or …rst di¤erences.

Beginning with the stocks in Figure 14, the spectra for growth-rate and HP …ltered data have peaks at frequencies associated with cycles of a little less than 4 years.[17] This is

[17] The spectra for stocks also appear to have other features associated solely with the detrending method. The …rst di¤erence operator, as evidenced by the mass near the zero frequency in the growth of the stock measures, fails to remove all the long-run variation in the raw stock data. Furthermore, the HP …lter appears to have a generated spurious cycles with a period of approximately 6.5 years. The existence of spurious cycles in simulated macroeconomic business cycle data has been previously noted in Cogley and Nason (1995b).

well less than the 9-9.5 year cycles present in the US data. To my knowledge, there are no studies of cattle supply that have been able to reproduce cycles of the length observed in the US data. RMS (1994) claim to have built a model that "…ts extremely well" and yet, admittedly, they are only able to produce similar length cycles of approximately 3.5 years. In my estimation, the most important future area of research in cattle supply will be to incorporate mechanisms into our existing models that "stretch out" the cycle in cattle stocks.

Arti…cial heifer and cow slaughter display somewhat mixed results on cycles. As shown in Figure 15, cow slaughter (whether in growth rates or passed through the HP …lter) display peaks at ! = 1:7, corresponding to a period of approximately 3.7 years. There is also a second peak at ! = 2:7 in the growth rate data, similar to the dual peaks displayed in the US growth rate data on cow slaughter. There is little evidence of cycles in the arti…cial heifer slaughter data. There is a very slight peak at the ! = 1:7 frequency in growth rates and a likely spurious cycle at the ! = 0:95 frequency in the HP …ltered data (see the previous footnote).

Arti…cial feed, calf and cow prices display little-to-no evidence of cyclical behavior. When measured in growth rates, the data approximate a white noise process with nearly "‡at line" spectra, and when passed through the HP …lter, the spectra only display peaks at the ! = 0:95 frequency. To the extent that this latter peak is spurious (as suspected), arti-…cial prices have weak-to-no cycles, which generally matches the impulse response behavior in Figures 7-13 (shocks to feed prices being the exception).

5 Conclusions

The primary goal of this paper was to build a more complete model of cattle supply, which could be used to both explain aggregate cattle dynamics and, ultimately, guide policy decisions. In the process, several interesting observations surfaced. First, it is shown that US cattle slaughter and prices do indeed exhibit cycles. The theoretical model provides mixed evidence with regard to slaughter and price cycles, with arti…cial slaughter data displaying more evidence of cyclical behavior than do arti…cial prices. To the extent that there are price cycles in the model, it is interesting to note that they are an equilibrium result from fully optimizing agents. As such, there is no opportunity to pro…t through countercyclical strategies (i.e., building up stocks when prices are near the trough of the cycle and selling when prices are near the peak of the cycle).

Second, the model does not exhibit the short-term negative supply response noted in Jarvis (1982), even when the shock is permanent in nature (see Figure 8). When ranchers are allowed to make decisions along both the calf and cow margins, the response to changes in relative prices will induce a positive short-run own supply response. The perverse supply response behavior noted in Jarvis instead shows up as a negative cross price response. That is, if the price of fed beef increases, ranchers optimally supply fewer cows and vice versa.

And third, as shown by the impulse response functions, the dynamic response to the various cattle time series depends on the nature of the shock driving the response, whether

it be a shock to retail demand, productivity, net exports, feed costs, etc. Therefore, when policymakers react to perceived changes in the cattle industry, it is critical that they understand the nature of the shock driving the dynamics.

In addition to the observations above, a fully calibrated and simulated version of the model replicates several key features of US cattle time series. The model (i) produces a similar volatility ordering to that found in the US data, (ii) replicates the sign of the contemporaneous correlations between key US cattle time series, and (iii) generates cycles in cattle stocks.

Although the model …ts the data well in these dimensions, it falls short in others. Most importantly, the model (i) understates the volatility of prices, (ii) understates the contemporaneous correlation between di¤erent stock measures, (iii) understates the length of the cycle in stocks, and (iv) only provides mixed evidence of slaughter and price cycles. In my estimation, it is these last two shortcomings that are the most pressing research items. By building in features to our existing models that "stretch" out the cattle cycle to replicate the observed cycle will be a major move forward in our understanding of cattle dynamics. The most promising extension in this regard is to formally model the age distribution of the stock of di¤erent animals, thereby allowing age e¤ects to contribute to cyclical dynamics (see also Rosen (1987) and Rucker et al. (1984)). Other promising extensions include credit constraints, rancher heterogeneity, variation in seasonal timing, noncompetitive behavior at the beef-packing level, and self-ful…lling prophecies.

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Table 1. Standard Deviations of U.S. and Artificial Cattle Time Series, 1930-1997

Series	Actual Data (in growth rates)	Simulated Data (in growth rates)
Calves	0.034	0.034**
Heifers	0.034	0.034**
Cows	0.032	0.034
Total Stock	0.030	0.024
Heifer Slaughter	0.120	0.135**
Cow Slaughter	0.148	0.175**
Feed Price	0.109	0.172**
Calf Price	0.175	0.102*
Cow Price	0.162	0.088

Notes: Calves, heifers, cows, total stock, and slaughter variables are measured in millions of animals. Actual feed price is an index of feed prices (1914 = 100). Actual calf and cow prices are measured in dollars per pound. The three price series are deflated by the consumer price index (1982-84 = 100).

* Failure to reject the null of equal variances at the 1% level

** Failure to reject the null of equal variances at the 5% level

Table 2. Contemporaneous Cross Correlations of U.S. and Artificial Cattle Time Series

Panel A. Growth Rate Data

	Calves	Heifers	Cows	Total Stock	Heifer Slaughter	Cow Slaughter	Feed Price	Calf Price	Cow Price
Calves	1	**0.257**	**0.257**	**0.611**	**0.187****	**0.192****	**0.078****	**-0.129**	**-0.129***
Heifers	0.725	1	**0.019**	**0.421**	**0.122****	**0.064**	**0.099****	**-0.112**	**-0.111**
Cows	0.736	0.689	1	**0.856**	**0.610**	**0.232**	**0.121****	**-0.163**	**-0.168***
Total Stock	0.922	0.834	0.927	1	**0.571**	**0.265***	**0.155****	**-0.203**	**-0.205***
Heifer Slaughter	0.255	0.171	0.345	0.307	1	**0.067**	**-0.005****	**-0.032****	**-0.034****
Cow Slaughter	0.371	0.375	0.514	0.476	0.637	1	**0.228****	**-0.017**	**0.003***
Feed Price	0.232	0.231	0.270	0.275	0.112	0.254	1	**-0.409**	**-0.358**
Calf Price	-0.452	-0.509	-0.532	-0.543	-0.199	-0.479	0.121	1	**0.994**
Cow Price	-0.431	-0.436	-0.392	-0.453	-0.182	-0.285	0.280	0.897	1

Panel B. HP Filtered Data

	Calves	Heifers	Cows	Total Stock	Heifer Slaughter	Cow Slaughter	Feed Price	Calf Price	Cow Price
Calves	1	**0.102**	**0.097**	**0.541**	**0.218***	**0.326****	**0.150****	**-0.011**	**0.006**
Heifers	0.758	1	**-0.425**	**0.042**	**-0.011****	**0.127***	**0.098****	**0.035**	**0.045**
Cows	0.759	0.736	1	**0.807**	**0.567**	**0.155**	**0.189****	**-0.090**	**-0.079**
Total Stock	0.923	0.863	0.938	1	**0.620**	**0.334****	**0.278****	**-0.074**	**-0.053**
Heifer Slaughter	-0.140	0.123	0.164	0.048	1	**0.022**	**0.034****	**0.032**	**0.038***
Cow Slaughter	0.331	0.411	0.607	0.510	0.561	1	**0.249****	**0.007**	**0.028**
Feed Price	0.288	0.241	0.306	0.313	-0.168	0.182	1	**-0.453**	**-0.390**
Calf Price	-0.506	-0.535	-0.644	-0.623	-0.390	-0.633	0.086	1	**0.997**
Cow Price	-0.485	-0.427	-0.512	-0.529	-0.323	-0.428	0.266	0.880	1

Notes: Bold correlations refer to the simulated data. The HP filter is set at $\lambda = 6.25$. The sample period is 1930 through 1997. Calves, heifers, cows, total stock, and slaughter variables are measured in millions of animals. Actual feed price is an index of feed prices (1914 = 100). Actual calf and cow prices are measured in dollars per pound. The three price series are deflated by the consumer price index (1982-84 = 100).

