Local durability and long-run habit persistence : An evaluation of the U.S risk premia.

Olivier Allais*

January 2000

Abstract

I study the empirical properties of a non-linear stochastic dynamic representative-agent model with rational expectations. The representative agent is assumed to have time non separable preferences. The time nonseparability in preferences is due to local substitution of consumption over time as well as to long-run habit persistence. Specifically, I investigate whether the dynamic model replicates the observed mean and the standard deviation of the U.S real returns in the 1965-1987 period. I use a projection method to solve the model and then I evaluate the intertemporal marginal rate of substitution (IMRS) as well as the asset returns implied by the dynamic model. First, I find that the IMRS implied by the model statistically fits the Hansen and Jagannathan bound. Secondly, I find that combined effects of substituion and complementarity over consumption nearly solve the equity premium and the risk-free rate puzzles. Finally, the model does also resolve the Campbell's stock market volatility puzzle.

Keywords : Habit formation and durability, equity premia, volatility of the intertemporal marginal rate of substitution, Hansen and Jagannathan bound, projection method.

JEL : G12, C63.

*EUREQua-université de Paris I.
Maison des Sciences Economiques, 106–112 Bd. de l'Hôpital, 75647 Paris Cedex 13, France. E-mail : *oac@univ-paris1.fr,* Tel : 01 44 07 82 13, Fax : 01 44 07 82 31.
I am indebted to Fabrice Collard who provided me helful comments about the computation of the model. I also thank Theptida Sopraseuth for useful suggestions.

1 Introduction

This paper is motivated by two empirical issues : the equity premium puzzle and the risk-free rate puzzle. Mehra and Prescott[15] show that Arrow-Debreu asset pricing model could not explain high equity premia, unless agents are extremely risk averse. One response to the equity premium puzzle is to accept these high values for the coefficients of relative risk aversion. However, Weil[18] argues that this leads to a second puzzle. He shows that a low riskless interest rate is possible only if agents have negative rate of time preference, in a standard consumption-based asset pricing model. Thus, they reduce their desire to borrow from the future. Weil calls this the risk-free rate puzzle. Number of authors have suggested that time nonseparability could help explain the poor empirical performances of the standard consumption-based asset pricing model[1]. Constantinides[6] argued that habit persistence could solve the equity premium puzzle of Mehra and Prescott. However, his model only displays complementarity over consumption. So, he does not take into account that consumption at dates local to time t should be relatively substitutable for consumption at time t. I choose to reexamine these puzzles by introducing notions of substitution (durability) and complementarity (habit) of consumption over time, as Heaton[12]. The details of the model are described in section 2. I use a projection method to solve this non-linear stochastic dynamic model with rational expectations. Then, I study whether the dynamic model replicates the observed mean and the standard deviation of the U.S real returns in the 1965-1987 period.

The computational aspects of my analysis are complementary to the study of Heaton[12]. Heaton considers an economy represented by seven state variables. In contrast, I derive the optimal portfolio rules implied by the model with only two state variables, which substantially improves the accuracy of the results. Thus, the projection method is implemented with a two state variables vector and the dividend variable is used as an exogenously given shock. I assume that the growth rate of dividend follows a first order Markov chain. The computational method is described in section 3. The resolution of the dynamic problem is done in two steps. In the first step, I compute the approximations of marginal utility of consumption and intertemporal marginal rate of substitution in consumption (IMRS). Then, I calculate the approximations of equity price and risk-free asset price. I finally deduce the returns on the equity and the risk-free security implied by the dynamic.

From these simulations I carry out two complementary experiments to see if this model could explain the U.S risk premia in the 1965-1987 period. First, I test if the model's implications concerning the volatility of the IMRS are satisfied. I find that the IMRS implied by the model statistically fits the Hansen and Jagannathan bound. Secondly, I analyze the time-series properties of the simulated model. Specifically, a constrained grid

[1] We could find other analyses of models with habit persistence in Abel[1], Campbell and Cochrane [3], Sundaresan[17], for example.

search is carried out to find the parameter values that fit the first two observed moments. They are chosen in order to meet the positivity of the marginal utility of consumption. For a set of parameter values, I find a simulated premium equals to 4.7% per annum. Though my estimate is below the 6% often cited in the literature on the equity premium puzzle, the model do a good approximation compared to the representative-agent model. In addition, I find that the introduction of local substitution substantially improves the model's ability to fit the volatility of risk-free rate, compared to the pure habit persistence model and it solve the Campbell's stock market volatility puzzle[2]. Finally, I conclude that the combined effects of substitution and complementarity over consumption nearly solve the equity premium and the risk free rate puzzles. These results are presented in section 4, and section 5 concludes the paper with some remarks about potential extensions.

2 The model.

I consider a single-agent economy with frictionless markets and no taxes. The assumption of a single-agent economy is standard and is made in the spirit of Lucas[14] and Cox, Ingersoll and Ross[7]. The representative agent has preferences over a good S_t, which are represented by the constant relative risk aversion (CRRA) utility function

$$U(S) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{S_t^{1-\gamma} - 1}{1-\gamma}, \qquad \gamma > 0, S \equiv \{S_t : t = 0, 1, 2, ...\}. \tag{1}$$

where β is the agent's subjective time discount factor, $E_0(.)$ is the mathematical expectation operator conditional on information in period zero.
I assume time nonseparability in preferences over the consumption goods C_t, introducing
i) local substitution of consumption over time,
ii) habit persistence,
as in Ferson and Constantinides[10] and Heaton[12].

They assume that in and every period the new expenditures C_t produce a flow of consumptions C_t^F given by

$$C_t^F = \sum_{\tau=0}^{\infty} \delta^\tau C_{t-\tau}, \text{ where } 0 \leq \delta \leq 1 \text{ and } \sum_{\tau=0}^{\infty} \delta^\tau = 1 \tag{2}$$

The parameter δ measures the degree to which consumption is substitutable over time. Finally, substitution effects and habit persistence are related according to :

$$S_t = C_t^F - \alpha(1-\theta) \sum_{j=0}^{\infty} \theta^j C_{t-1-j}^F, \quad 0 \leq \alpha \leq 1, \quad 0 \leq \theta \leq 1, \text{ and } \sum_{j=0}^{\infty} \theta^j = 1 \tag{3}$$

The good S_t is composed of two elements. The first element represents the substitution effect over the consumption good C_t, and the weighted sum of lagged consumption flows $(1-\theta)\sum_{j=0}^{\infty}\theta^j C_{t-1-j}^F$ measures the habit stock. The parameter α gives the proportion of habit stock that enters the preferences. The introduction of habit persistence effects makes consumption complementary over time. If $\theta = 0$, then the model is just a one-period habit model, as it was studied by Ferson and Constantinides[10]. So, when $\alpha = 0$ the utility function is time-separable in consumption flow, and the model reflects only substitution of consumption over time. When $\alpha = \delta = \theta = 0$, the model reduces to the case of the time-separable preferences.

Combining (2) and (3) implies that S_t is given by

$$S_t = \frac{1-\phi L}{(1-\delta L)(1-\theta L)} C_t = A(L)C_t, \tag{4}$$

where $\phi = \theta + \alpha(1-\theta)$, and L is the lag operator. We can write S_t as $S_t = A(z)C_t$, for $z \in \Re$. As shown by Ferson and Constantinides, and Heaton, this model displays substitutability for low z and habit persistence for high z. Thus, if $\delta < \theta$, and if α is not too large, $\frac{1-\phi L}{(1-\delta L)(1-\theta L)}$ will be positive for low z and as far as z will become higher habit persistence will dominate durability. That is the reason why these authors present the model as a model which exhibits *local* substitution and *long-run* habit persistence.

Now, I suppose an environment in which the representative agent trades on securities markets. Two kinds of assets are traded : a risk-free asset, and a risky asset. Thus, the representative agent is faced with the following budget constraint

$$p_{f,t}f_t + p_{e,t}e_t + C_t = (d_t + p_{e,t})\, e_{t-1} + f_{t-1}, \tag{5}$$

where e_t, f_t are respectively the number of shares of equity and risk-free asset purchased by the agent at time $t-1$ and held until period t, $p_{e,t}$, $p_{f,t}$ are respectively the price of a unit of the corresponding assets at time t, and d_t is the stochastic dividend paid for each unit of equity held between periods $t-1$ and t.

