A COMPUTATIONAL APPROACH TO NEIGHBOURHOOD STRUCTURES IN THE SIMULATION OF DICHOTOMOUS DEVELOPMENT

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Abstract

This paper addresses to the specific topic of simulation of interaction among individuals, by means of spatial connections.

An overlapping generation system is here in order: at each step the agent chooses how to allocate potential labour between work and study, and hence, how to divide wages between consumption and savings, which will be in turn, invested into physical capital, to be used in the second part of his life. New generations acquire from the elders previous situation, with changes induced by learning and neighbourhood effects.

The efforts of this study have been concentrated in the task of modelling such learning and neighbourhood effects, as well as in the analysis of different choices, both for neighbourhood amplitude, and the topology applied.

In this way, the emergence of dichotomous growth has been explored, analysing the impact of spatial and psychological relationships, when they are simulated through connectivity structures, well known to topology representation theory, and strictly related to the approach of self-organisation in cortical brain models.

I. Introduction

Computer simulation is nowadays a key technique to model economic dynamics [4].

Many reasons can explain the actual interest on such topic: this work takes as the most remarkable the one outlined in Kirman (1997)[9], who emphasised the importance of viewing to the economy as to an evolving network.

In this context, interaction is a leading aspect in modern economy, where individual behaviour arises as synthesis of both previous personal experience and partnership effects: our behaviour influences others, as well as others can affect our welfare.

Such interdependence, together with its implications have been powerfully summarised in the illustration of the so called "social dilemmas" [11] as expressions of complex dynamic processes.

Social dilemmas are interpersonal and inter-group situations, which occur when potentially advantageous patterns introduced into a system become unsatisfactory, because they drive the agents in the community to outcomes which should have been better off by individuals running separately.

Hence, plausible simulations of interaction should take into account at least three interrelated levels of issue:

(a) the individual level, driven by personal interest;

(b) the aggregate level, where global behaviour not necessarily emerges as simply cumulative from the individual stage;

(c) the level of the bi-directional flow, linking individual to aggregate behaviour, and viceversa, so that the former stage affects the dynamics of the whole, as well as the macro level, in turn, may influence the micro one.

A common approach documented in the contemporary literature is to model those interplays by means of cellular automata [6] , genetic algorithms [2], or even hybridisations of both previous methods [14],[15].

The discussion of such techniques, however, goes beyond the scope of this paper, that addresses to the specific field of simulation of interactions by means of spatial connections.

In particular, the emergence of dichotomous growth will be explored, when the spatial distribution is implemented through connectivity structures, well known to topology representation theory, and strictly related to the approach of self-organisation in cortical brain models [8],[10] , [12], [13] .

Simulations have been performed starting from the assumption of a two sector growth model, similar to that in McCain [14], but for a few modifications.

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As in McCain, an overlapping generation system has been considered. Each individual lives for two periods; at each step agents choose how to allocate potential labour between work and study, and hence how to divide wages between consumption and savings. Those, in turn, will be invested in physical capital, to be used in the second part of individual life.

The production function for tangible goods is a Cobb-Douglas function:

$$Q = (L\ K\ H)^{1/3}$$

where Q ,L , K , H are, respectively, production, labour services, tangible and human capital.

New generations acquire from elders previous situation, with changes induced by learning and neighbourhood effects.

Our efforts have been focused in the task of modelling such effects, adding to the original model the impact over decision variables coming from the existence of notable spatial connections.

In our model, in fact, connections coming from different topology assumptions are considered, so that they are able to represent both neighbourhood effects in strictly geographical sense, and also collective behaviours related to affinity and other psychological motivations.

In such sense, linkings have been thought relevant both to condition the level of human capital (and hence production), and also propensities to save and to study.

The plausibility of this approach is investigated through the adoption of classical cross neighbourhood with various radius amplitude and various clique typologies [3] In particular, the impact of radius amplitude will be explored both when it is maintained unvarying along the whole simulation, and when it is decided by each cell according to fitness principles.

The remaining of this paper is as follows: section II briefly introduces to the theory of cortical brain maps, focusing on the way information is tuned by elementary artificial brain units (neurons), and on its mathematical formalisation. In this context, the implications of such approach for the simulation of complex systems will be remarked. Section III describes the assumptions for the economic model under examination. Section IV discusses simulation results, and section V contains some conclusive remarks and outlooks for future works.

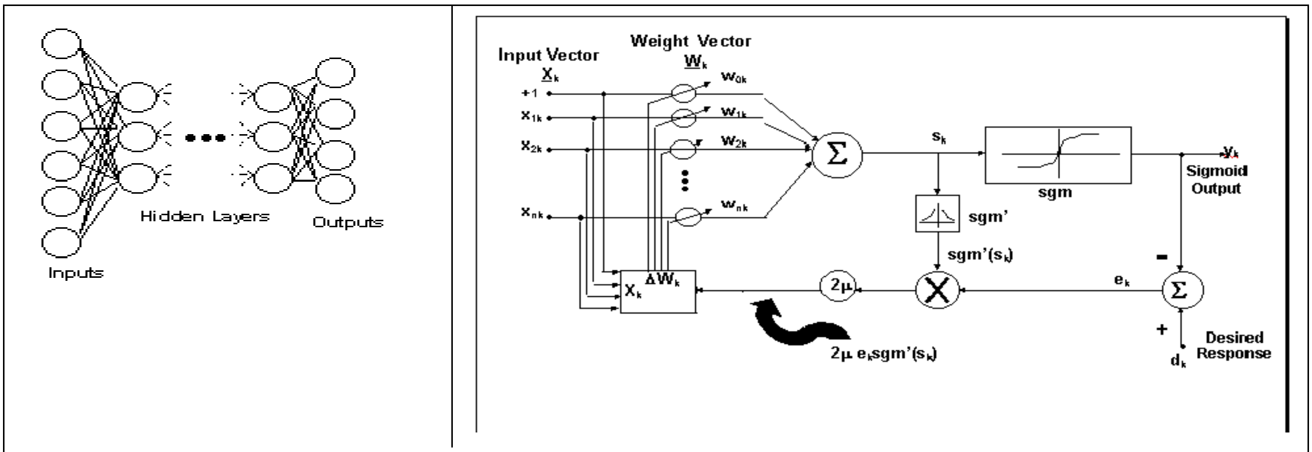
II. Self Organisation and Cortical Brain Models.

Artificial Neural Networks (ANN since now on), have gained increasing popularity over the past two decades, because of their ability to model input/ output relationships through plastic linking, which can evolve and adapt over time.

Their formalism seems to be inspired by the principles of nervous system architecture: nodes (i.e. neurons) are modelled as I/O elements, with connections (corresponding to biological synapses), which generally assume values in the open interval (-1,1), as to say the strength and significance of their activation respect on input patterns.

