Time-Consistency of Optimal Fiscal Policy in an Endogenous Growth Model

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Abstract

This paper analyses the time-consistency of optimal fiscal policy in a model with private capital and endogenous growth achieved via public capital. If a full-commitment exists, the optimal policy is obviously sustainable. Nevertheless, in the absence of such a commitment, it is well known that debt restructuring cannot make the optimal fiscal policy time-consistent in economies with private capital. Under a zero-tax rate on capital income, we prove that debt restructuring can solve the time-inconsistency problem of fiscal policy. We find that the policy under debt-commitment is quite close to the full-commitment policy both in growth and in welfare terms.

Keywords: Endogenous Growth; Optimal Fiscal Policy; Time Consistency.

JEL classification: E61, E62, H21, O41.

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1 Introduction.

This paper investigates under which conditions an optimal fiscal policy can be made time-consistent in an economy with both private and public capital, and how far in terms of growth and welfare this policy would be from the full-commitment policy. The economy is modelled in an endogenous growth framework where public capital is not only essential for production, but also the engine of growth. This provides an interesting setting for addressing time-consistency issues. Clearly, future allocations are mostly driven by the future possibilities of growth. Moreover, in an endogenous growth model, future allocations play a more important role for welfare. Thus, given the relevant role that government plays in the growth process, it will be crucial to make fiscal policy time-consistent at a small welfare cost.

A benevolent government chooses both public spending and taxation plans for the current and future periods so as to maximise the welfare of the representative individual. Most studies on optimal taxation assume that there exists a full-commitment that enables the current government to bind the actions of future governments. Nevertheless, actual policy design is better described as a policy plan that is selected sequentially through time by a succession of governments. In this case, the resulting policy will in general not coincide with the announced optimal policy. Therefore, as shown by Kydland and Prescott (1977), the optimal policy will be time-inconsistent.

An optimal policy selected by a government at a given period is said to be time-inconsistent when it is no longer optimal when reconsidered at some later date, even though no relevant information has been revealed. The time-inconsistency problem of optimal fiscal policy arises under very general conditions; it appears in dynamic economies populated by individuals with rational expectations. In particular, in a representative agent model with a benevolent government, the optimal policy will be time-inconsistent when the government has no lump-sum taxes at its disposal. As Chamley (1986) showed, capital taxation illustrates this problem clearly. In order to encourage investment, a government should promise low future taxes on capital income. In contrast, once this investment has taken place, capital is in inelastic supply and should be taxed heavily. This is known as the capital levy problem. Then, in view of these incentives to renege on the previously selected policy, governments face a credibility problem. Thus, in the absence of full-commitment, the optimal policy cannot be implemented and this time-inconsistency implies a welfare loss.

In a barter economy without capital, Lucas and Stokey (1983) showed that an optimal fiscal policy could be made time-consistent through debt restructuring when governments commit to honouring public debt, and are allowed to issue this debt with a sufficiently rich maturity structure.[1] This analysis was extended to an open economy by Persson and Svensson (1986) and by Faig (1991) and to a monetary economy by Persson, Persson and Svensson (1987). In a model with endogenous government consumption and public capital, Faig (1994) made the optimal fiscal policy time-consistent through this method. Nevertheless, in economies with privately owned capital, debt restructuring is found unable to solve the time-inconsistency of fiscal policies because of the capital levy problem.[2] Notice that debt restructuring consists of selecting debt so that the incentives to abandon the previously selected policy are neutralised. For example, taxing the labour income that is received at a given period has a different degree of distortion depending on the planning date. Debt can neutralise such a change in the degree of distortion. In contrast, capital income taxation at t is distorsionary when it is planned at $s < t$, but lump-sum when considered at t. Debt cannot change the non-distortionary nature of the initial tax on capital income. Hence, in economies with private capital, debt restructuring cannot solve the time-inconsistency of optimal policy, and thus full-commitment policies and sustainable ones differ. This problem has been widely recognised by the literature, and it has led to limit the debt restructuring method to quite simple models which, for instance, have no private capital and display no growth. However, no solution and no appropriate measure of how important this problem is have been provided yet.

[1] By debt restructuring is meant the redesigning of the level and maturity calendar of the public debt that will be inherited by next period government.

[2] Zhu (1995) solves the time-inconsistency problem in an economy with private capital, but this capital is assumed to have an endogenous rate of utilisation, so it is never in inelastic supply.