The representative agent maximizes his intertemporal utility function subject to the equations (4) and (5). The agent solves the maximization problem by determining contingency plans for C_t, S_t, e_t, f_t. The Euler equation governing the equity price is given by

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \left(\frac{p_{e,t+1} + d_{t+1}}{p_{e,t}} \right) \right]. \tag{6}$$

The left hand side of (6) is the marginal utility cost of consuming one unit of numeraire good less at time t ; the righ-hand side is the expected marginal utility benefit from investing the unit in the risky asset at time t, selling it at time $t+1$ for $(\frac{p_{e,t+1}+d_{t+1}}{p_{e,t}})$ units, and consuming the proceeds. The agent equates marginal cost and marginal benefit, such as (6) describes the optmum. If we divide both the left and the right hand sides of (6) by λ_t, we get the familiar form

$$1 = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} . (\frac{p_{e,t+1} + d_{t+1}}{p_{e,t}}) \right] = E_t \left[m_{t+1} . R_{e,t+1} \right], \tag{7}$$

where m_{t+1} is the intertemporal marginal rate of substitution (IMRS) or the stochastic discount factor, and $R_{e,t+1}$ is the gross real rate of equity return.The first-order condition describing the risk-free rate price is

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \frac{1}{p_{f,t}} \right], \tag{8}$$

and the interpretation is the same as the Euler's risky asset equation.

In this model the marginal utility of consumption is a function of the marginal utility and the expected marginal utilitity of S_t. We get the following Euler equation

$$\lambda_t = \mu_t - \beta \phi E_t \left[\mu_{t+1} \right], \tag{9}$$

where μ_t, the marginal utilitity of S_t, is a function of μ_{t+1} and μ_{t+2} such as

$$\mu_t = S_t^{-\gamma} + \beta \left(\delta + \theta \right) E_t \left[\mu_{t+1} \right] - \beta^2 \left(\delta . \theta \right) E_t \left[\mu_{t+2} \right]. \tag{10}$$

Market clearing further imposes $C_t = d_t, \forall t$.

To empirically investigate the properties of this model, I must assess the marginal utility of C_t, and the price of equity. Unfortunately, I cannot analytically solve the model. I have to simulate the dynamic model. The new methods for simulating nonlinear rational-expectations equilibrium models make the exploration of these issues feasible. Nonetheless, the marginal utility of consumption depends on current expectations of marginal utility of S_t more than one period ahead, which do not make the computational resolution easier. In the next section I present the computational method that I use to assess the U.S real asset returns.

3 Solving method

In this section, I present how I solve the non-linear stochastic dynamic model presented in the previous section. The economy is described by a vector of two state variables $z_t = (S_t, Y_t)$, where $Y_{t+1} = S_t$, and a variable of an exogenously given shock d_t. The stochastic dividend evolves through time according to

$$d_{t+1} = x_{t+1} d_t$$

where x_t is the gross growth rate of dividend. It follows a first order Markov structure whose density of x_{t+1} conditional on x_t is given by $p_{x'}(x_{t+1} \mid x_t)$. This process allows the apparent non-stationarity I observe in the per capita consumption stream over the sample period. I assume two states for the Markov chain such as :

$$\begin{aligned} x_M &= 1+\phi+\sigma, \qquad x_m = 1+\phi-\sigma \\ p_{mm} &= p_{MM} = p, \qquad p_{mM} = p_{mM} = 1-p \end{aligned}$$

where ϕ is the average real growth rate of per capita consumption, σ is the standard deviation of the real growth rate of per capita consumption and $p = \frac{(1-\rho)}{2}$, where ρ is the first-order serial correlation of this growth rate. The US economic data in the 1965-1987 period imposes the following parameter values : $\phi = 0.0018$, $\sigma = 0.036$, and $p = 0.43$.

The model is a function of the non stationnary variable d_t, in the previous section. So the model is first deflated for dividend growth to express the model as a function of the stationnary variable x. Specifically, the marginal utilities of S and C are deflated by $d^{-\gamma}$, and the state variables are deflated by the dividend[2]. The deflated variables are in lowercase letters. I find the following first-orders equations :

$$\tilde{\lambda}_t = \tilde{\mu}_t - \beta\phi E_t \left[x_{t+1}^{-\gamma}.\tilde{\mu}_{t+1}\right], \tag{11}$$

$$\tilde{\mu}_t = s_t^{-\gamma} + \beta\left(\delta+\theta\right) E_t \left[x_{t+1}^{-\gamma}\tilde{\mu}_{t+1}\right] - \beta\left(\delta.\theta\right) E_t \left[x_{t+2}^{-\gamma}\tilde{\mu}_{t+2}\right], \tag{12}$$

where $\tilde{\lambda}_t$ and $\tilde{\mu}_t$ represent the deflated policy variables. The recursive equation of S_t is also transformed such as there are only stationnary variables in its expression. Applying the market clearing restriction, this equation is actually of order 2 since it includes s_t, s_{t-1}, and it depends also on $x_{t+1}$and x_t. Specifically, the recursive proporties of s_t are governed by the following functions

$$\begin{aligned} s_{t+1} &= (\delta+\theta)x_{t+1}^{-1}s_t - (\delta.\theta)y_t(x_{t+1}x_t)^{-1} - \phi x_{t+1}^{-1} + 1 = f(s,y,x,\tilde{x}) \\ y_{t+1} &= s_t \end{aligned} \tag{13}$$

I still have to transform the asset prices. In fact, the equity price alone is concerned by the modification, as far as a bond pays no dividend. The equity price is deflated by the dividend. The new Euler equations are now in function of x_{t+1} and are given by :

$$\tilde{p}_{e,t} = \beta E_t \left[\left(\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}\right) x_{t+1}^{-\gamma+1}(\tilde{p}_{e,t+1}+1)\right] \tag{14}$$

[2]Analytically, the deflations are such as $s_t = \frac{S_t}{d_t}$, $y_t = \frac{Y_t}{d_{t-1}}$, $\tilde{\mu}_t = \frac{\mu_t}{d_t^{-\gamma}}$, $\tilde{\lambda}_t = \frac{\lambda_t}{d_t^{-\gamma}}$, and $\tilde{p}_{e,t} = \frac{p_{e,t}}{d_t}$.

$$p_{f,t} = \beta E_t \left[\left(\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \right) x_{t+1}^{-\gamma} \right] \tag{15}$$

The model is solved relying on a method of weighted residuals (see Judd[13], and Christiano, Fisher [5]). I now describe the details of the method. The domain of approximation is restricted to $[s_m, s_M] \times [s_m, s_M] \times [x_m, x_M]$. The implementation of the computational method must consider that the marginal utility of s_t depends on the expected $\tilde{\mu}_{t+1}$ and $\tilde{\mu}_{t+2}$. This computational issue is undertaken using the weighted residuals projection method twice time. The resolution of the dynamic problem is done in two steps. In the first step, I compute an approximation of $\tilde{\mu}_t$, in order to restrict our attention particularly on the double projection. The approximation of $\tilde{\mu}_t$ is denoted by $\hat{\tilde{\mu}}$. Then, I compute the full model. I approximate $\tilde{p}_{e,t}$ by taking into account $\hat{\tilde{\mu}}$, and deduce an approximation of $p_{f,t}$.