The more investigated kind of ANN (both at theoretical and applicative level) is the so called Feed Forward Neural Network (FFNN), where neurons are organised into one or more layers, with connections exclusively between adjacent layers [1], [5], [7].

Those nets are generally driven to learn by the Back-Propagation algorithm, which makes use of steepest descent techniques to minimise the error between the network and a given desired response (a schematic illustration of this principle is given in Figure 1 (b)).



(a) (b)

Figure 1. A Feed Forward Neural Network (a) and a snapshot on the Backpropagation algorithm (b) usually employed to train them.

Despite of their large diffusion, FFNNs have been source of criticism for almost a couple of reasons:

- the procedure seems to be not efficient, in the sense that it is sensitive to local minima;
- connections among layers doesn't seem plastic enough, to take advantage of the parallelism of the architecture.

From this side a plausible alternative comes from the theory of cortical brain maps, and the related Self-Organising Maps (SOM) algorithm [10].

Self Organising Maps are unsupervised neural models consisting of a number of neurons generally arranged into a two-dimensional grid, driven to preserve topological relationships over the input, while performing a dimensionality reduction of the representation space.

SOM basic functioning may be summarised as follows.

The initial stage starts with a topological map M defined into a discrete bi-dimensional output space $\mathbb{Z}^2$, where neurons are arranged in a disordered manner. Let $\underline{w}_i \in \Omega \subset \mathbb{R}^d$ the pointer associated to neuron *i* in the map.

At each step, an input $\underline{u}$ from a continuous space $\mathbb{R}^d$ is presented to the net, then the algorithm describe a mapping Φ from $\mathbb{R}^d$ to $\mathbb{Z}^2$, according to which a winner neuron n or leader is selected in the map when it satisfies to:

$$n = \arg\min_{k \in M} \left\| \underline{w}_k - \underline{u} \right\|$$

This makes possible to order neurons according to both their similarity with input and among themselves:

(1) $$p(i)>p(j) \Leftrightarrow (\|\underline{u}-\underline{w}_i\|>\|\underline{u}-\underline{w}_j\|) \vee (\|\underline{u}-\underline{w}_i\|=\|\underline{u}-\underline{w}_j\|) \wedge (i>j)$$

where p(i) is the position in M of neuron n_i at time t+1.

Hence, both the pointer $\underline{w}_n$ associated to leader neuron, and all the pointers $\underline{w}_j$ belonging to a convenient (according to (1)) neighbourhood in the map are modified with the following rule:

(2) $$\Delta \underline{w}_j = h_{ji}(\alpha, d_{map}(p(i),j,i))(\underline{u}-\underline{w}_j)$$

where:

α is a fixed constant, $d_{map}(p(i),j,i)$ is a distance function which takes into account the ordering on neurons coming from (2), $(\underline{u}-\underline{w}_j)$ is the error between input and each pointer, and $h_{ji}(.)$ is the neuron interaction function between two neurons: it depends on the distance in the map d_{map} between each node.

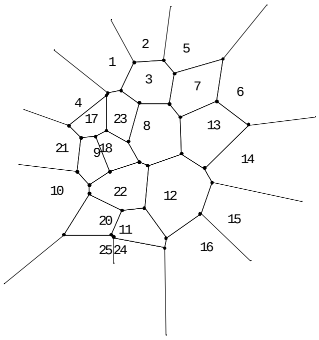
On following, I'll assume that:

$$h_{ji}(\alpha, d_{map}(p(i), j, i)) = e^{-\alpha \cdot d_{map}(p(i), j, i)}$$

The learning phase is completed after presenting a large number of inputs to the map.

Two points need to be remarked.

1. It appears immediately that SOM algorithm takes into account spatial relationships twice, in quite different ways: once neurons are ordered among themselves according to (1), at each step a sort of preprocessing (respect on weights adjustment) stage is performed, through which the Voronoi tessellation of input space (better its changes over time) is captured. This makes possible to gain more information about the way input is captured by the net.



(a)

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

(b)

Figure 2. Voronoi tessellation (a) and "traditional" neighbourhood structure (b).

Hence, in the proper learning phase (see (3)) information is retrieved, by considering linkages among neurons according to conventional neighbourhood formalisms (cross shaped, Moore,…)

2. It is easy to see that by keeping α closer to 0 (e.g. α<0.01), the impact of additional information coming from input tends to be spanned at highest level over its nearest neurons.

   On the contrary, if α is maintained nearer to 1 (e.g. a>0.1), then neurons within a k-width neighbourhood from input will be lesser sensitive to news than in prior case.

Figure 3 shows these ideas in a more intuitive fashion.

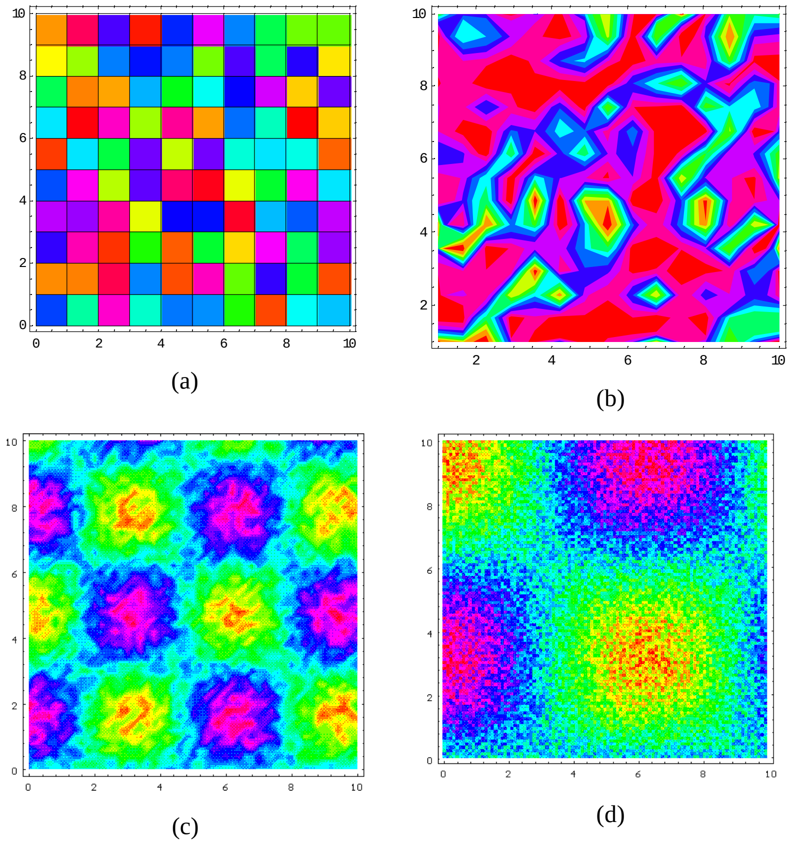


Figure 3. Self Organising Maps.