* Failure to reject the null of equal correlations at the 1% level

** Failure to reject the null of equal correlations at the 5% level

Figure 1. U.S. Cattle Stocks (1930-1997)

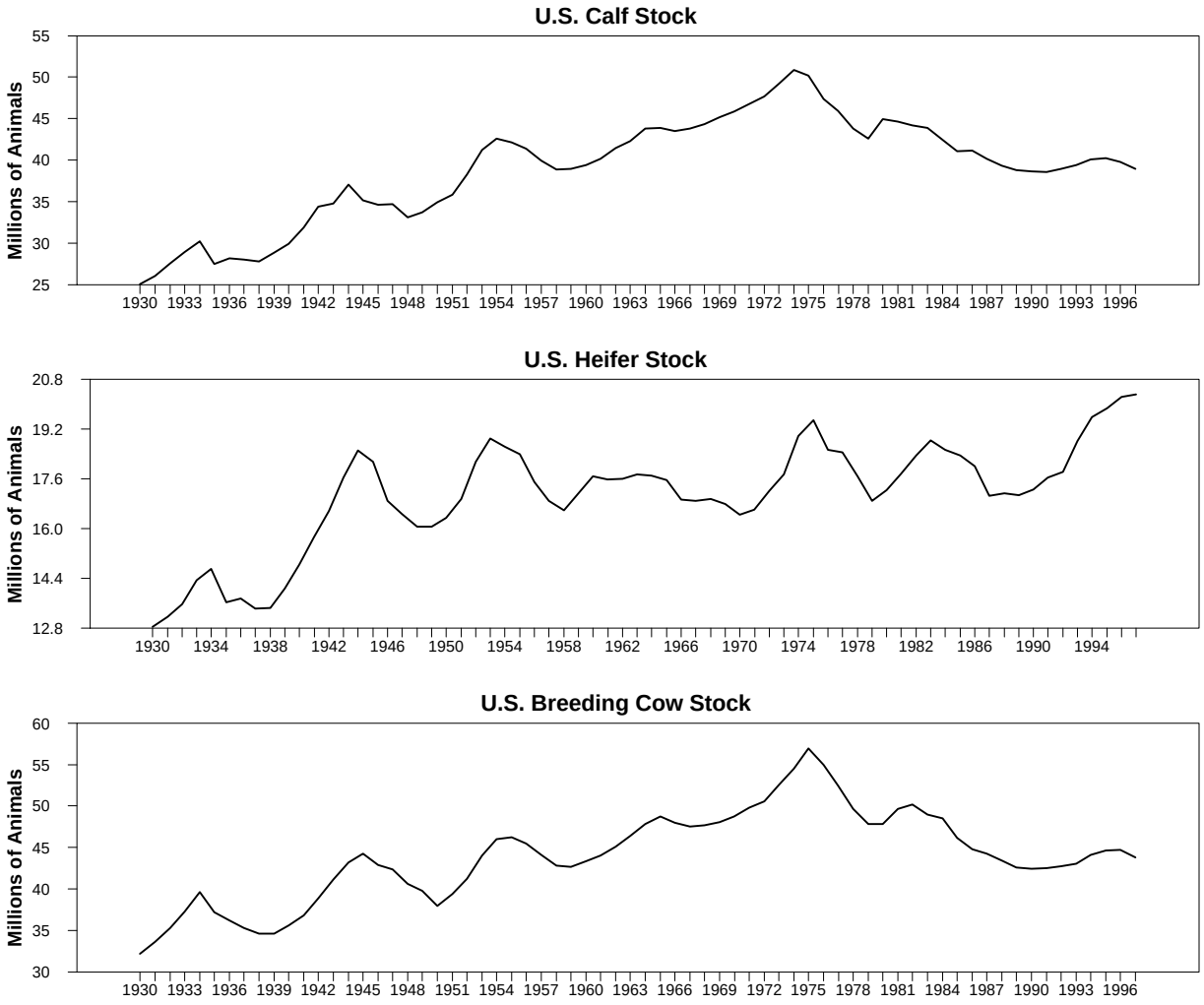


Figure 2. U.S. Cattle Slaughter (1930-1997)

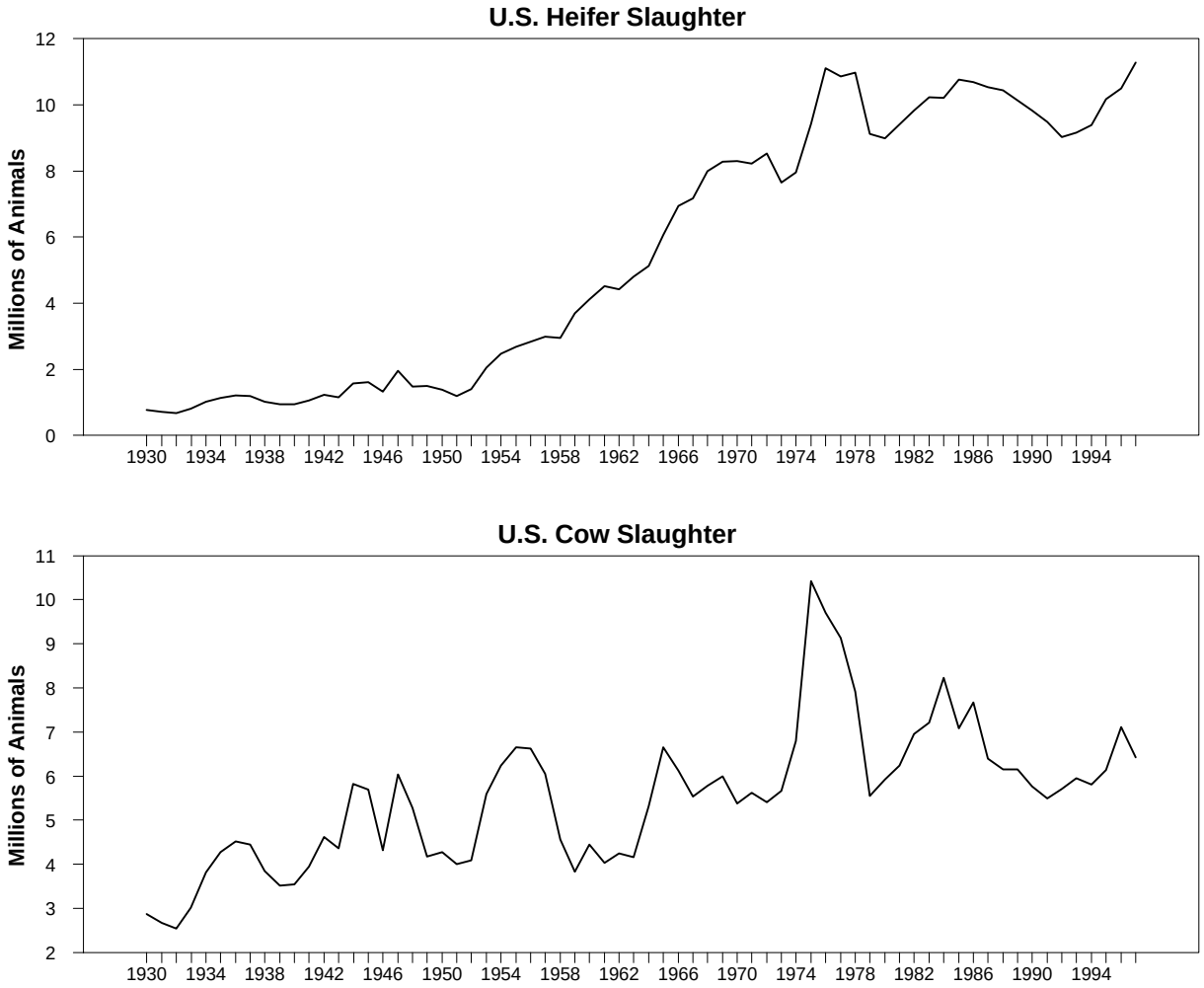


Figure 3. U.S. Feed and Beef Prices (1930-1997)