First, we have to approximate the marginal utility of s. It is given by the following equation

$$\tilde{\mu}(s, y, x) \approx \hat{\tilde{\mu}}(s, y, x; \mathbf{a}_x) = \sum_{i=1}^{n_s} \sum_{j=1}^{n_y} a_{ij,x} \psi_{ij}(s, y), \quad \text{for } x = x_m, x_M \tag{16}$$

where $\psi_{ij}(s, y) \equiv T_{i-1}(2((s - s_m)/(s_M - s_m)) - 1) T_{j-1}(2((y - s_m)/(s_M - s_m)) - 1)$, $T_i(.)$ and $T_j(.) : [-1, 1] \to [-1, 1]$, are Chebychev polynomials, and $n_s \times n_y$ is the order of approximation. I use the linear transformation $2((s - s_m)/(s_M - s_m)) - 1$ in order to take into account that Chebichev polynomials are defined in $[-1, 1]$. I denote $\mathbf{a}_x$ the $n_s \times n_y$ dimensional column vector that has to be solved for the two states x_m, and x_M. These functions must satisfy the following residual function :

$$R(s, y, x; \tilde{\mu}) = 0, \text{ for all } s, y, x \in [s_m, s_M] \times [s_m, s_M] \times [x_m, x_M]$$

where the residual function is just defined by the residuals of the Euler equation (12)[3]. We form projection functions to approximate $R(s, y, x; \tilde{\mu})$. I implement the Galerkin method to compute the vector $\mathbf{a}_x$. The Galerkin method computes the $2 \times n_s \times n_y$ following projections :

$$P_{ij}(\mathbf{a}_x) \equiv \int_{s_m}^{s_M} \int_{s_m}^{s_M} R\left(s, y, x; \hat{\tilde{\mu}}_{\mathbf{a}_x}\right) \psi_{ij}(s, y)\, ds dy, \quad x = x_m, x_M, \tag{17}$$

and choose $\mathbf{a}_x$ so that $P_{ij}(\mathbf{a}_x) = 0$ for all $i = 1, ..., n_s$ and $j = 1, ..., n_y$. Here the difficulty is that each $P_{ij}(\mathbf{a}_x)$ is an integral which need to be numerically computed. Since the ψ_{ij} are Chebichev polynomials, we numerically compute the $P_{ij}(\mathbf{a}_x)$ using $(m_s \times m_y)$-points Gauss-Chebyschev quadrature. To do this, we need the $(m_s \times m_y) \geqslant (n_s \times n_y)$

[3]The residual functions are reported in the appendix A.

grid points, which are the n roots of the n^{th} order Chebychev polynomials. The approximation is done for each state x. To evaluate the $P_{ij}(\mathbf{a}_x)$ in matrix terms, we form the $(m_s \times m_y, n_s \times n_y)$ matrix X, which are composed of Chebyschev polynomials. The matrix X is given by

$$X = \left[\psi_{11}\left(s,y\right), \psi_{12}\left(s,y\right), ..., \psi_{1n_y}\left(s,y\right), ..., \psi_{n_s n_y}\left(s,y\right)\right]$$

where

$$\psi_{ij}\left(s,y\right) = \sum_{l_s=1}^{m_s}\sum_{l_y=1}^{m_y}\psi_{ij}\left(s_{l_s}, y_{l_y}\right), \text{for } i = 1,...,n_s \text{ and } j = 1,...,n_y$$

Then, for the state x, we form the Gauss-Chebyschev quadrature approximation of the $(m_s \times m_y, 1)$ vector of the residual functions such as :

$$R\left(s,y,x;\hat{\tilde{\mu}}_{\mathbf{a}_x}\right) = \left[R\left(s_1,y_1,x;\hat{\tilde{\mu}}_{\mathbf{a}_x}\right), ..., R\left(s_1,y_{m_y},x;\hat{\tilde{\mu}}_{\mathbf{a}_x}\right), ..., R\left(s_{m_s},y_{m_y},x;\hat{\tilde{\mu}}_{\mathbf{a}_x}\right)\right]'$$

Finally, the approximation of the equation (17) has the following form :

$$\hat{P}_{ij}(\mathbf{a}_x) = X'.R\left(s,y,x;\hat{\tilde{\mu}}_{\mathbf{a}_x}\right) = 0, \quad \text{for } x = x_m, x_M. \tag{18}$$

(18) represents a nonlinear system of $2.n_s.n_y$ equations in the $2.n_s.n_y$ unknowns $\mathbf{a} = [\mathbf{a}_{x_m}, \mathbf{a}_{x_M}]'$. This system can be solved using the versions of Newton-Raphson method implemented in the GAUSS routine, $NLSYS$. I denote $\hat{\mathbf{a}}_{0,x}$ the solution of the system, in the first step, when the state x is considered. This vector of solutions is used as a vector of initial values in the second step.

Secondly, I define an other residual function to determine an approximation of the price dividend ratio, $\tilde{p}_{e,t}$. As previously, the approximation of $\tilde{p}_{e,t}$ is given by :

$$\tilde{p}_e\left(s,y,x\right) \approx \hat{\tilde{p}}_e\left(s,y,x;\mathbf{b}_x\right) = \sum_{i=1}^{n_s}\sum_{j=1}^{n_y} b_{ij,x}\psi_{ij}\left(s,y\right), \quad \text{for } x = x_m, x_M \tag{19}$$

and the approximation has to satisfy (14), and the residual function $\Re$ is just defined by the residuals of (14)[4]. Then, as previously, I solve the system of $4.n_s.n_y$ equations in the $4.n_s.n_y$ unknowns $\mathbf{a}$ and $\mathbf{b}$

$$\left\{\begin{array}{r} X'.R\left(s,y,x;\hat{\tilde{\mu}}_{\hat{\mathbf{a}}_x}\right) = 0 \\ X'.\Re\left(s,y,x;\hat{\tilde{\mu}}_{\mathbf{a}_x},\hat{\tilde{p}}_{e,\mathbf{b}_x}\right) = 0 \end{array}\right., \quad \text{for } x = x_m, x_M$$

[4]The residual functions are reported in the appendix A.

I denote $\hat{\mathbf{a}}$, and $\hat{\mathbf{b}}$ the solutions of the previous system. Then, I deduce the approximation of the price of the risk-free asset for the state x

$$\hat{p}_f\left(s,y,x;\hat{\mathbf{a}}_x\right)=\beta E\left[\left(\frac{\hat{\tilde{\lambda}}\left(f\left(s,y,x,\tilde{x}\right),s,\tilde{x};\hat{\mathbf{a}}_x\right)}{\hat{\tilde{\lambda}}\left(s,y,x;\hat{\mathbf{a}}_x\right)}\right)\tilde{x}^{-\gamma}\mid x\right] \tag{20}$$

If the current state is x, the equity and the risk-free returns are respectively

$$R_{e,x}=E\left[\left(\tilde{x}.\left(\frac{\tilde{p}_e\left(f\left(s,y,x,\tilde{x}\right),s,\tilde{x};\hat{\mathbf{b}}_x\right)}{\hat{\tilde{p}}_e\left(s,y,x;\hat{\mathbf{b}}_x\right)}\right)\right)\mid x\right]-1 \tag{21}$$

$$R_{f,x}=E\left[\left(\frac{1}{\hat{p}_f\left(f\left(s,y,x,\tilde{x}\right),s,\tilde{x};\hat{\mathbf{a}}_x\right)}\right)\mid x\right]-1 \tag{22}$$

and the expected returns are

$$R_i=\pi.R_{i,x_M}+(1-\pi).R_{i,x_m},\quad \text{for } i=e,\ f.$$

In this paper, we suppose a symetric matrix. So π is equal to 0.5.

4 The results

In this section, I carry out two complementary studies to see if the previous model could explain the U.S risk premia in the period 1965-1987. First, I test if the model's implications concerning the volatility of the IMRS are satisfied. Second, I analyze the time-series properties of the simulated model. Specifically, I compute the first and the second empirical moments for a particular set of parameter values, and I compare these moments with their empirical counterpart. Then, I choose the set of parameter values that better replicate the observed moments. I compute the moments with a simulated draw of 5500 observations, discarding the first 500 simulations. I set $n_s=n_y=2$, and $m_s=m_y=8$. I use the annual U.S CRSP value weighted real return and the annual U.S real return on Treasury Bills with one months to maturity for the period 1965-1987. In the first part of this section, I present the results of using the methodology of Hansen and Jagannathan and the tests of the volatility bound restrictions. In the second part of the section, I present the results of the constrained grid search.