TOP: in clockwise sense, a 10x10 SOM map at initial step (a) and after 2000 iterations (b). Neurons are coloured according to their similarity to neighbours in an Euclidean sense. Note that from original disordered state the map has evolved to ordered state, with similar cells aggregated into neighbourhoods.

BOTTOM: Neighbourhood Effects over a 10x10 Self – Organising Map when α is maintained closer to 1 (c), or to 0 (d).

In this way, by properly varying both α values and the $d_{map}(p(i),j,i)$ function it is possible to control the learning phase, so that:

- neighbourhood with same shapes (e.g. cross) have different sensitivity to information spanned over them (α varying) ;
- equal intensity of information (α unchanged) may be spent over different shaped areas, thus enforcing (or penalising) the effect of original input over the map.

These aspects make SOM a quite promising instrument to model human behaviour and interactions into an economic system. In particular, the extreme flexibility which is possible to gain by operating over α and d_{map} offers the opportunity to reproduce swarm effects, as well as its antonym that is the individual specification as sole identity

III. The economic model

In this section the economic assumptions underlying the model under examination are briefly introduced and discussed.

As earlier discussed in McCain [14], a two-sector overlapping generation model is here in order.

Youngs begin their life with an equal amount of potential labour s , which has to be divided between work and study. Wages, in turn, will have to be employed in first-period consumption and savings. Those latter will be used in the second stage of life, when individuals retire and live of profits generated by capital.

The function ruling the production of tangible good at time t is a Cobb-Douglas function of the type:

(3) $$Q_t^{(i)} = \left(L_t^{(i)} K_t^{(i)} H_t^{(i)}\right)^{1/3}$$

where $Q_t^{(i)}, L_t^{(i)}, K_t^{(i)}, H_t^{(i)}$ are, respectively, production, labour services, tangible and human capital for the i^{th} agent.

Labour services $L_t^{(i)}$ depend on initial potential labour disposal, as well as on propensity to study $v_t^{(i)}$. Simulations have been run assuming L as follows:

(4) $$L_t^{(i)} = s\left(1 - v_t^{(i)}\right)$$

Analogously, physical capital K is a function of propensity to invest into physical capital z_t and residual propensity to study from previous step:

(5) $$K_t^{(i)} = s^2 z_t^{(i)}\left(1 - v_{t-1}^{(i)}\right)$$

Human capital available to agent is given by:

(6) $$H_t^{(i)} = (1 - t)H_{t-1}^{(i)} + g\left(v_t^{(i)}\right)H_t^{*(i)}$$

where t is a constant value in the interval [0,1), $H_t^{*(i)}$ allows labour externalities into the model, being the average human capital into the spatial neighbourhood of each agent, and $g\left(v_t^{(i)}\right)$ is a conditional function which ensures diminishing returns to efforts in human capital formation.

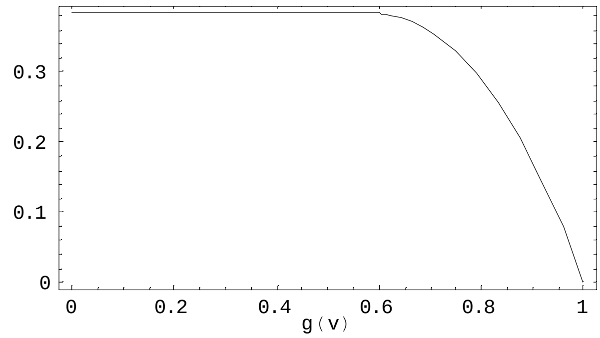


Figure 4. An admissible shape for g(v).

Finally, utility function is given by:

(7) $$U_t^{(i)} = \left(s\ v_t^{(i)}\right)^{2/3} Q_t^{(i)} \left(z_t^{(i)}\left(1 - z_t^{(i)}\right)\right)^{1/2}$$

Starting from those assumptions, a computational model involving Self-Organising Maps has been developed.

A rectangular grid of agents is considered, with border joined together, so that a torus has been formed. Hence, from now on agents will have to be thought lying over a continuous surface.

Each individual is associated to a reference vector

(8) $$\underline{u}_t = \left\{ v_t^{(i)}, z_t^{(i)}, Q_t^{(i)}, H*_t^{(i)}, H_t^{(i)}, U_t^{(i)} \right\}$$

representing agent condition respect on variables previously defined.

In this context, v and z have been assumed as control variables, i.e. parameters whose evolution can influence decisively the behaviour of production, work and hence the formation of utility profile.

The procedure is briefly summarised at a generic step t.

Definition 1 *(Best performer).*

A best performer(BF) is said to be agent whose utility has resulted at highest level in previous step.

The best performer is selected over the net:

(9) $$BF_t = \max_{i \in M} U_t^{(i)}$$

At this point, a Voronoi tessellation of neural space is performed, by ordering neurons according to their distance from the couple $\left\{ v_t^{(BF)}, z_t^{(BF)} \right\}$, hence this order is retrieved through learning procedure (2).

Therefore, each agent acquires new propensities to study and to save which are used to calculate step values for production Q, utility U and labour H.

Respect on the original formulation which has been referred to, a random perturbation $\xi \in (-1,1)$ has been introduced, so that the tuning to BF position might not be perfect at all. Therefore (2) in our model becomes:

(10) $$\Delta \underline{w}_j = h_{ji}(\alpha, d_{map}(p(i),j,i))(\underline{u} - \underline{w}_j) = e^{-\alpha \cdot d_{map}(p(i),j,i)}(\underline{u} - \underline{w}_j) + \xi$$

By properly choosing the kind of relationships among neighbours, it is possible to force the net to give more or less emphasis to the proximity of neurons.

This study has considered three different types of neighbourhood:

(a) von Neumann or cross neighbourhood;

(b) Moore neighbourhood;

(c) “elastic” neighbourhood, as to say a system of clique typologies among which each neuron can decide to refer to, according to its fitness condition over the system.

From now on those types of proximities will be referred as VN(r), MN (r), EN (r) for von Neumann, Moore, and elastic neighbourhood respectively, where “r” stands for the radius amplitude under examination.

The same kind of neighbourhood activity has been assumed running in the way (eventual) labour externalities might impact the system.

IV. Simulation results.

In this section simulation results will be discussed.

A Self Organising Map of 20x20 neurons arranged into a rectangular lattice has been considered. Each neuron is structured as mentioned in previous section, i.e. it is associated to a 6^{th}-uple like that in (8).

Various neighbourhood have been tested over 1000 runs.

Parallel runs have been performed, using the same pseudo-random initialisation for each type of neighbourhood, and average results have been reported for each situation.

System parameters have been maintained unvarying all the simulation long, and are reported in Table 1, together with the kind of neighbourhood used.