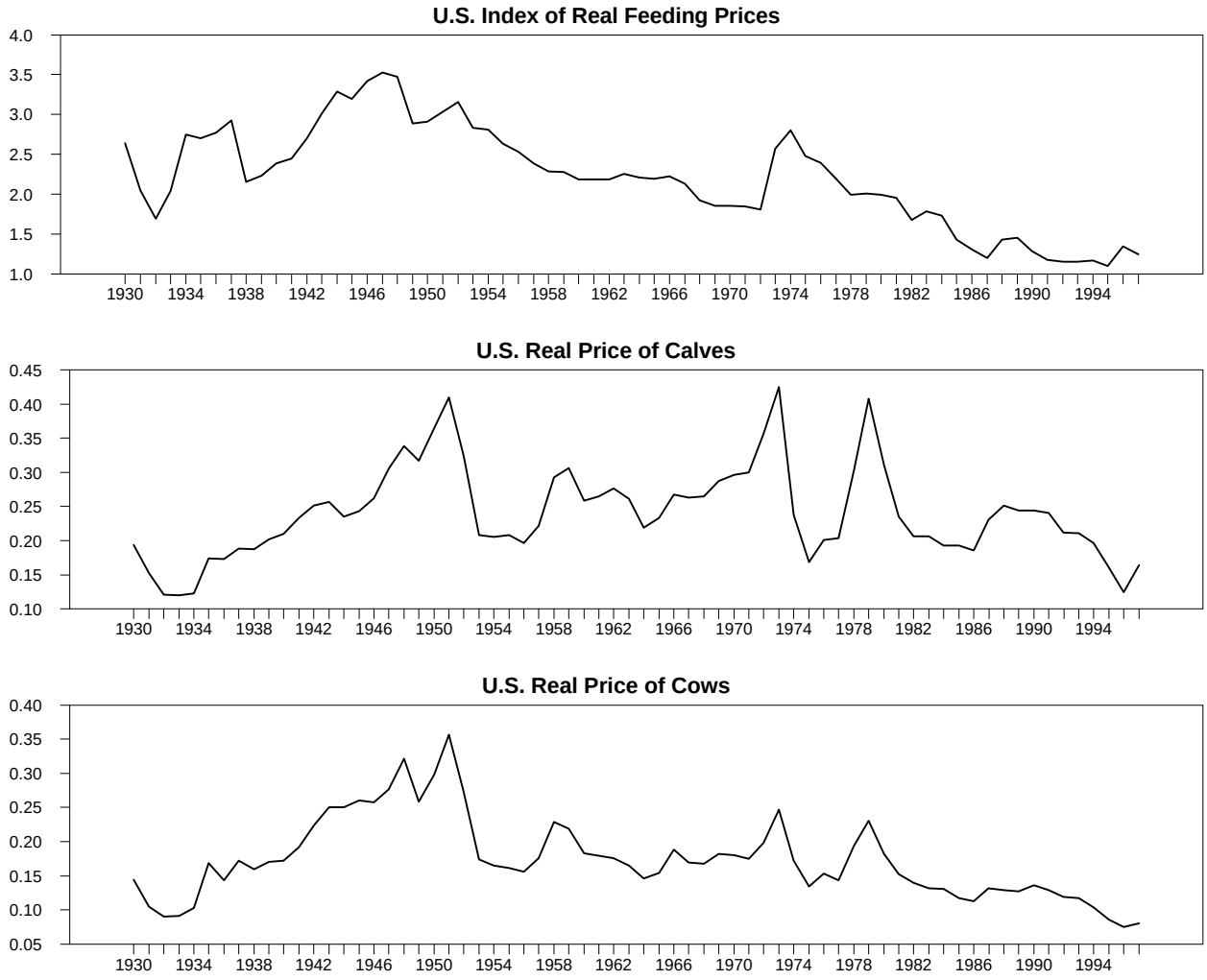
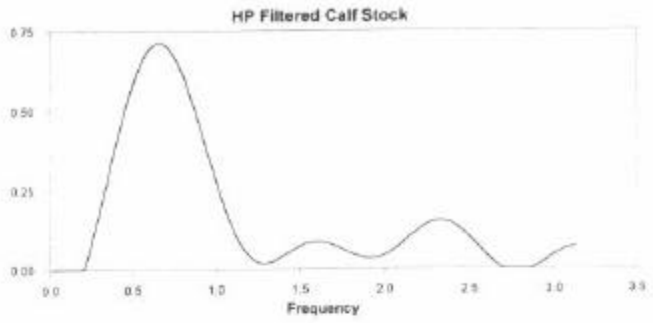
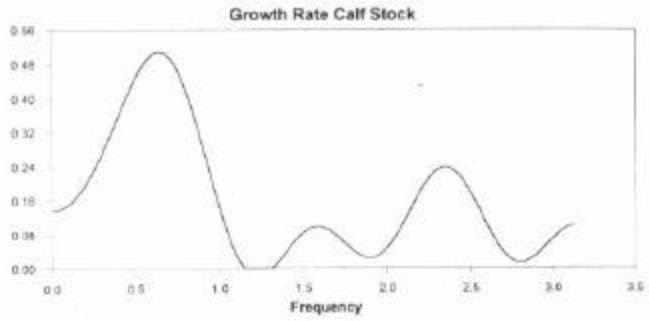
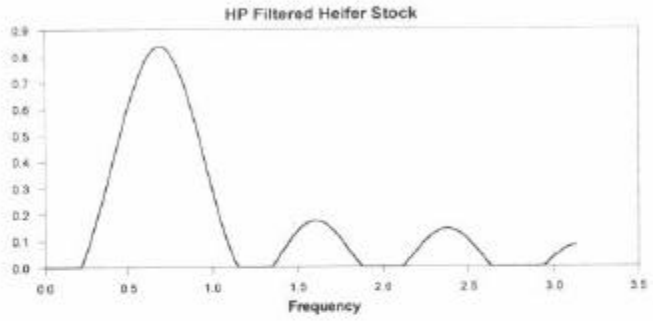
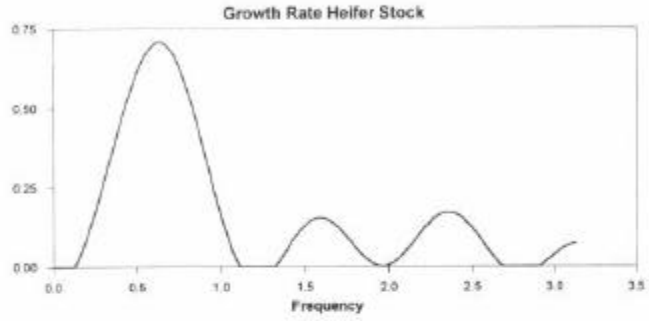


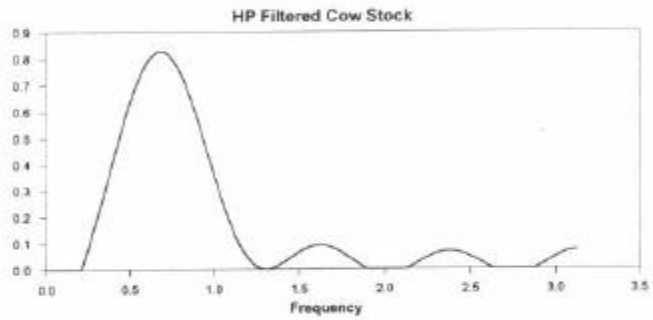
Figure 4. Spectra for U.S. Cattle Stocks