4.1 The Hansen and Jagannathan bound.

Hansen and Jagannathan[11] derived a lower bound on the volatility of the intertemporal marginal rate of substitution (IMRS) that correctly prices the assets. They compute their bound by taking a standard form of the Euler equations of consumption-based asset pricing models. One advantage of their procedure is that the bound they construct makes no reference to a particular model. It is solely calculated from returns data. In this paper, I only consider the bound that does not impose the positivity restriction on the IMRS. To estimate the bound, I use the US treasury Bills, $r_{f,t}$, and the U.S CRSP value weighted return, $r_{e,t}$, and two additional artificial returns, such as $r_{e,t-1}.r_{f,t}$, $r_{f,t-1}.r_{e,t}$, which prices are respectively $r_{e,t-1}$, $r_{f,t-1}$. We denoted by $x_t = (r_{e,t}, r_{f,t}, r_{e,t-1}.r_{f,t}, r_{f,t-1}.r_{e,t})$ the vector of asset returns and $q_t = (1, 1, r_{e,t-1}, r_{f,t-1})$ the vector of asset prices. I extend this visual method by implementing a statistical procedure for judging whether the model of section 1 is able to fit this lower bound. The test also provides one means of taking into account the sampling error. I use here the methodology of Cecchetti, Lam and Mark[4] to perform the statistical inference. Their statistic measures the vertical distance, labeled Δ, between a sample pair (μ_v, σ_v) and the lower bound σ_x, where μ_v and σ_v respectively represent the empirical mean and standard error of a particular $IMRS$. The candidate IMRS is rejected if its sample pair significantly lies below the bound. In order, to evaluate whether the difference is large, they compute the following statistic :

$$\begin{aligned} H_0 &: \Delta \leq 0 \\ \frac{\Delta}{\hat{\sigma}_\Delta} &= \left(\frac{\hat{\sigma}_v - \hat{\sigma}_x}{\hat{\sigma}_\Delta}\right) \\ \hat{\sigma}_\Delta &= \left(\frac{\partial \Delta}{\partial \theta'}\right)_{\hat{\theta}} \widehat{\Sigma}_\theta \left(\frac{\partial \Delta}{\partial \theta}\right)_{\hat{\theta}} \end{aligned}$$

where Δ has asymptotically gaussian distribution with mean 0 and variance σ^2_Δ, and $\widehat{\Sigma}_\theta$ is the estimated covariance matrix of the parameter θ, such as $\theta = \left(\mu_q, \mu_x, \Sigma_x\right)'$. Here, μ_q is the mean vector of the four asset prices, and μ_x, Σ_x are respectively the mean vector and covariance matrix of the 2×2 assets payoffs. In practice, I compute $\hat{\theta}$, and $\hat{\Sigma}_\theta$ by generalized method of moments using the first two moments of asset returns and the first moment of asset prices[5]. The covariance matrix $\hat{\Sigma}_\theta$ is the Newey and West[16] covariance matrix estimator.

[5] The moment conditions used in estimation are $E\left[x_t - \mu_x\right] = 0$,

$$E\left[q_t - \mu_q\right] = 0,$$
$$E\left[vec\left(x_t x_t'\right) - vec\left(\Sigma_x\right) + vec\left(\mu_x \mu_x'\right)\right] = 0,$$

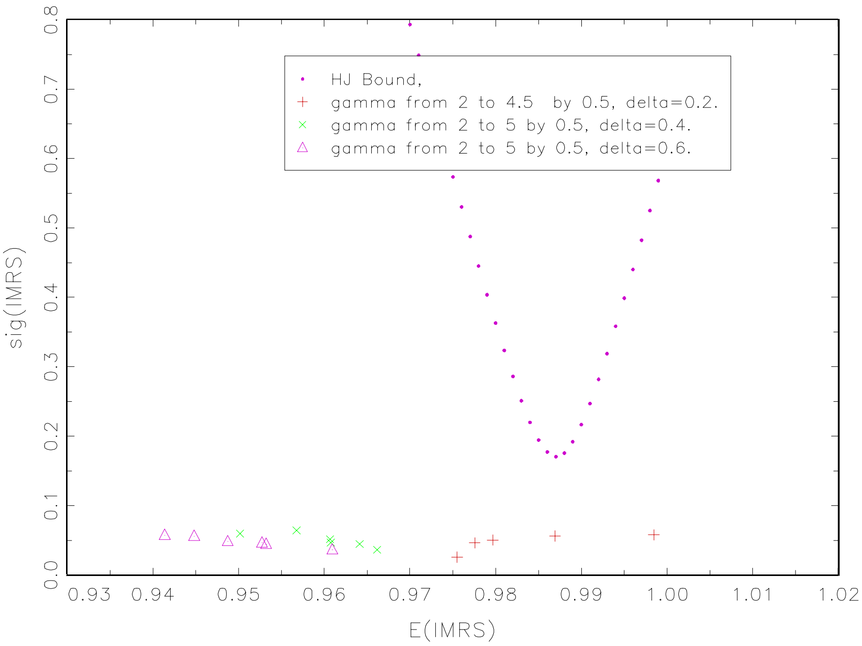

Figure 1 : HJ bound and the simulated mean-standard deviation pairs.

Note : These moments are computed with a simulated draw of 5500 observations, discarding the first 500 simulations and with β, θ, α respectively equal to 1, 0.8, and 0.6, for δ equal to 0.2,.0.4, 0.6, and γ ranging from 2 to 4.5 by 0.5.

Figure 1 plots the Hansen and Jagannathan bound, denoted by HJ and represented by the cup shaped region in the domain $(E(IMRS), std(IMRS))$. It is a function of μ_v. The figure 1 also plots the simulated mean-standard deviation pairs of the $IMRS$, for different values of β, γ, δ, θ, α. These moments are computed with a simulated draw of 5500 observations, discarding the first 500 simulations and with β, θ, α respectively equal to 1, 0.8, and 0.6, for δ equal to 0.2,.0.4, 0.6, and γ ranging from 2 to 4.5 by 0.5. The figure 1 shows that the volatility of the $IMRS$ increases, but the mean decreases as γ increases, for δ equals to 0.4, 0.6. Therefore, the triangles and the stars move away from the admissible region. Whereas, for δ equals to 0.2, the plus get nearer to the HJ bound. Nevertheless, the model for this specification do not generate enough volatility in the $IMRS$, ignoring sampling error. The tables $1, 2, 3$ report the results of the volatility bound test, for different values of γ, and δ. These tables are reported in the appendix B1. The results of these tables are summarized in the figure 2. Figure 2 plots the t-ratio of the HJ bound test for δ equals to $0, 0.2, 0.4$,0.6, and for differents values of γ. I only display the t-ratio whose estimated values of μ_v are below unity. As far as we know that the risk free rate puzzle is not solve for μ_v values upper than one.