Parameter	σ	τ	α	ξ
Value	10	0.02	0.6	0.01

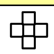
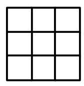
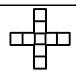
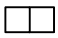
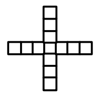
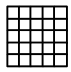
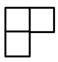
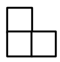
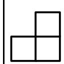
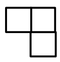
Neighbourhood	**Von Neumann (cross-shaped)**		**Moore**		**Elastic**				
Amplitude	**1**		**1**		**0**				
	2				**1**				
	3		**2**		**2**				

Table 1. Parameters values and types of neighbourhood used.

The congruence of SOM approach has been investigated under various aspects.

As former stage, the natural question which arises is: *does neighbourhood matter in the emergence of dichotomous development?*

Note that SOM approach can stress on this feature, in the measure it insists on the importance of linkages among individual both in strictly “regional” sense (relevance of traditional neighbourhood), and in “affinity” sense (Voronoi tessellation), that is by adjusting the shape of d_{map} in (4).

In those simulations, the general impact of both regionality and affinity has been observed in terms of production level, whilst affinity have been monitored mainly by looking at the evolution of v and z, and regionality through the behaviour of H* (labour externalities).

For sake of simplicity, results are shown separately, according to the type of regional neighbourhood in use: figures 5-7 show results for Von Neumann neighbourhood for radius amplitude from one up to three; figures 8-9 show results for Moore neighbourhood for radius amplitude equal to one and two; Figures 10-11 show results for elastic neighbourhood. In this case, simulations were driven so that although all possible, a particular neighbourhood choice could prevail over others: findings for single agent[2] and two agent width are reported, as to say situations where "egoistic" politics prevail.

Each figure shows respectively:

(a) the localization of agents with higher/low fitness in terms of production. The situation is fixed at initial step and hence after 500 and 1000 iterations.

(b) the dynamics of the distribution of propensities to study *v*, to save *z*, and that of the levels of externalities over human capital *H** and human capital itself. Also in this case the evolution of significant variables is fixed at different steps.

(c) the distribution (in percentage terms) of various levels of agents with high/low welfare.

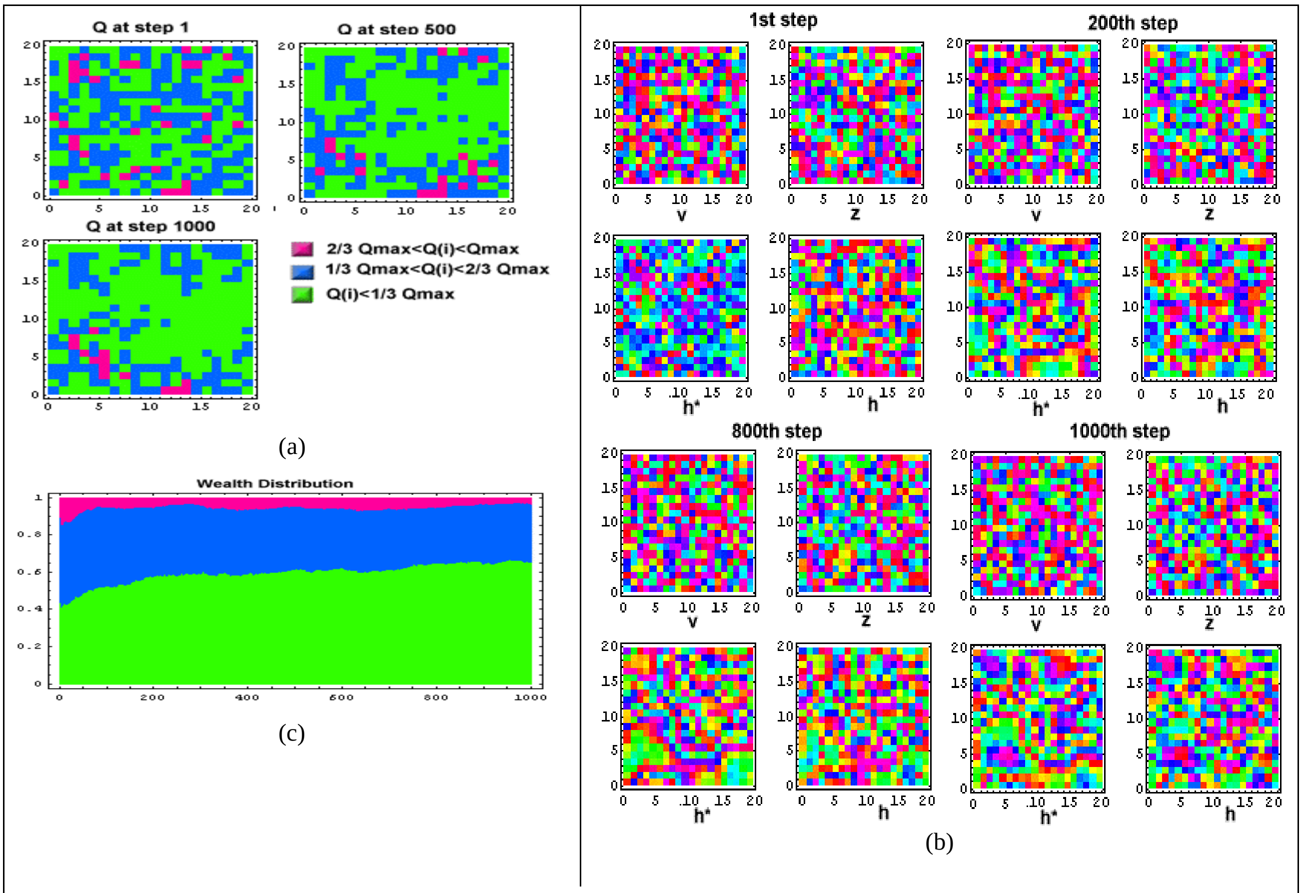


Figure 5. Results for Von Neumann Neighbourhood with radius 1, VN(1).

[2] Note that this condition is equal to assume a Moore or Von Neumann Neighbourhoods with zero radius amplitude.