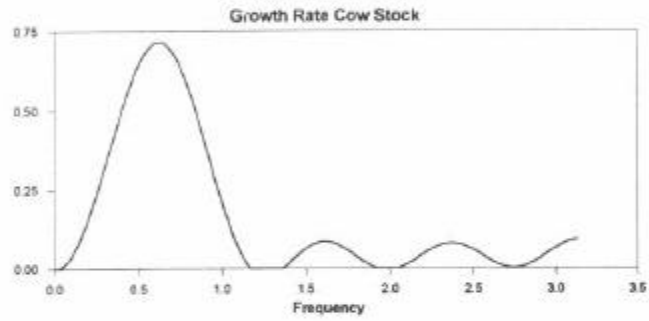
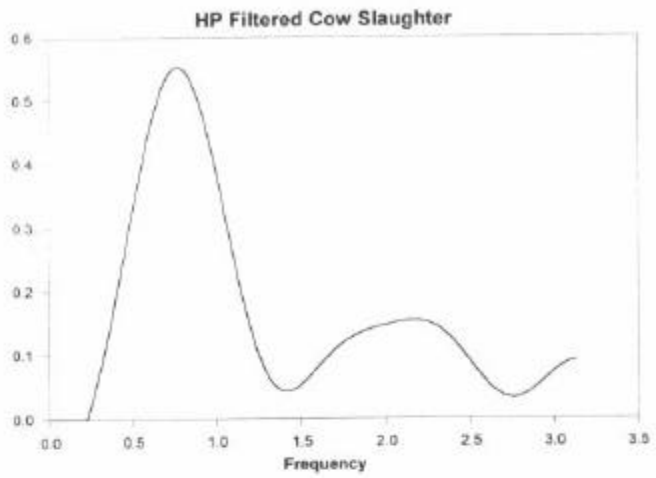
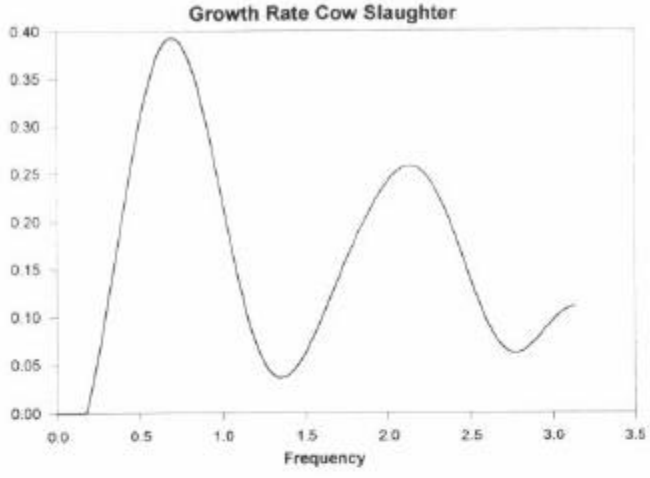
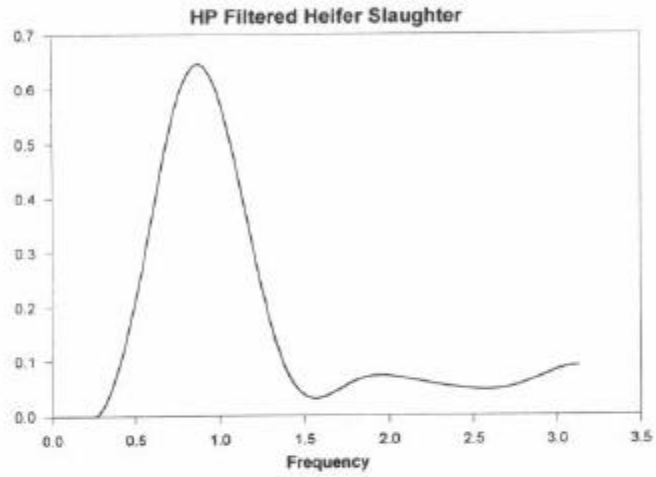


Figure 5. Spectra for U.S. Cattle Slaughter







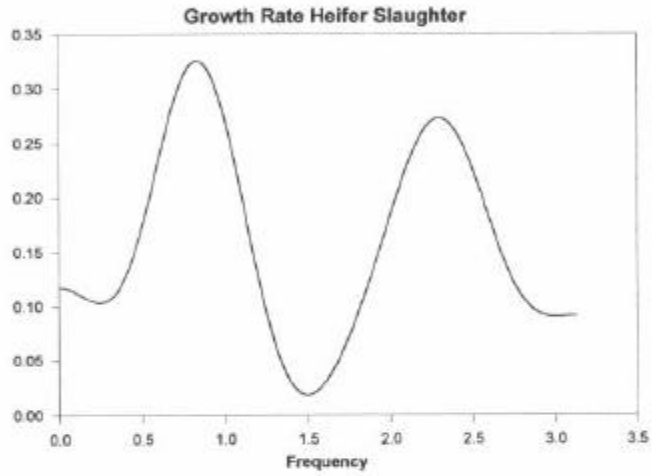
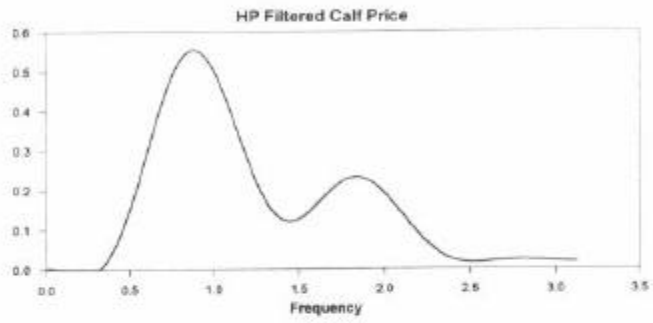
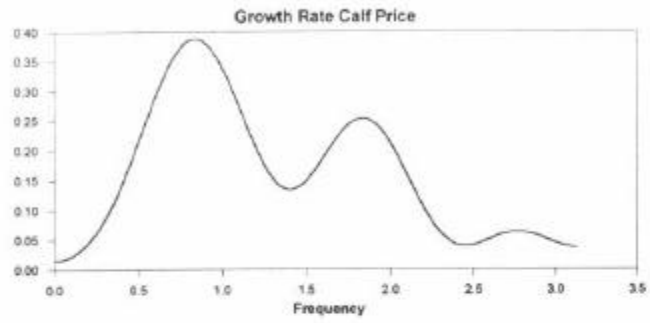
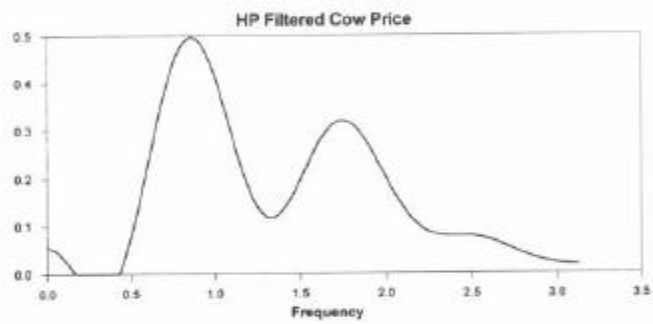