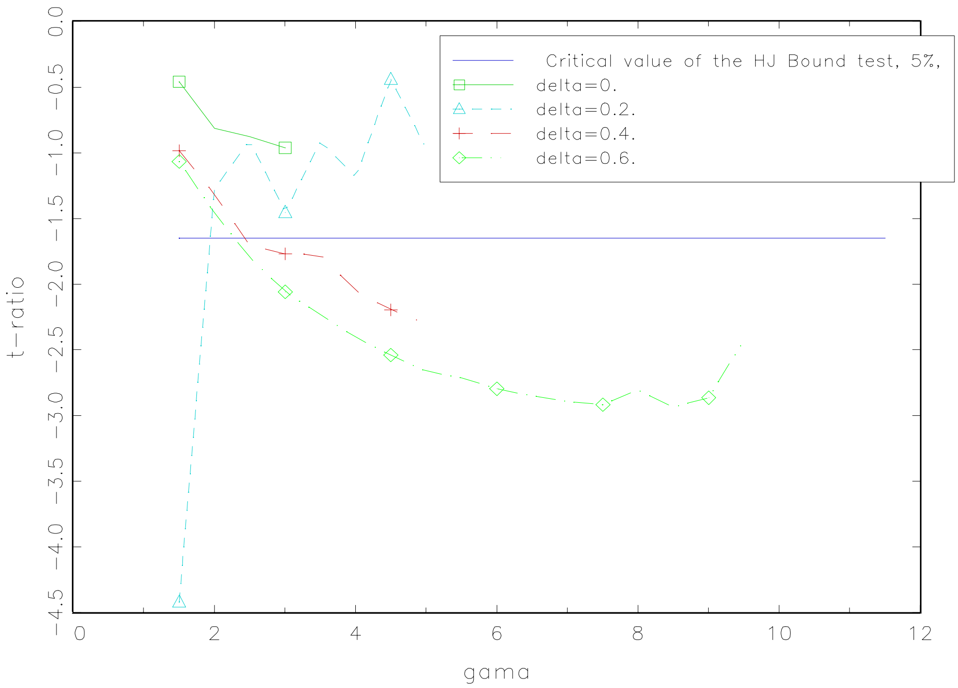

Figure 2 : Hansen and Jagannathan bound test for different δ.

Note : Figure 2 plots the t-ratio of the HJ bound test for δ equals to $0, 0.2$, 0.4,0.6, and for differents values of γ. I only display the t-ratio whose estimated values of μ_v are below unity.

In the figure 2, we see that the null hypothesis is never rejected by the pure habit persistence model ($\delta = 0$). In addition, for δ equals to 0.2, I accept the hypothesis that the simulated points are in the HJ region, for all the γ. Although, these points do lie below the bound, the distance is not significant. On the contrary, as the level of substituability grows, $H0$ is easier rejected by the HJ bound test, except for γ equals to 1.5 and 2. Therefore, as noted by Hansen and Jagannathan[11], the durability substantially reduces the volatility of the IMRS. In addition, a low degree of durability could fit the Hansen Jagannathan bound.

Secondly, I choose to fix δ to 0.2, and to compute mean-standard deviation pairs for different values of θ. I respectively set β and α to 1 and 0.6. I recall that θ measures the persistence of the habit effect and when $\theta = 0$, the model is just a one-period habit model. The figure 3 represents the simulated moments of the discount factor, for θ equal to $0.4, 0.6, 0.8$. In fact, there are no significant changes for the different values of θ.

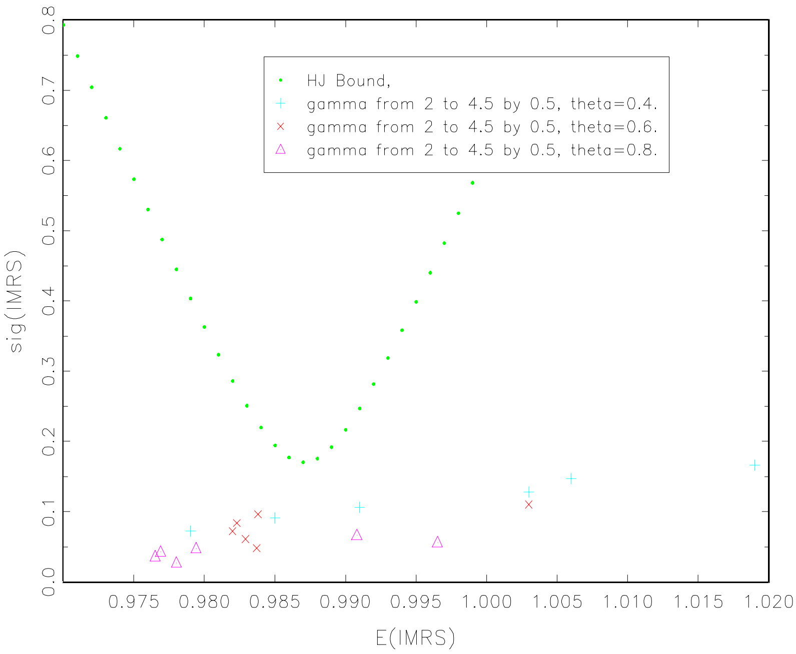

Figure 3 : HJ bound and the simulated mean-standard deviation pairs

Note : These moments are computed with a simulated draw of 5500 observations, discarding the first 500 simulations and with β, δ, α respectively equal to 1, 0.2, and 0.6, for θ equal to 0.4,.0.6, 0.8, and γ ranging from 2 to 4.5 by 0.5.

I have the same results when I compare the tables 4, 5, 6. These tables are reported in the appendix B2. The results of these tables are summarized in figure 4. I consider four cases. We see that for a model without habit persistence the Hansen and Jagannathan bound is statistically accepted, even for low level of courbature. For the three over cases, the HJ bound test does not seem very sensitive to θ. Nonetheless, this figure shows that as far as θ is high, the null hypothesis is longer accepted. One of the features of the one-period habit persitence model is its low level for γ. Ferson and Constantinides[10] found a value under unity for γ. However, this value does not seem very realistic for Deaton[8]. So, we need a minimum level of habit persistence to have a coherent value of γ. To summarized these two figures, we find that the results do not seem very sensitive to the persistence of the habit effect. Nonetheless, we need a minimum level of habit stock to fit the Hansen and Jagannathan bound for an high level of γ.

In the figure 5, I plot the simulated means and the standard deviations of the $IMRS$, with β, δ, θ respectively equal to 1, 0.2, and 0.8. The parameter α can take three values : $0.4, 0.6, 0.7$. We recall that the parameter α gives the proportion of the habit stock that is compared to the current level of the good s. I see that for a low level of α, $\alpha = 0.4$, the plus move away from the HJ bound. Whereas as α increases, I need a lower level of γ to be closer to the admissible region. Therefore, it seems that I need a high level of α to fit the bound.

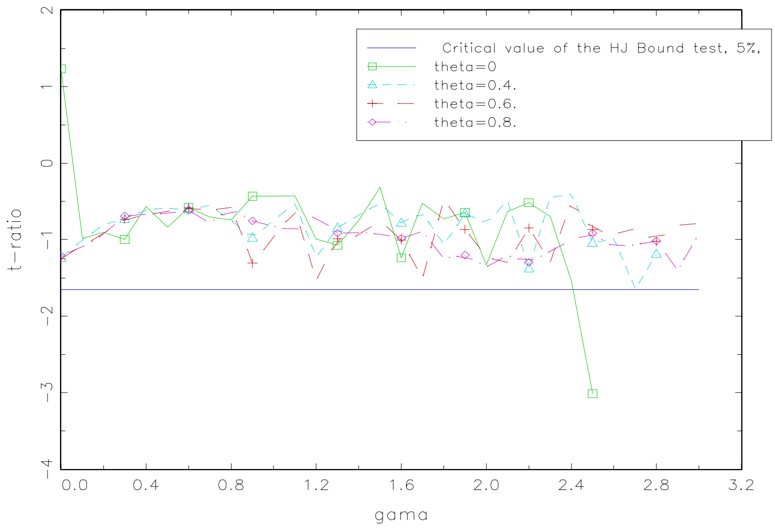

Figure 4 : Hansen and Jagannathan bound test for different θ.

Note : Figure 4 plots the t-ratio of the HJ bound test for θ equals to $0, 0.4, 0.6$,0.8, and for differents values of γ. I only display the t-ratio whose estimated values of μ_v are below unity.