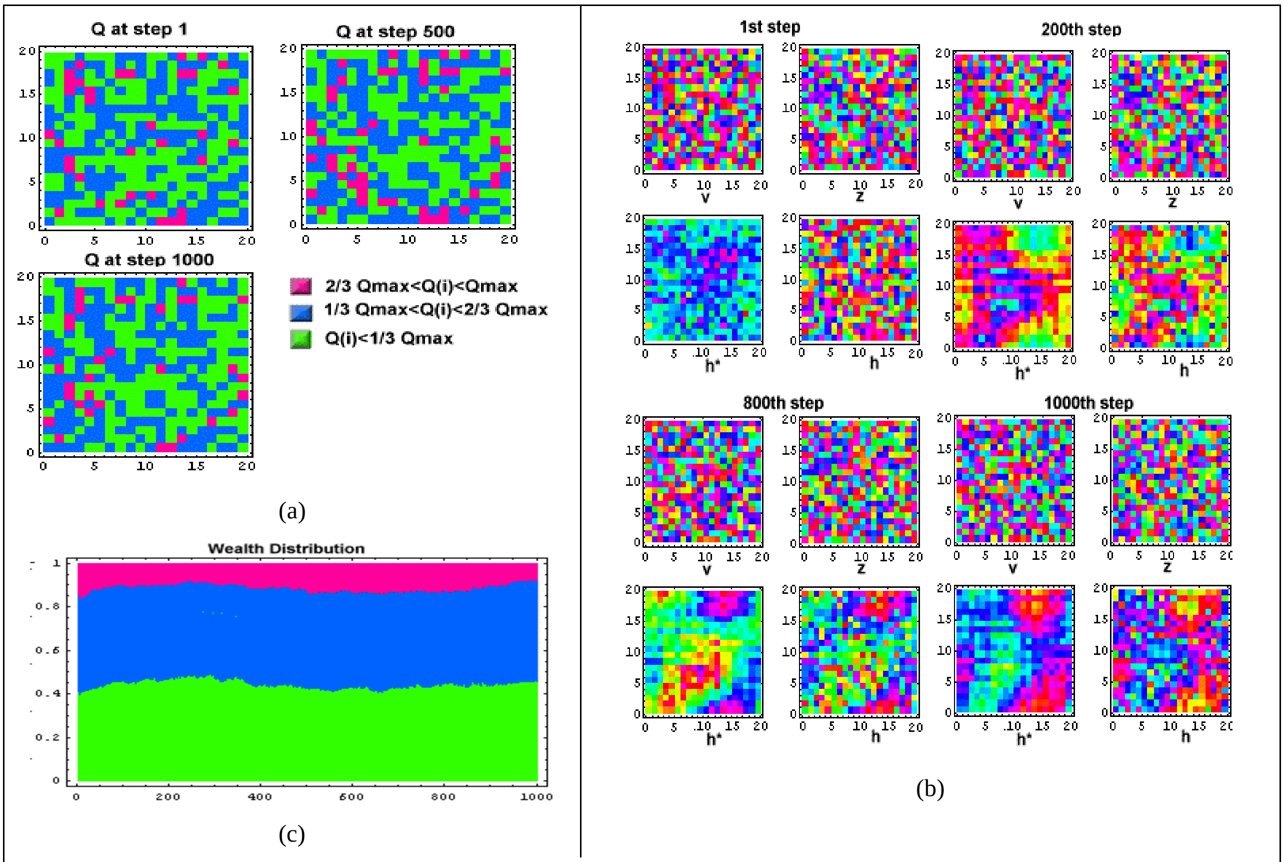


Figure 6. Results for Von Neumann Neighbourhood with radius 2, VN(2).

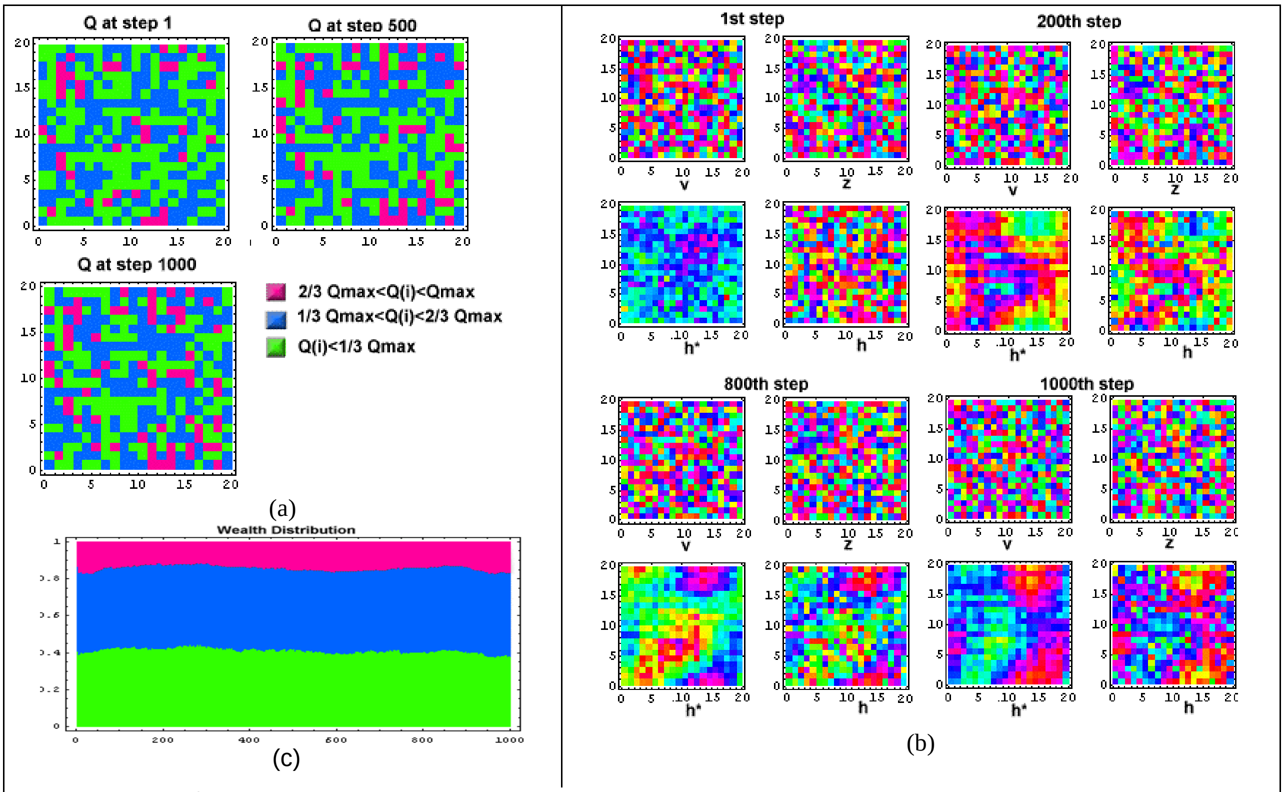


Figure 7. Results for Von Neumann Neighbourhood with radius 3, VN (3).