Figure 6. Spectra for U.S. Feed and Cattle Prices







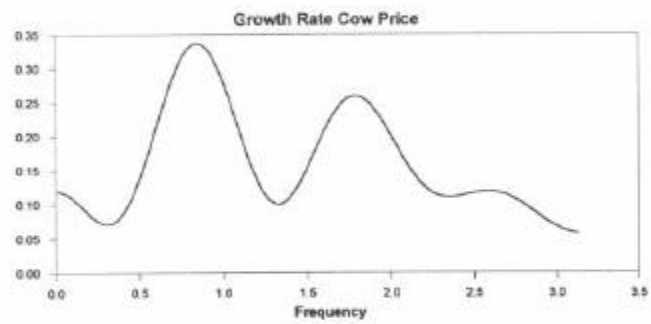


Figure 7. Responses to a Unit Increase in Calf Prices

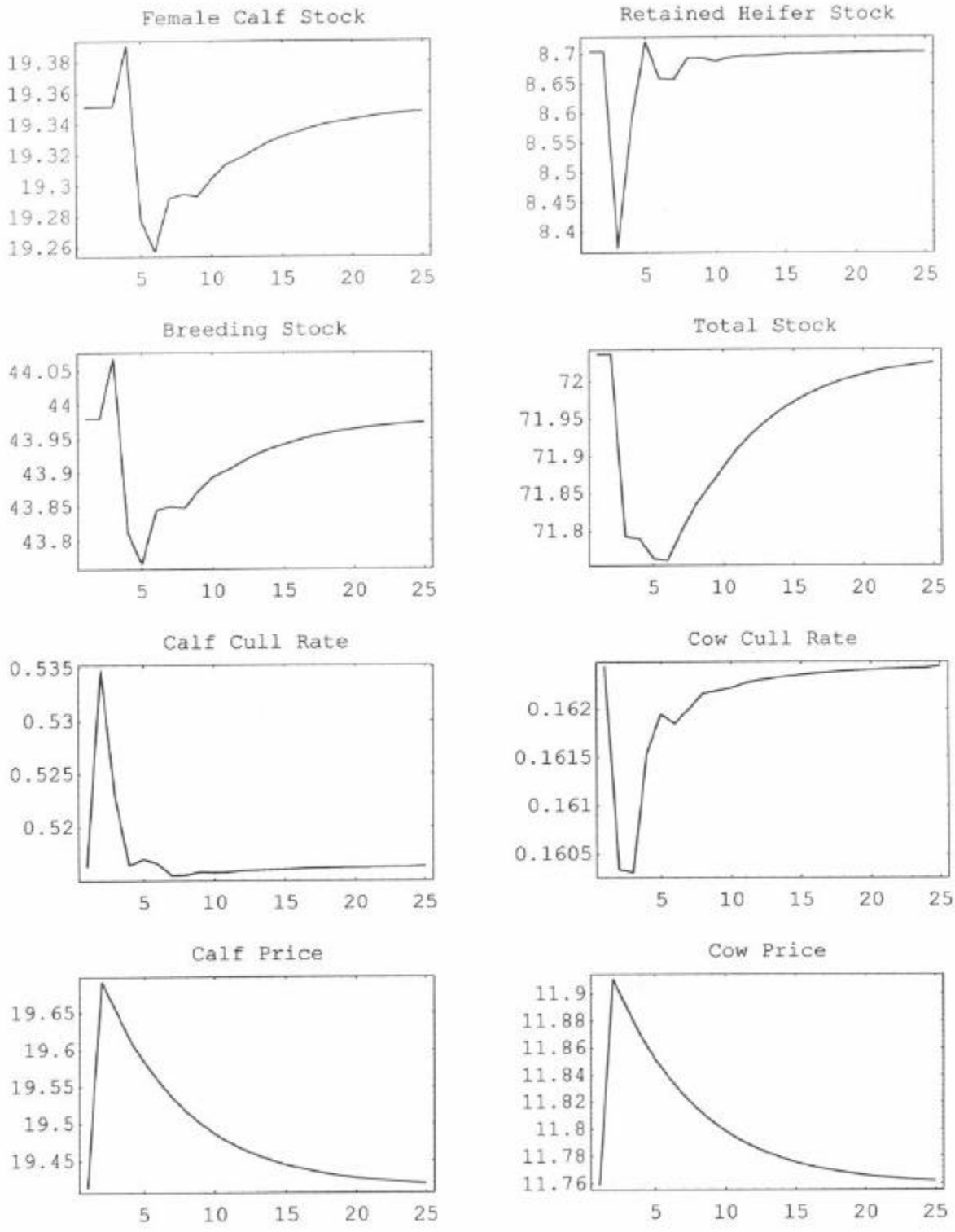


Figure 8. Responses to a Unit Increase in Calf Prices (rhok = 1)