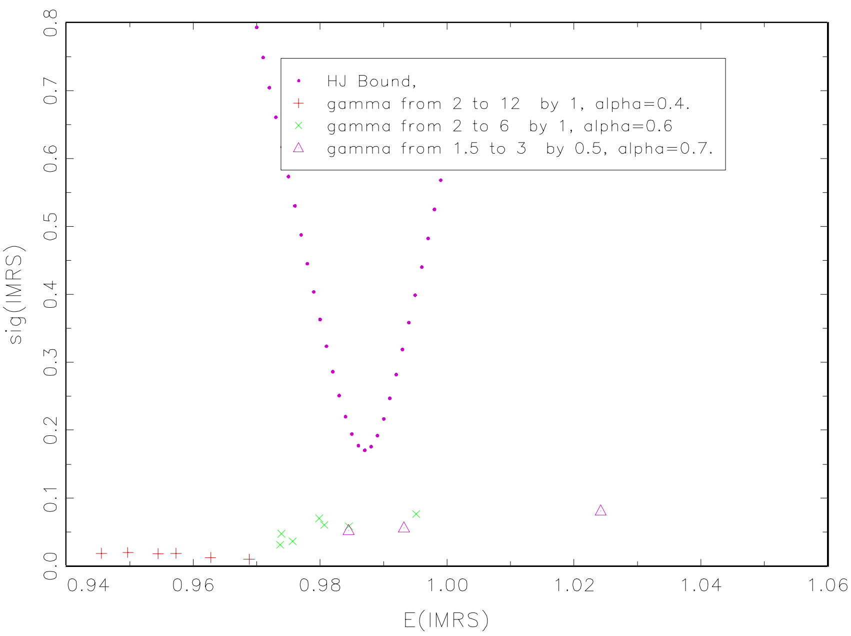

Figure 5 : HJ bound and the simulated mean-standard deviation pairs

Note : These moments are computed with a simulated draw of 5500 observations, discarding the first 500 simulations, and with β, δ, θ respectively equal to 1, 0.2, and 0.8, for α equal to 0.4,.0.6, 0.8, and different values of γ.

The tables 7, 8, 9 report the results of the volatility bound test. These tables are reported in the appendix B3. The results of these tables are summarized in the figure 6.

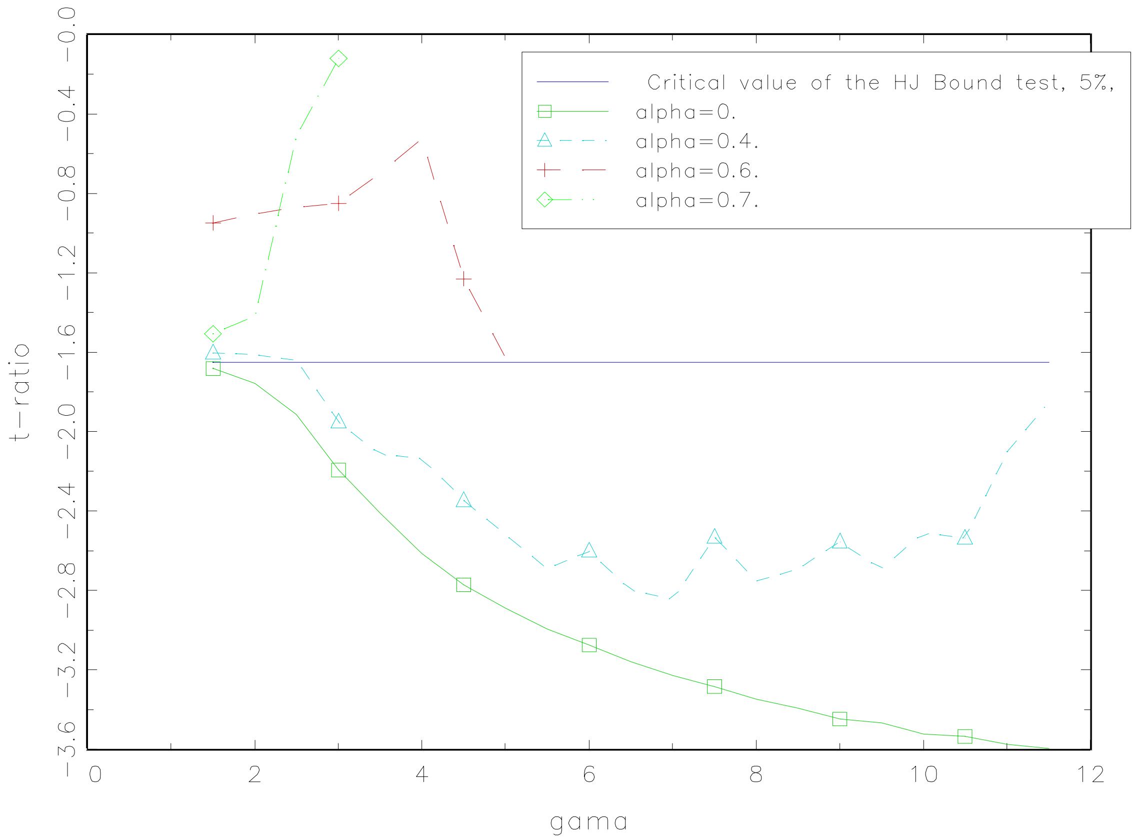

Figure 6 : Hansen and Jagannathan bound for different value of α.

Note : Figure 6 plots the t-ratio of the HJ bound test for α equals to $0, 0.4, 0.6$,0.7, and for differents values of γ. I only display the t-ratio whose estimated values of μ_v are below unity.

Figure 6 plots the t-ratio of the HJ bound test for α equals to $0, 0.4, 0.6$,0.7, and for differents values of γ. For α equals to 0, the model reflects only the durability or substituability of consumption over time. The HJ bound test reject this model. In addition, for α equals to 0.4, the vertical distances between the sample pairs (μ_v, σ_v), for the set of parameters considered, and the lower bound σ_x are significant, even for a high value of γ. The distance is still significant for a γ equal to 12. While for higher level of α, the null hypothesis is accepted. So, I need a high proportion of habit stock to meet the Hansen and Jagannathan restrictions.

This section allows us to have an idea of parameter values for the sensitivity analysis. Thus, a model which fits the Hansen and Jagannathan bound is a model with a relatively low degree of substitutability and a high proportion of habit stock.

4.2 The equity premium and the risk-free rate puzzles.

I implemented a grid search subject to the constraints that the risk-free return mean is between 0 and 3 percent, its volatility is smaller than 3 percent and the equity return is larger than 5 to find the values of the parameters of interest. Futhermore, the parameter values are chosen in order to meet the positivity of the marginal utility of consumption. I find that γ, δ, α and θ are respectively equal to 2.9, 0.17, 0.82 and 0.61, for a discount factor sets to unity. These parameter values imply a high proportion of habit stock and a low substitution effect, as it was shown in the previous section. The habit persistence effect is also strong which implies a relatively high courbature level (see figure 4). The first two moments implied by these parameter values are reported in the table 12. In this table, I respectively denote by r_i^{obs} and r_i^{sim} the observed and the simulated asset i return. The mean and standard deviation of the real returns are reported on annualized basis. First, for these parameter values the vertical distance between the sample pair $(\mu_v = 0.981, \sigma_v = 0.041)$ and the lower bound σ_x is not significant. I obtain a $t - ratio$ equals to -0.8543. So, the admissible region is fitted for these parameters values. Therefore, this consumption-based asset pricing model generates enough volatility for the IMRS to correctly price the assets. The model is not rejected by this nonparametric method. Second, the simulated mean real returns on equity and bond are coherent compare to the sample mean. Mehra and Prescott[15] have argued that the representative-agent models yield average equity returns that are much too low relative to historical observed returns. Our model still undervalues mean equity return. But the simulated mean equity return is now comparable to its observed return. In addition, the simulated mean risk-free rate is very close to observed mean bond return.

Table 12 : The moments implied by the grid search.

	r_e^{obs}	r_e^{sim}	r_e^{obs}	r_e^{sim}	$(r_e - r_f)^{obs}$	$(r_e - r_f)^{sim}$
mean	0.078	0.065	0.020	0.018	0.058	0.047
std	0.186	0.14	0.052	0.047	0.1845	0.0173

Note : This table displays the sample and the simulated means and second moments of the asset returns and risk premia. The simulated moments are obtained for γ, δ, α and θ respectively set to 2.9, 0.17, 0.82 and 0.61,and for a discount factor sets to unity. 5000 observations of the simulated series are used to calculate the simulated moments of IMRS.