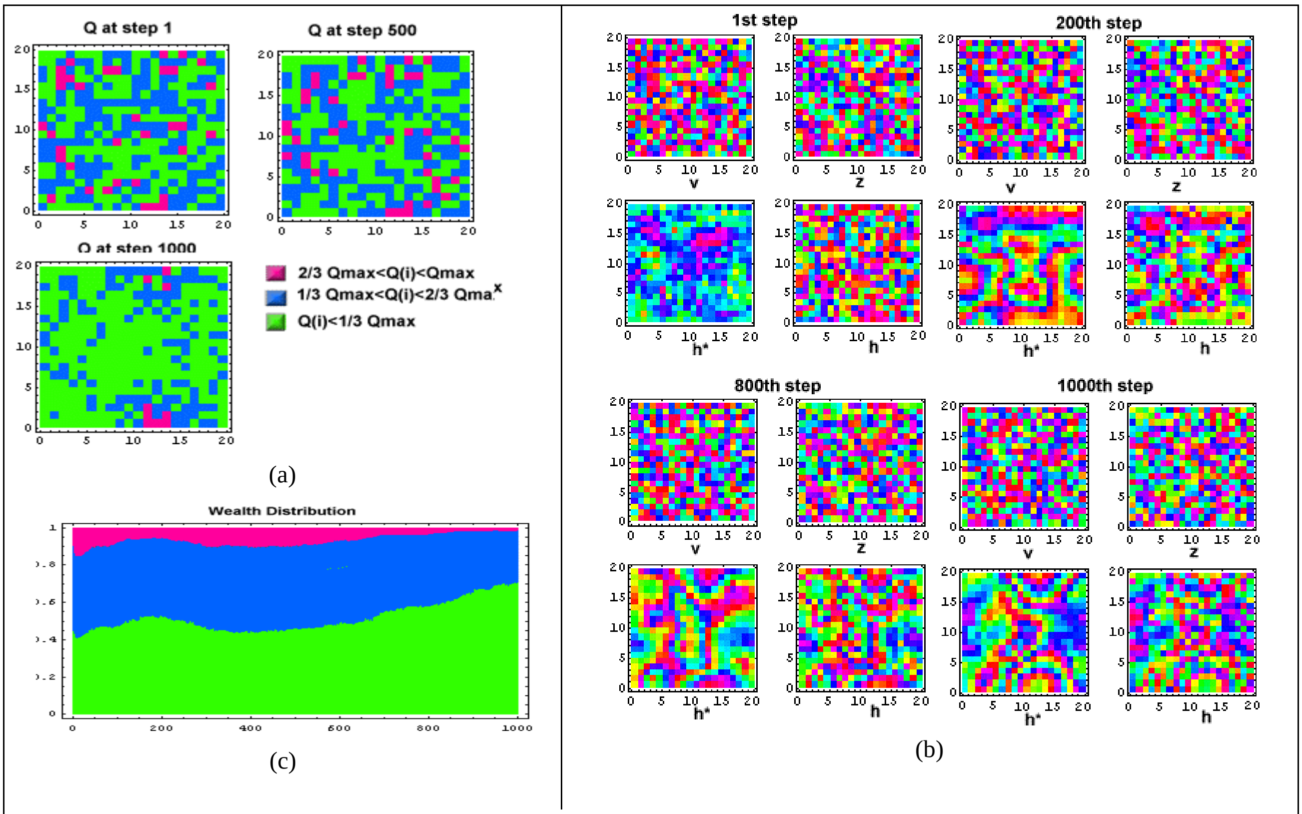


Figure 8. Results for Moore Neighbourhood with radius 1, MN(1).

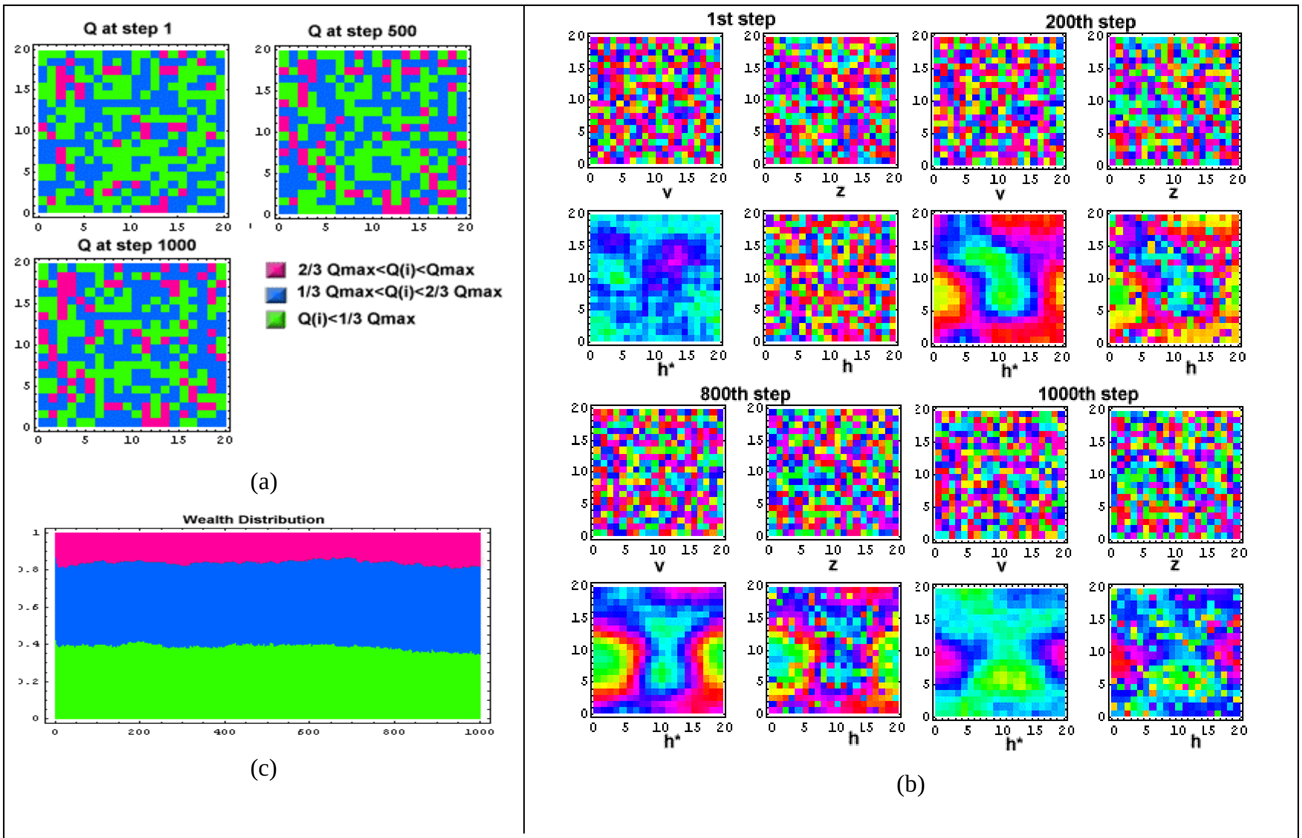


Figure 9. Results for Moore Neighbourhood with radius 2, MN(2).