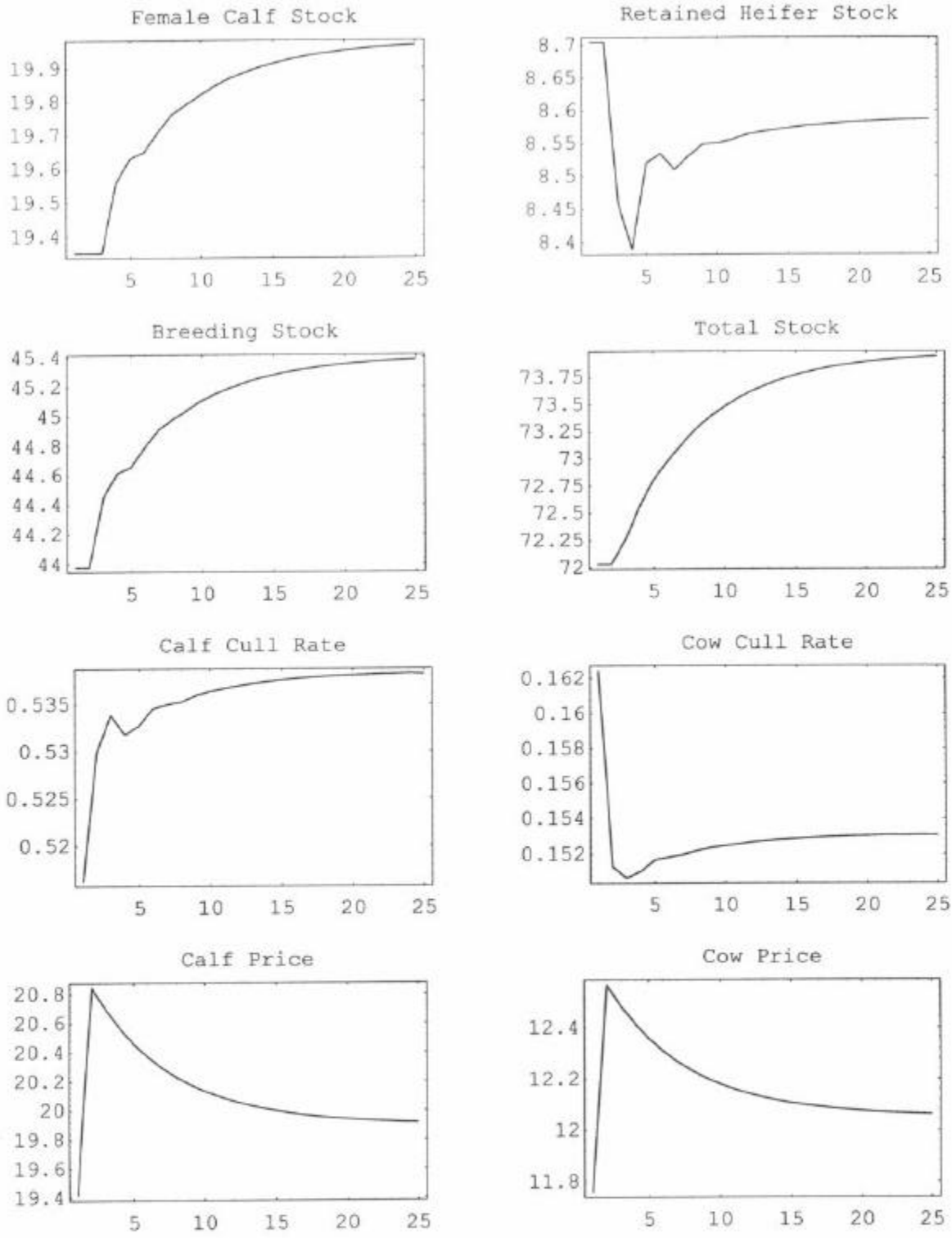


Figure 9. Responses to a Unit Increase in Calf Prices

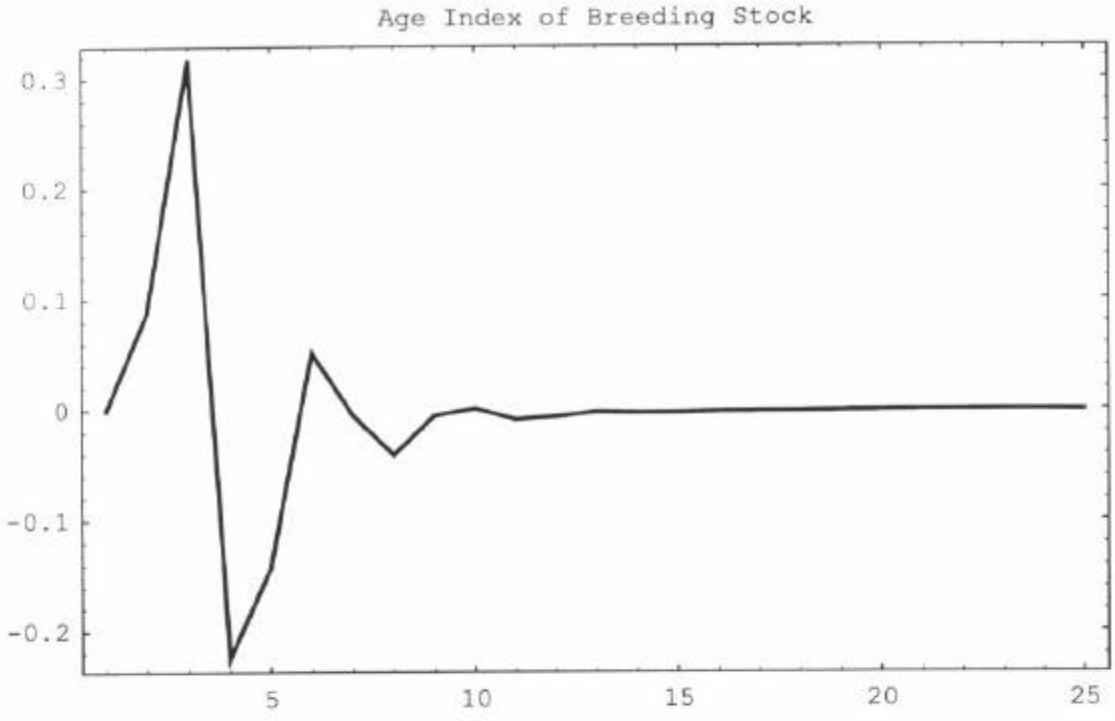


Figure 10. Responses to a Unit Increase in Cow Prices

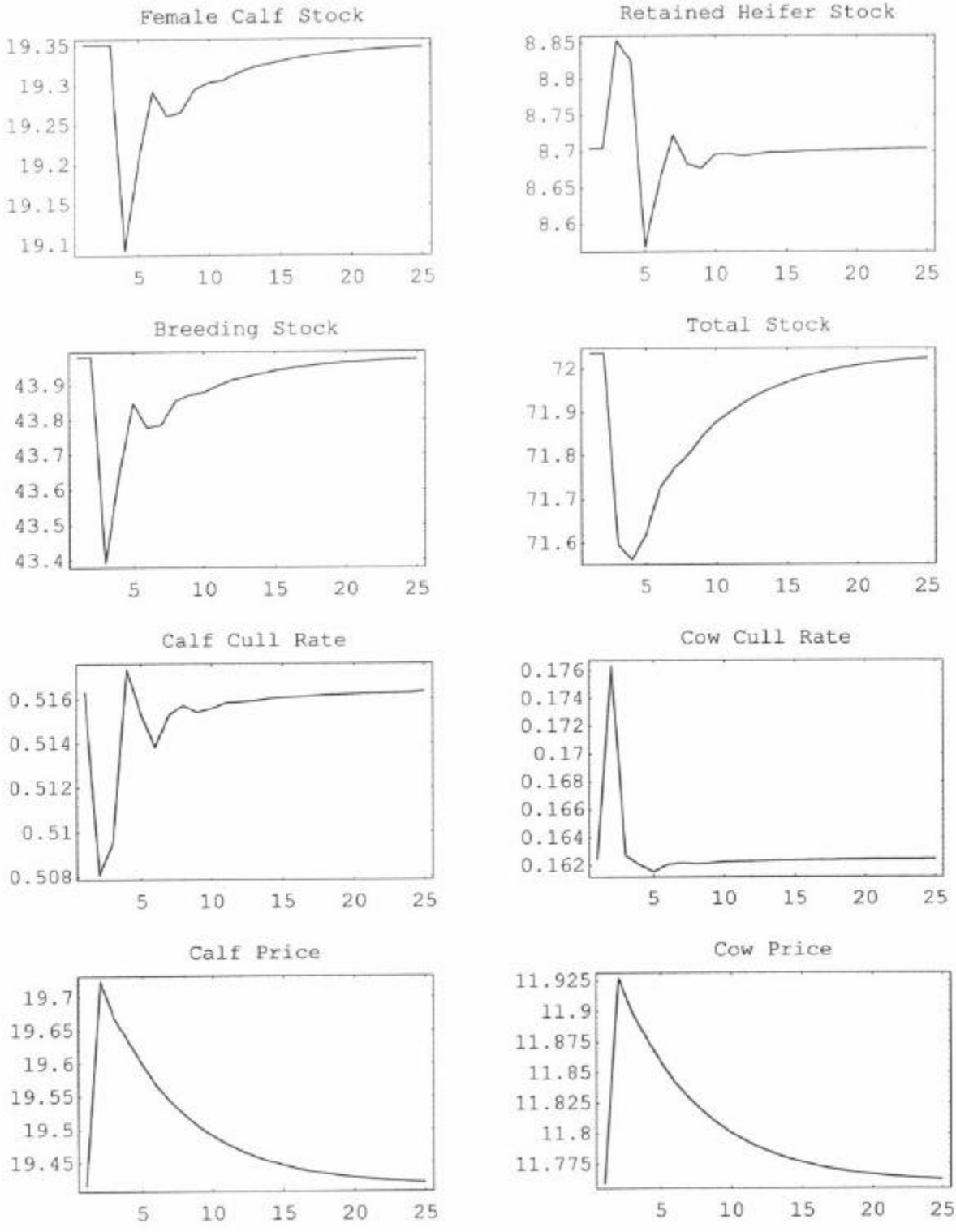


Figure 11. Responses to a Unit Increase in Feed Costs

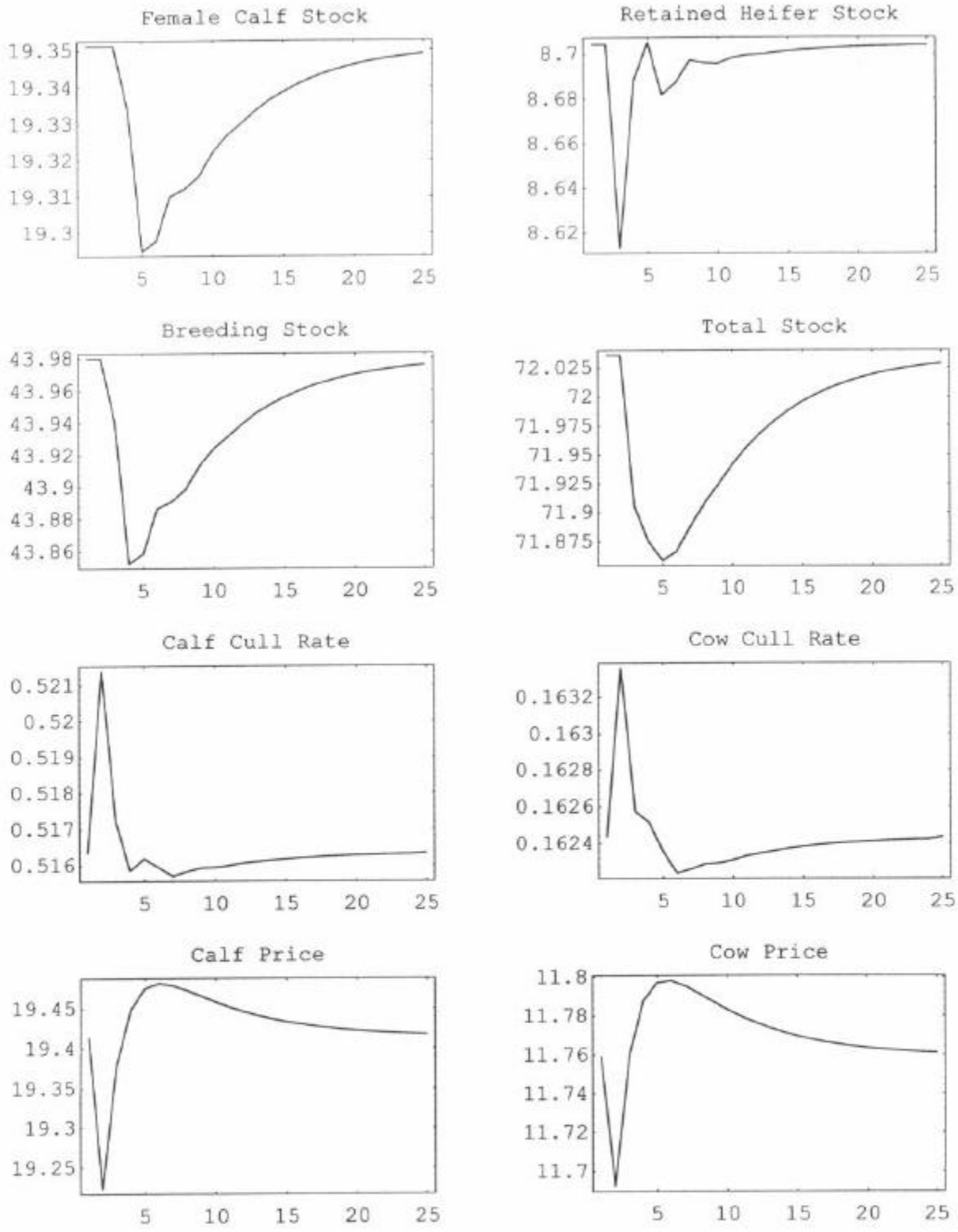