The third column of table 12 displays the average equity premium, denoted by $(r_e - r_f)^{obs}$ for the observed and $(r_e - r_f)^{sim}$ for the simulated. Simulated premium is 4.7% per annum in the model, compared to 0.39% in the standard representative-agent model, which is the larger premium obtainable according to the simulations of Mehra and Prescott[15]. Though my estimate is below the 6% often cited in the literature on the equity premium puzzle, my model do a good job compared to the standard representative-agent model. Therefore, I can conclude that combined effects of substitution and complementarity over consumption nearly solve the equity premium and the risk free rate puzzles.

However, the second order moments are undervalued in the both cases. Nevertheless, the standard deviation of the risk-free rate seems in line with the data. It is a good result compared to the pure habit persistence model. In fact, this model implies extremely volatile stochastic discount factor to explain the equity premium puzzle. Therefore, it generates a very volatile risk free rate. For example, Cecchetti, Lam and Mark[4] found a standard deviation for risk-free rate of 11.5%, using monthly data, for γ equals to 5 and an habit parameter value sets to 0.5. Our estimates of the standard deviation for the bond is much lower and is equal to 4.7%, using annual data. I can conclude that the introduction of local substitution substantially improves the model's ability to fit the volatility of risk-free rate. Yet, it is not the case for the real equity returns. Our model is not able to generate enough volatility for the equity return. Campbell[2] calls this *the stock market volatility puzzle*[6]. Nonetheless, my results are better than those of Heaton[12]. My volatility estimate is equal to 14% compares to 4.2% for Heaton. Therefore, I substantially improve the accuracy of the model's results, with this computation method.

5 Conclusion

The purpose of this paper was to examine the empirical properties of a non-linear stochastic dynamic model with rational expectations, in which the representative agent is assumed to display time non separable preferences. Specifically, I carried out two complementary studies to check the empirical relevance of the model. First, I used the Hansen and Jagannathan bound to check if the consumption based asset-pricing model with local substitution and long run habit persistence over consumption correctly prices the assets. I found that the IMRS implied by the model statistically fits the Hansen and Jagannathan bound if the degree of substitutability is relatively low and the proportion of habit stock is high. Secondly, I compared the simulated two first order moments with those observed. I concluded that combined effects of substitution and complementarity over consumption nearly solve the equity premium and the risk free rate puzzles. In addition, I found that the introduction of local substitution substantially improves the model's ability to fit the volatility of risk-free rate, compared to the pure habit persistence model. Finally, the model does resolve the Campbell's stock market volatility puzzle, with our computational

[6]Campbell shows that the volatility of stock returns is too high to be readily explained by the consumption-based asset pricing with power utility. Since the stock return should equal current consumption growth in this model. The result is true if we suppose that log dividend equals log consumption, and lognormality and homoskedasticity of asset returns and consumption. He also shows that the Epstein, Zin[9] and Weil [18] utility does not provide a solution to this puzzle.

method of asset returns. Therefore, I found that I substantially improve the accuracy of the model's results, compares to Heaton's results.

Nonetheless, these results may be improved in three ways. First, I studied a partial equilibrium representative-agent model. It would be interesting to consider the same preferences in a general equilibrium model. Secondly, I also maintain the assumption of homogeneous agents. One other possibility would be to investigate a non-linear stochastic dynamic model with heterogeneous agents. Thirdly, I suppose a complete-market economy. However, the implications of equilibrium incomplete-market economy deserve to be studied, because agents will be limited in their ability to smooth consumption.

References

[1] A Abel. Asset prices under habit formation and catching up with the joneses. *A E R papers and proceedings*, 80:38–42, 1990.

[2] J Campbell. *Asset Prices, Consumption, and the Business Cycle.* Taylor, J and Woodford, M, 1999.

[3] J Campbell and J Cochrane. By force of habit : A consumption-based explanation of aggregate stock market behavior. Technical report, Harvard University et NBER, 1997.

[4] S Cecchetti, P Lam, and N Mark. Testing volatility restrictions on intertemporal marginal rates of sustitution implied by euler equations and assets returns. *The Journal of Finance*, XLIX(1):123–152, mars 1994.

[5] L Christiano and J Fisher. Algorithms for solving dynamic models with occasionally binding constraints. Technical Report 218, NBER, 1997.

[6] G Constantinides. Habit formation : a resolution of equity premium puzzle. *Journal of Political Economy*, 90(3):519–543, 1990.

[7] J Cox J, and Ingersoll and S Ross. An intertemporal general equilibrum model of asset prices. *Econometrica*, 53(2):385–407, 1985.

[8] A Deaton. *Understanding Consumption.* Oxford University Press : New York, NY, 1992.

[9] Zin S Epstein, L. Substitution, risk aversio end the temporal behavior of consumption and asset return : An empirical analysis. *Journal of Political Economy*, 99(2):261–286, 1991.

[10] W Ferson and G Constantinides. Habit persistence and durability in aggregate consumption. *Journal of Financial Economics*, 29:199–240, 1991.

[11] L Hansen and R Jagannathan. Implications of security market data for models of dynamic economies. *Journal of Political Economy*, 99(2):225–262, 1991.

[12] J. Heaton. An empirical investigation of asset pricing with temporally dependent preference specifications. *Econometrica*, 63(3):681–717, May 1995.

[13] K Judd. *Numerical Methods in Economics.* MIT Press, 1998.

[14] R Lucas. Asset prices in an exchange economy. *Econometrica*, 34(6):1429–1445, 1978.

[15] R Mehra and E Prescott. The equity premium : a puzzle. *Journal of Monetary Economics*, 15:145–161, 1985.

[16] W Newey and K West. A simple, positive definite, heteroscedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55(3):703–708, 1987.

[17] S Sundaresan. Intertemporally dependent preferences and the volatility of consumption and wealth. *Rewiew of Financial Studies*, 2:73–89, 1989.

[18] P Weil. The equity premium puzzle and the risk-free rate puzzle. *Journal of Monetary Economics*, 24(2):401–421, 1989.

A The residual functions

A.1 The residual function of the marginal utility of S

The residual function is given by the Euler equation (12) and as the following form

$$R\left(s,y,x;\hat{\hat{\mu}}_{\mathbf{a}_x}\right)=\hat{\hat{\mu}}\left(s,y,x;\mathbf{a}_x\right)-s^{-\gamma}-\beta\left(\delta+\theta\right)E\left[\tilde{x}^{-\gamma}.\hat{\hat{\mu}}\left(f\left(s,y,x,\tilde{x}\right),s,\tilde{x};\mathbf{a}_x\right)\mid x\right]$$
$$+\beta^2\left(\delta.\theta\right)E\left[E\left[\tilde{\tilde{x}}^{-\gamma}.\hat{\hat{\mu}}\left(f\left(\tilde{s},\tilde{y},\tilde{x},\tilde{\tilde{x}}\right),f\left(s,y,x,\tilde{x}\right),\tilde{\tilde{x}};\mathbf{a}_x\right)\mid\tilde{x}\right]\mid x\right],\quad\text{for } x=x_m,x_M$$

A.2 The residual function of the equity price

The residual function is given by the Euler equation (14) and as the following form

$$\Re\left(s,y,x;\hat{\hat{\mu}}_{\mathbf{a}_x},\hat{\hat{p}}_{e,\mathbf{b}_x}\right)=\hat{\hat{p}}_e\left(s,y,x;\mathbf{b}_x\right)-$$
$$\beta E\left[\left(\frac{\hat{\hat{\lambda}}\left(f\left(s,y,x,\tilde{x}\right),s,\tilde{x};\hat{\mathbf{a}}_{0,x}\right)}{\hat{\hat{\lambda}}\left(s,y,x;\hat{\mathbf{a}}_{0,x}\right)}\right)\tilde{x}^{-\gamma+1}(\tilde{p}_e\left(f\left(s,y,x,\tilde{x}\right),s,\tilde{x};\mathbf{b}_x\right)+1)\mid x\right]$$

where the approximation of the marginal utility of consumption is such as ,given (11)

$$\hat{\hat{\lambda}}\left(s,y,x;\hat{\mathbf{a}}_{0,x}\right)=\hat{\hat{\mu}}\left(s,y,x;\hat{\mathbf{a}}_{0,x}\right)-\beta\phi E\left[\tilde{x}^{-\gamma}.\hat{\hat{\mu}}\left(f\left(s,y,x,\tilde{x}\right),s,\tilde{x};\hat{\mathbf{a}}_{0,x}\right)\mid x\right]\qquad(23)$$

B The Hansen and Jagannathan bound test

In this appendix, I present the results of the HJ bound test. We consider 3 cases. In these tables, the vertical distance between a sample pair (μ_v,σ_v) and the lower bound σ_x, and the statistic of the test are respectively labeled *dev* and *tratio*.