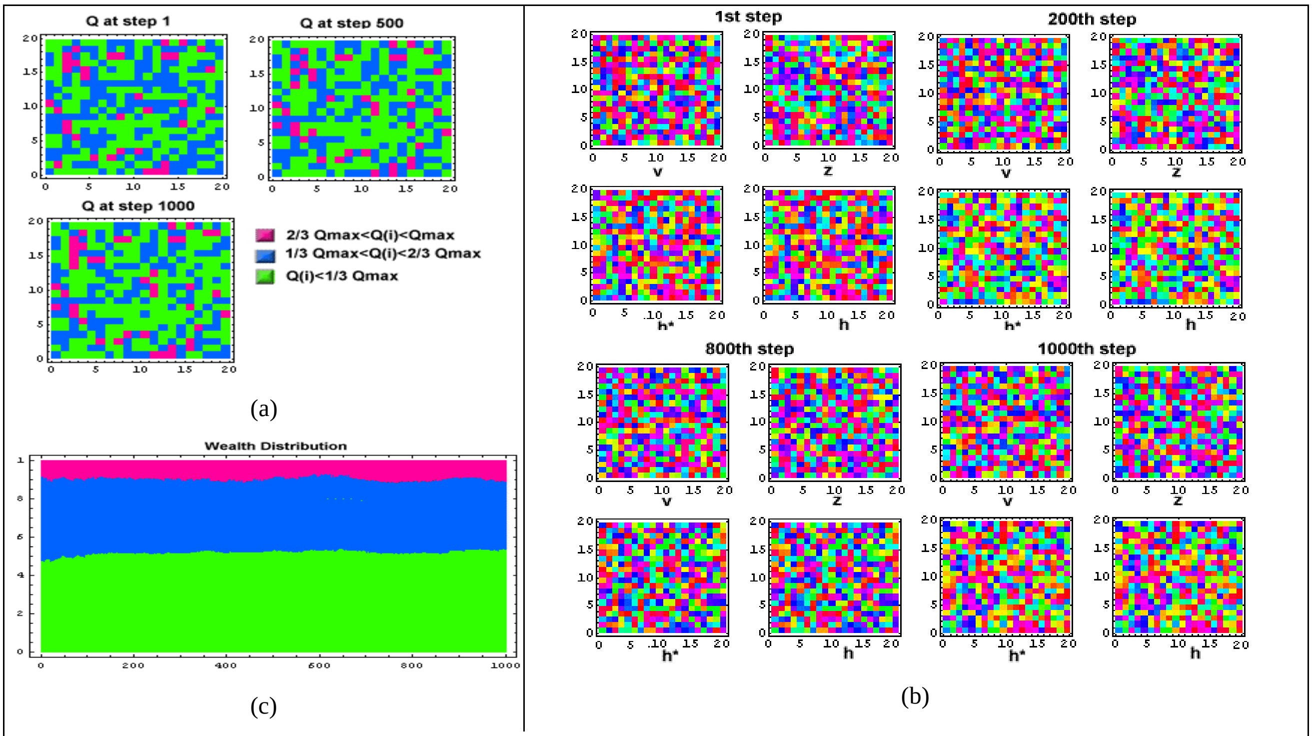


Figure 10. Results for elastic Neighbourhood with radius 1, EN(1).

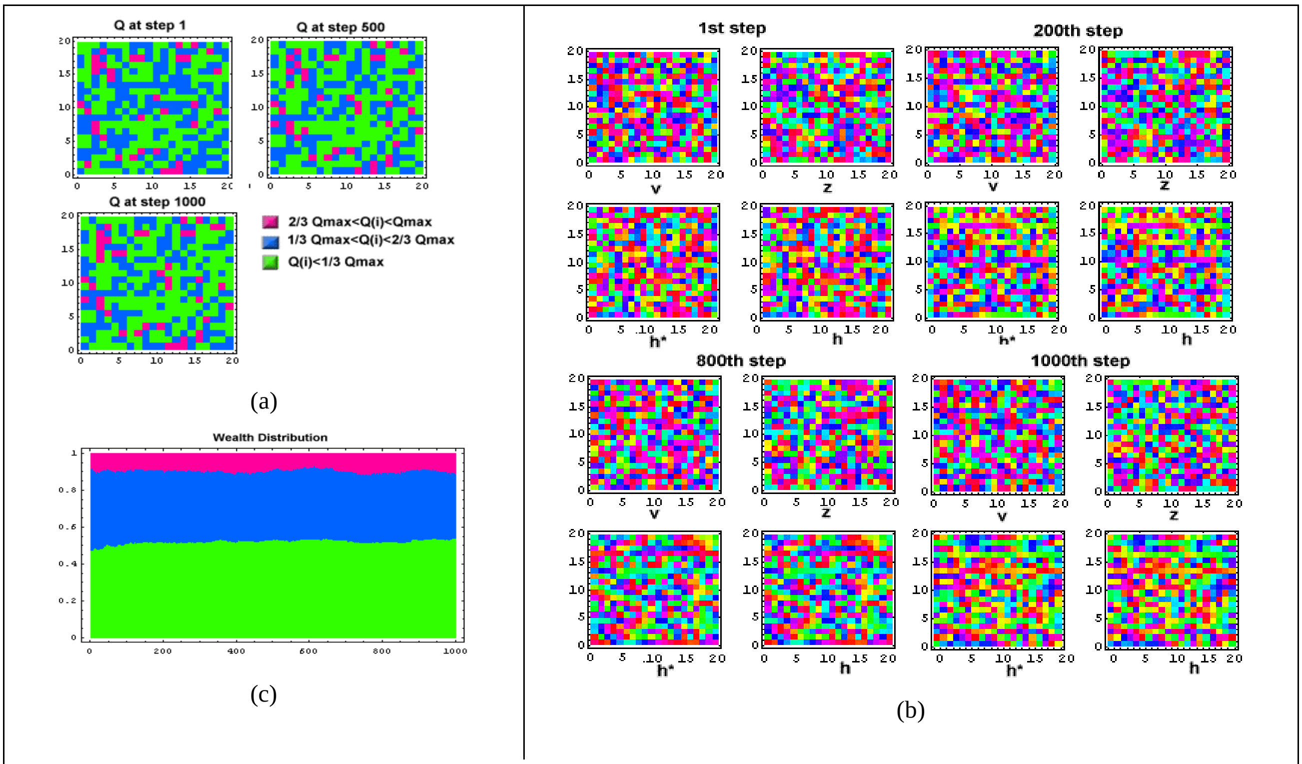


Figure 11. Results for elastic Neighbourhood with radius 2, EN(2).