Figure 12. Responses to a Unit Decrease in Net Exports of Fed Beef

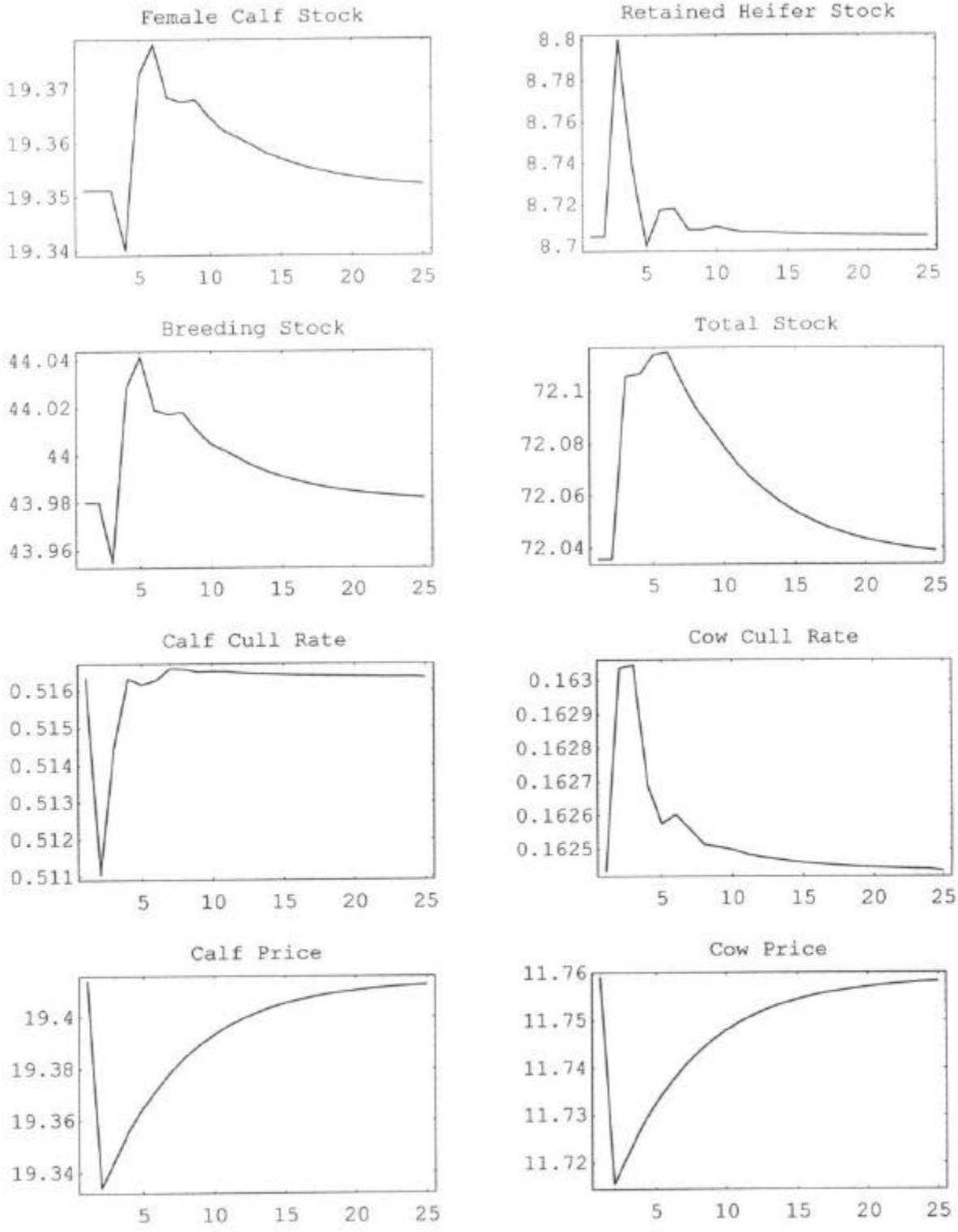


Figure 13. Responses to a Unit Decrease in Chicken Prices

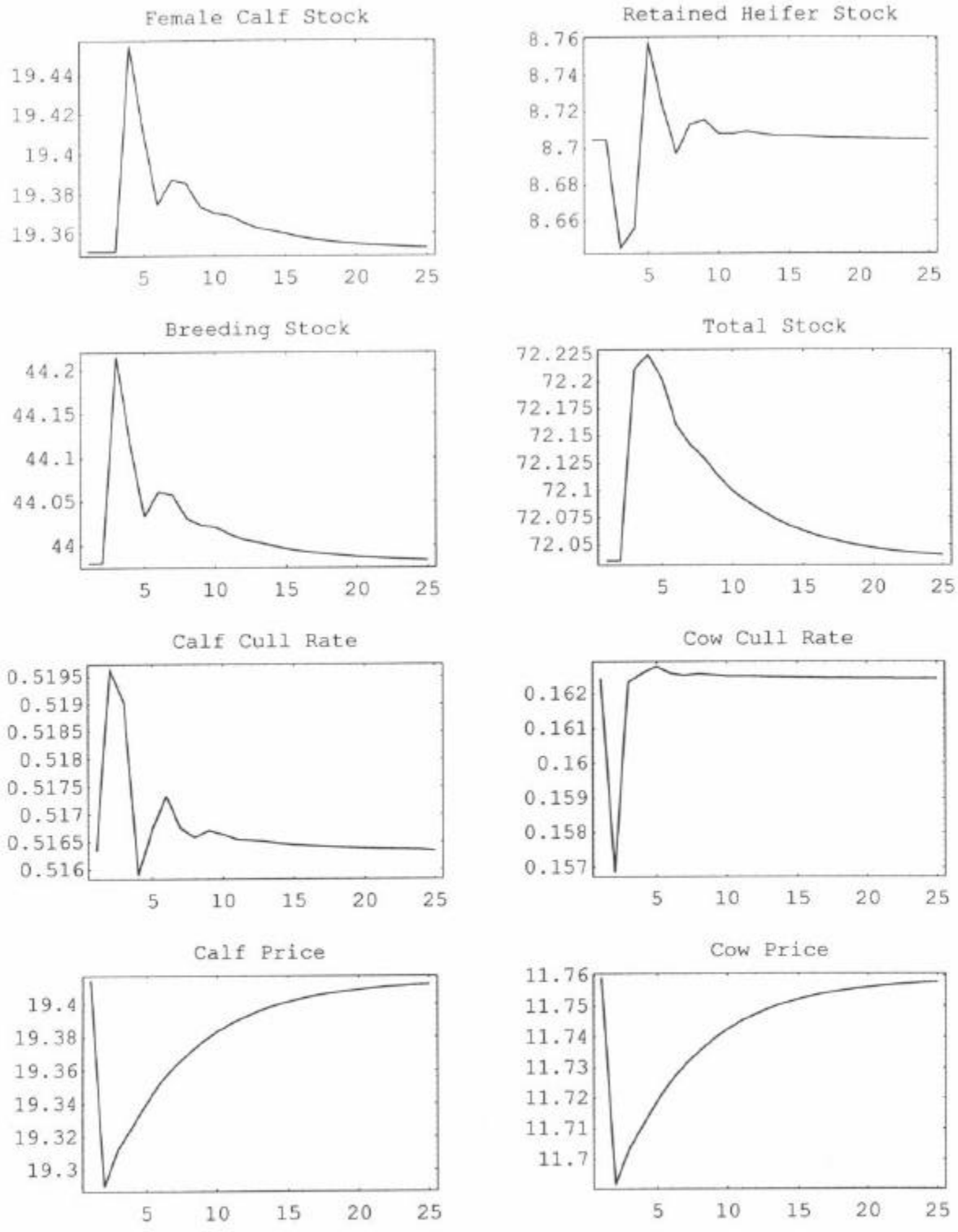
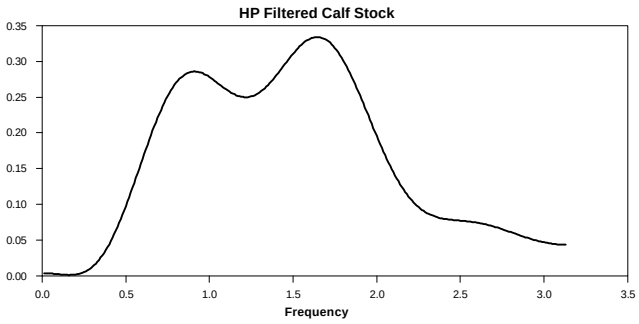
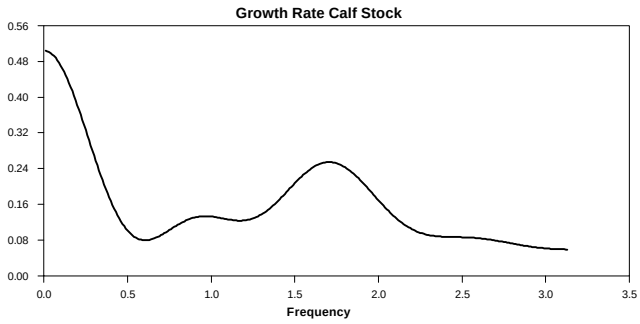
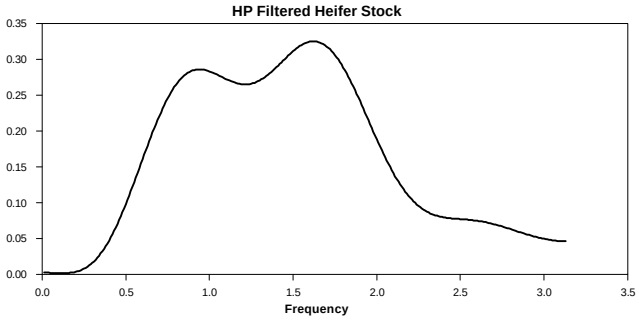
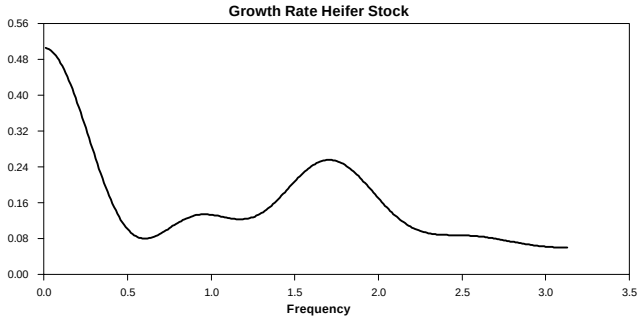