B.1 Changes in substituability

The tables $1,2,3$ report the results of the volatility bound test, for different values of γ, and δ.

Table 1 :Results of tests of the volatility bound restrictions.β=1, δ=0.2, θ=0.8, α=0.6

γ	μ_v	σ_v	σ_x	*dev*	*tratio*
2	0.979	0.0257	0.4246	-0.3989	-0.960
2.5	0.9764	0.0377	0.5346	-0.4969	-1.0605
3	0.9773	0.0420	0.4960	-0.4540	-1.0038
3.5	0.9850	0.0480	0.2194	-0.1714	-1.4939
4	0.9890	0.0599	0.2153	-0.1554	-0.4818
4.5	0.9941	0.0670	0.3798	-0.3128	-0.6887

Notes : 5000 observations of the simulated series are used to calculate the moments of IMRS.

Table 2 :Results of the tests of the volatility bound restrictions β=1, δ=0.4, θ=0.8, α=0.6

γ	μ_v	σ_v	σ_x	dev	$tratio$
2	0.9720	0.0287	0.7286	-0.6999	-1.3176
2.5	0.9640	0.0360	1.0911	-1.0551	-1.7093
3	0.9623	0.0420	1.1688	-1.1268	-1.7775
3.5	0.9621	0.0506	1.1780	-1.1274	-1.7730
4	0.9550	0.0580	1.5042	-1.4462	-2.0556
4.5	0.9580	0.0650	1.3662	-1.3012	-1.9271

Notes : 5000 observations of the simulated series are used to calculate the moments of IMRS.

Table 3 :Results of tests of the volatility bound restrictions. β=1, δ=0.6, θ=0.8, α=0.6

γ	μ_v	σ_v	σ_x	dev	$tratio$
2	0.9690	0.0280	0.8636	-0.8356	-1.4772
2.5	0.9620	0.0360	1.1826	-1.1466	-1.8004
3	0.9569	0.0410	1.4167	-1.3757	-2.0066
3.5	0.9511	0.0470	1.6840	-1.6370	-2.2117
4	0.9440	0.0520	2.0119	-1.9599	-2.4294
4.5	0.9408	0.0600	2.1599	-2.0999	-2.5093

Notes : 5000 observations of the simulated series are used to calculate the moments of IMRS.

B.2 Changes in habit stock

Tables 4, 5, 6, 7 diplay the results of the volatility bound test, for different values of γ, and θ.

Table 4 :Results of the tests of the volatility bound restrictions β=1, δ=0.4, θ=0, α=0.6

γ	μ_v	σ_v	σ_x	dev	$tratio$
0.1	0.9984	0.006612	0.5603	-0.5537	-1.233
0.3	0.9954	0.02121	0.4326	-0.4114	-0.9030
0.5	0.9912	0.03596	0.2735	-0.2376	-0.5639
0.7	0.9922	0.05090	0.3077	-0.2568	-0.5836
0.9	0.9874	0.06344	0.1970	-0.1335	-0.7435
1.1	0.9889	0.07723	0.2131	-0.1358	-0.4329
1.3	0.9979	0.09132	0.5389	-0.4475	-0.9931
1.5	0.9957	0.1045	0.4458	-0.3413	-0.7495
1.7	1.001	0.1196	0.6644	-0.5448	-1.235

Notes : 5000 observations of the simulated series are used to calculate the moments of IMRS.

Table 5 :Results of tests of the volatility bound restrictionsβ=1, δ=0.2, θ=0.4, α=0.6

γ	μ_v	σ_v	σ_x	dev	$tratio$
2	0.9790	0.0722	0.4246	-0.3524	-0.8481
2.5	0.9850	0.0910	0.2194	-0.1284	-1.1192
3	0.9910	0.1060	0.2672	-0.1612	-0.3873
3.5	1.0030	0.1280	0.7641	-0.6361	-1.4686
4	1.0060	0.1470	0.8995	-0.7525	-1.7757
4.5	1.0190	0.1660	1.4947	-1.3287	-3.2480

Notes : 5000 observations of the simulated series are used to calculate the moments of IMRS.

Table 6 :Results of tests of the volatility bound restrictionsβ=1, δ=0.2, θ=0.6, α=0.6

γ	μ_v	σ_v	σ_x	dev	$tratio$
2	0.9837	0.0480	0.2522	-0.2042	-0.9385
2.5	0.9829	0.0610	0.2770	-0.2160	-0.8047
3	0.9820	0.0720	0.3079	-0.2359	-0.7493
3.5	0.9823	0.0838	0.2973	-0.2135	-0.7105
4	0.9838	0.0960	0.2493	-0.1533	-0.7283
4.5	1.0030	0.1100	0.7641	-0.6541	-1.5101

Notes : 5000 observations of the simulated series are used to calculate the moments of IMRS.

Table 8 :Results of tests of the volatility bound restrictionsβ=1, δ=0.2, θ=0.8, α=0.6

γ	μ_v	σ_v	σ_x	dev	$tratio$
2	0.9780	0.0270	0.4663	-0.4393	-1.0024
2.5	0.9765	0.0360	0.5303	-0.4943	-1.0589
3	0.9769	0.0427	0.5131	-0.4704	-1.0232
3.5	0.9794	0.0473	0.4082	-0.3609	-0.8904
4	0.9965	0.0560	0.4787	-0.4227	-0.9306
4.5	0.9908	0.0660	0.2609	-0.1949	-0.4752

Notes : 5000 observations of the simulated series are used to calculate the moments of IMRS.

B.3 Changes in the proportion of habit stock

Tables $8, 9, 10$ diplay the results of the volatility bound test, for different values of γ, and θ.

Table 8 :Results of tests of the volatility bound restrictionsβ=1, δ=0.2, θ=0.8, α=0.4

γ	μ_v	σ_v	σ_x	dev	$tratio$
2	0.9690	0.0096	0.8636	-0.8540	-1.5097
4	0.9490	0.0190	1.7809	-1.7619	-2.3188
6	0.9390	0.0200	2.2432	-2.2232	-2.6039
8	0.9301	0.0365	2.6553	-2.6188	-2.7915
10	0.9413	0.0460	2.1368	-2.0908	-2.5126
12	0.9580	0.0630	1.3662	-1.3032	-1.9301

Notes : 5000 observations of the simulated series are used to calculate the moments of IMRS.

Table 9 :Results of tests of the volatility bound restrictions.β=1, δ=0.2, θ=0.8, α=0.6

γ	μ_v	σ_v	σ_x	dev	$tratio$
2	0.9825	0.0280	0.2904	-0.2624	-0.9038
3	0.9819	0.0390	0.3115	-0.2725	-0.8532
4	0.9914	0.0560	0.2801	-0.2241	-0.5261
5	1.0034	0.0780	0.7821	-0.7041	-1.6305

Notes : 5000 observations of the simulated series are used to calculate the moments of IMRS.

Table 10 :Results of tests of the volatility bound restrictions.β=1, δ=0.2, θ=0.8, α=0.7

γ	μ_v	σ_v	σ_x	dev	$tratio$
1.5	0.9867	0.0440	0.1974	-0.1534	-1.4282
2	0.9910	0.0540	0.2672	-0.2132	-0.5123

Notes : 5000 observations of the simulated series are used to calculate the moments of IMRS.