A number of considerations may be drawn down:

1) Monitoring system level for control variables, both v and z seem to converge to plausible values with neoclassical hypotheses (Figure 12)

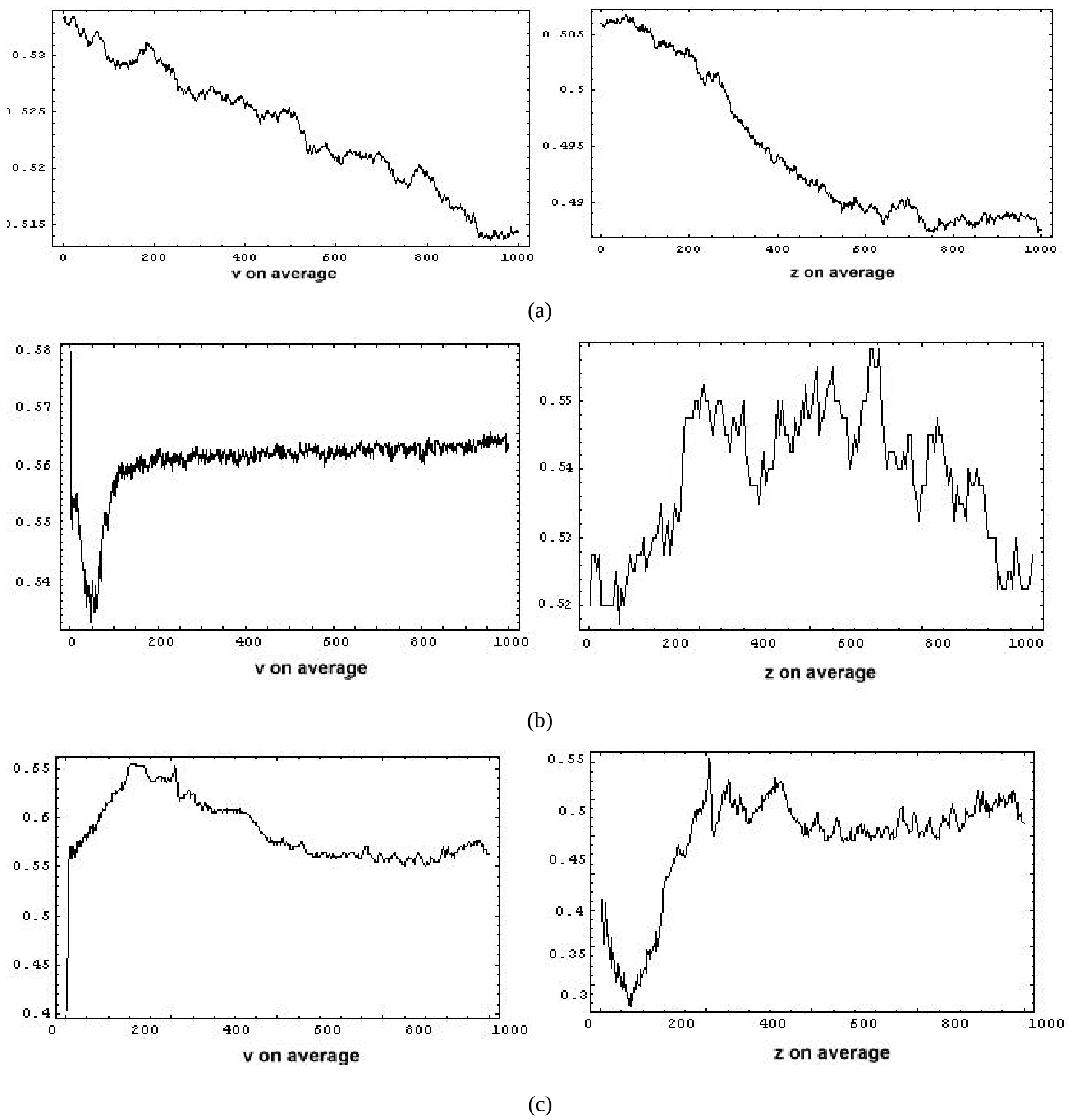


Figure 12. Average Behaviour of system values for propensity to study (v) and to save (z).

These results appear stable, in the sense that they are limit values to which simulations converge independently by the kind of neighbourhood in order, as well as by particular initialisations (different startings have been chosen, using equally distributed random numbers -Figure 12 (a)- , or initial maps with random values into proper ranges, closer to 0 or to unit –Figure 12 (b) and (c)).

2) At first impact, it is possible to observe that affinity seems to be not really relevant in the development dynamics: agents having low, median or high propensity to study are present in equal parts throughout the simulation. This agrees with observation already present in [14] , that is equally trained agents (equal coloured cells in v, z grids, figures 5-11 (b)) tend to have different productivity, according to the particular regional context they are placed in.

3) At the same time, wealth distribution (more properly welfare condition, plot (c) in every figures) appears variously structured accordingly to the different regional neighbourhood prevailing in the simulation.

   Since the regional dimension has been inflated in the model through the labour externalities (H*), it should be reasonable to deduce that the major impact on the welfare level is played by the presence of labour externalities, when the shape of neighbourhood is wide enough.

However, that is not ever true, and contradictory signals are offered for dichotomisation.

For example, dichotomisation appears strongly both in VN(1) and MN(1), while it is not so evident in all other cases.

This should make possible to think that there exist a sort of “optimality” in radius neighbourhood amplitude allowing for externality effects and hence for the emergence of dichotomous growth.

More properly, dichotomisation seems to appear in presence of really localised neighbourhood (radius equal to one), whilst too wide proximity soften dichotomy effects.

V. Conclusions

This work has focused on the plausibility of computer simulation to reproduce the dynamic of economic systems. In particular, Self Organising Maps have been introduced as instrumental tools to perform this task: thanks to their extreme flexibility, by properly varying control parameters, it is possible to drive them to reproduce a wide variety of situations, which cab be useful to emulate (obviously in a simplified way) real world dynamics. To this purpose, it has been pointed how Self Organising Maps inherited features could be use to represent both affinity among individuals (and hence their psychology), and regional proximity.

Starting from this point, various simulations were implemented, using a variety of possible neighbourhood, in order to test the emergence of dichotomous growth, and some possible explications of such phenomenon.

From those simulations, it has emerged, that, although psychologically similar, agents are strongly influenced by regional factors, under the conditions these latter, in turn, must be really localized. In particular it has emerged that there exist an “optimality threshold” for radius neighbourhood amplitude beneath which proximity effects tend to be soften.

This conclusion has been supported by looking at the dynamics of the simulated artificial world with proximities affinities represented through propensities to study v and to invest into physical capital z, and regional neighbourhood structure inflated through the presence of labour externalities. The evidence of dichotomous growth has arisen in presence of really localized neighbourhoods (unit radius for Von Neumann and Moore neighbourhood), whilst the use of too spanned neighbourhoods (radius greater than two) has produced a better distribution of different levels in welfare and production.

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