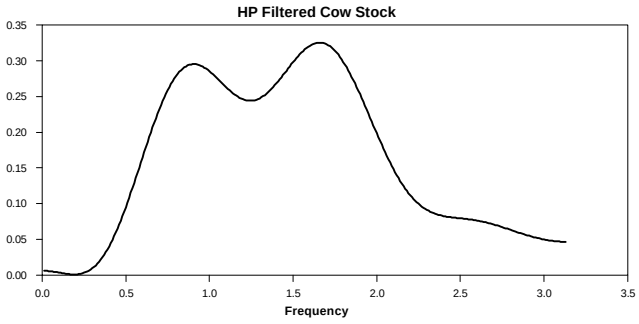
Figure 14. Spectra for Simulated Cattle Stocks











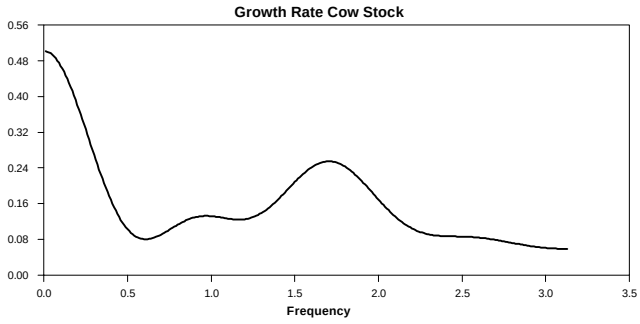
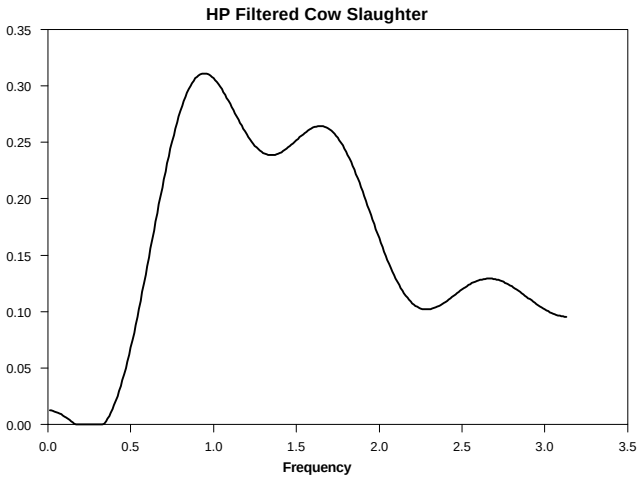
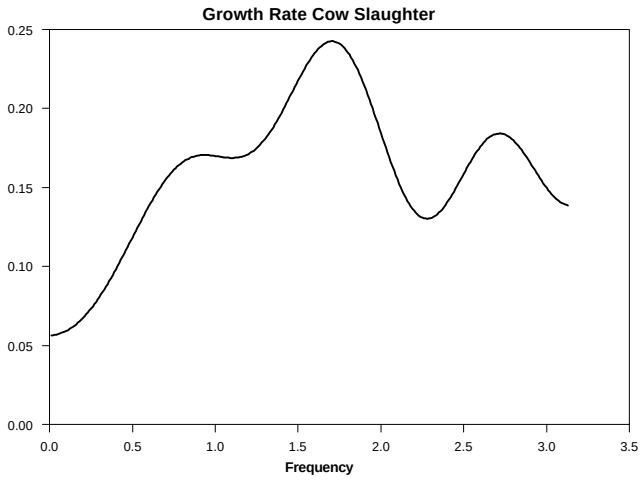
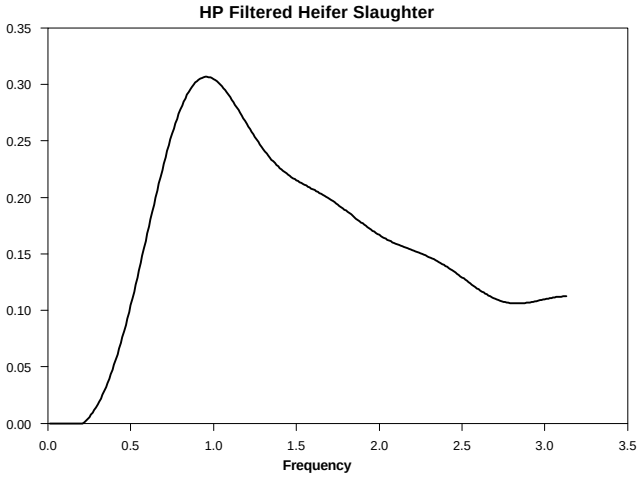


Figure 15. Spectra for Simulated Cattle Slaughter







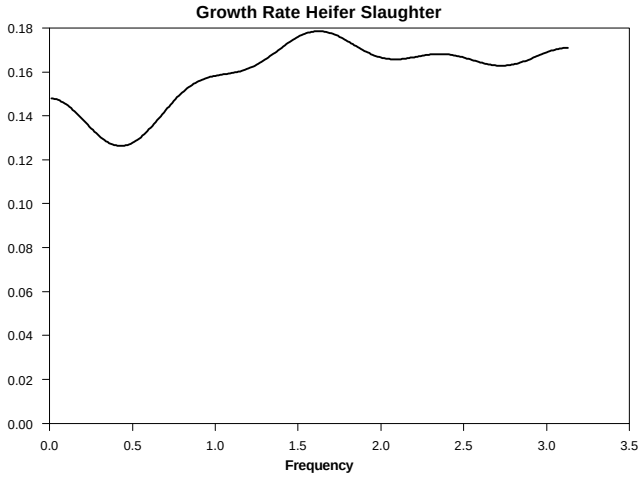


Figure 16. Spectra for Simulated Feed and Cattle Prices