In this paper we investigate under which conditions an optimal fiscal policy can be made time-consistent in an economy with private capital, abstracting from reputational issues, and what effects on both growth and welfare this policy would have. In order to do so, an economy with private capital and endogenous growth achieved via public capital is modelled.[3] First, the policy under full-commitment is studied. However, in the absence of such a commitment, this policy cannot be made time-consistent because of the capital levy problem. A natural solution would be to exclude capital income taxation from the policy selection. In this context, we show that a restricted optimal fiscal policy, subject to a zero-tax rate on capital income, can be made time-consistent through debt restructuring *à-la-Lucas and Stokey*. On the one hand, this restriction allows us to find a policy plan that is sustainable. But on the other hand, when it is compared with the full-commitment policy, it implies a welfare loss. Studies on tax reform may give rise to the idea that this welfare loss could be large. In a model with exogenous government spending and no growth, Chari, Christiano and Kehoe (1994) showed that about 80% of the welfare gains in a Ramsey system comes from high tax rates on old capital income. Thus, since the policy under debt-commitment does not make use of capital taxation, it may imply a dramatic welfare loss. This argument raises the need for measuring the welfare differential. Then, in order to compare this policy with the one under full-commitment, we use a numerical solution method for non-linear rational expectations models, in particular the eigenvalue-eigenvector decomposition method suggested by Novales et al. (1999), which is based, in turn, on Sims (1998). Both models are solved and we find that the policy under debt-commitment is quite close to the full-commitment policy both in growth and in welfare terms.

The remainder of the paper is organised as follows. Section 2 presents the model. In section 3, the policy plans under full-commitment and under debt restructuring are solved, and compared. Section 4 concludes with a summary of the main findings. Finally, the appendices include proofs and explain the numerical solution method.

[3] The importance of public capital in private production has been supported by several empirical studies. See, for example, Aschauer (1989).

2 The model.

Our economy is a version of the endogenous growth model with public spending presented by Barro (1990). Our version departs from the original model in the policy selection. We shall also consider a second best framework, i.e. no lump-sum taxation is available, but in our model government spending will take the form of a public investment that can be financed through time-variant tax rates on labour and capital income and through debt.[4] We assume that public debt can be issued with a sufficiently rich structure in terms of maturity calendar and debt-type variety. More precisely, the government at date t can issue sequences $\{ {}_{t+1}b_s^c,\ {}_{t+1}b_s^w \}_{s=t+1}^{\infty}$ of claims, that exist at $t+1$ and must be paid at $s \geq t+1$, on debt indexed to consumption and to after-tax wage respectively.[5] Under such a variety of debt,[6] governments promise debt payments, interest and principal, that are valued a consumption good and a unit of time devoted to production in a future period respectively.[7]

Consider an economy populated by identical infinitely-lived individuals endowed with a given initial capital k_0, initial debt claims maturing at $t \geq 0$, and one unit of time per period that can be either devoted to leisure $1 - l_t$, or to output production l_t. The representative individual derives utility from consumption c_t, and leisure so that its objective is to choose both goods and investment on assets for every period in order to maximise the sum of discounted utilities,

$$\sum_{t=0}^{\infty} \beta^t U\left(c_t, 1 - l_t\right), \tag{1}$$

[4]The first-best allocation would be attainable if the government could levy taxes on consumption, capital income and labour income. In that case, the time-inconsistency problem would obviously disappear. As usual in this literature, consumption taxes are excluded so as to work in a second best framework.

[5]If debt were indexed to before-tax wage, a government could default in debt payments through changing the labour income tax rate. Hence, another source of time-inconsistency would appear.

[6]This variety of debt can be also found in Faig (1994) and Zhu (1995).

[7]Debt indexed to consumption can be identified with Treasury Inflation-Protected Securities that are issued since 1997 by the U.S. Treasury and which vary with the consumer price index. We may identify debt indexed to after-tax wage with the promise of future social security pensions that are closely linked to the wage rate.

with $\beta \in (0,1)$, and $U(\cdot,\cdot)$ takes the following functional form:

$$U(c_t, 1-l_t) = \theta \ln c_t + (1-\theta)\ln(1-l_t), \tag{2}$$

where $\theta \in (0,1)$ reflects preferences between leisure and consumption. Taking sequences of prices and policy instruments as given, the consumer maximises welfare (1) subject to the budget constraint,

$$p_t\left[c_t + k_{t+1} + \sum_{s=t+1}^{\infty}\frac{p_s}{p_t}\left({}_{t+1}b_s^c - {}_t b_s^c\right) + \sum_{s=t+1}^{\infty}\frac{p_s}{p_t}q_s\left({}_{t+1}b_s^w - {}_t b_s^w\right)\right] \leq$$
$$p_t\left[{}_t b_t^c + (1-\tau_t^l)w_t\left[l_t + {}_t b_t^w\right] + R_t k_t\right], \tag{3}$$

and to the no-Ponzi-game condition on assets,

$$\lim_{t\to\infty}\sum_{s=t}^{\infty} p_s\, {}_t b_s^c = 0, \lim_{t\to\infty}\sum_{s=t}^{\infty} p_s q_s\, {}_t b_s^w = 0, \lim_{t\to\infty} p_t k_{t+1} = 0, \tag{4}$$

where p_t is the price of a final good in period t, w_t is the real wage received for the fraction of time that the individual devotes to work at t, τ_t^l is the labour income tax rate at t, R_t is the gross return on capital k_t, after tax τ_t^k, and depreciation rates δ_k, and r_t is the net return on capital at t, that is, $R_t = \left\{1 + \left(1-\tau_t^k\right) r_t - \delta_k\right\}$. Finally, q_t is the price of a bond indexed to after-tax wage in terms of final goods in period t. The first order conditions for this optimisation problem are the following:

$$\frac{U_x(c_t, 1-l_t)}{U_c(c_t, 1-l_t)} = \left(1-\tau_t^l\right) w_t, \tag{5}$$

$$\frac{U_c(c_t, 1-l_t)}{U_c(c_{t+1}, 1-l_{t+1})} = \beta\left\{1 + \left(1-\tau_{t+1}^k\right) r_{t+1} - \delta_k\right\}, \tag{6}$$

$$\beta^t \frac{U_c(c_t, 1-l_t)}{U_c(c_0, 1-l_0)} = \frac{p_t}{p_0}, \text{ and } q_t = \left(1-\tau_t^l\right) w_t, \tag{7}$$

where $U_c(c_t, 1-l_t)$ and $U_x(c_t, 1-l_t)$ denote the marginal utility with respect to consumption and to leisure at t respectively. Following this notation, second-order derivatives of the utility function will be denoted by $U_{c_t c_t}$, $U_{c_t x_t}$ and $U_{x_t x_t}$.

In the present model, public investment accumulates over time and amounts to a stock of public capital g_t that depreciates at a rate δ_g. This public capital is a publicly-provided good subject to complete congestion. For the purpose of ongoing growth, public investment is tied to production in the following way:

$$g_{t+1} - (1-\delta_g)\, g_t = \varphi_t y_t, \tag{8}$$

with $0 \leq \varphi_t \leq 1$ set optimally by the government. This condition does not only preclude extreme growth rates, but also provides an accumulation rule for public capital that is separated from the accumulation of private capital. Thus, the government cannot control all resources. Additionally, public capital must satisfy the government intertemporal budget constraint, that is

$$\sum_{t=0}^{\infty} p_t z_{0t} \geq 0, \tag{9}$$

where

$$z_{0t} \equiv \left[\tau_t^l w_t l_t + \tau_t^k r_t k_t - \left(g_{t+1} - (1-\delta_g)\, g_t\right) - {}_0b_t^c - q_t\, {}_0b_t^w\right], \tag{10}$$

which is the government "cash-flow" that at date 0 becomes

$$\sum_{s=1}^{\infty} \frac{p_s}{p_0} \left({}_0b_s^c - {}_1b_s^c\right) + \sum_{s=1}^{\infty} \frac{p_s}{p_0} q_s \left({}_0b_s^w - {}_1b_s^w\right), \tag{11}$$

that can be defined either as the excess of tax revenues over public spending and debt payments or as the real value of the net issue of new debt. In order to allow for a balanced growth path (BGP), initial inherited debt must satisfy that $\frac{{}_0b_t^c}{c_t}$ and ${}_0b_t^w$ take constant values in the long run. Given that $\frac{{}_0b_t^c}{c_t}$ is an endogenous variable, we shall rather assume that $\lim\limits_{t\to\infty} {}_0b_t^w = \kappa_w$ and $\lim\limits_{t\to\infty} {}_0b_t^c = \kappa_c$, where κ_c and κ_w are arbitrary constant values. The latter is assumed so as to satisfy that $\frac{{}_0b_t^c}{c_t}$ becomes constant under any possible growth process.

In this economy there is a final good that can be either consumed or invested. This good is produced through the following technology:

$$y_t = f(k_t, l_t, g_t) = A k_t^{\alpha} \left(l_t g_t\right)^{1-\alpha}, \tag{12}$$

where $A > 0$ and $\alpha \in (0, 1)$. This production function is subject to diminishing returns with respect to each factor, but it exhibits constant returns returns with respect to k_t and g_t together. Hence, public capital is introduced as an essential input that enhances both private capital and labour marginal products, and that allows for endogenous growth. Thus, this technology includes two state variables and makes our model exhibit transitional dynamics. Feasible allocations are described by the resource constraint,

$$c_t + k_{t+1} + g_{t+1} \leq A k_t^{\alpha} \left(l_t g_t\right)^{1-\alpha} + (1-\delta_k)\, k_t + (1-\delta_g)\, g_t. \tag{13}$$

A representative firm produces the final good and maximises profits given factor prices. Necessary conditions for this optimisation program are

$$r_t = f_{k_t} \text{ and } w_t = f_{l_t}, \tag{14}$$

where f_{k_t} and f_{l_t} denote marginal products of capital and labour at t respectively. Likewise, a similar notation will be followed by the marginal product of public capital and by the second-order derivatives of the production function.

A competitive equilibrium for this economy is defined as follows:

Definition 1 *For a given policy* $\left\{g_{t+1}, \tau_t^k, \tau_t^l\right\}_{t=0}^{\infty}$ *and initial values of public capital* g_0*, private capital* k_0*, and initial debt sequence* $\{\, {}_0b_t^c,\ {}_0b_t^w\}_{t=0}^{\infty}$*, an allocation* $\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}$ *is a competitive equilibrium allocation if and only if there exists a price sequence* $\{p_t, q_t, r_t, w_t\}_{t=0}^{\infty}$ *such that the representative individual maximises* (1) *subject to* (3) *and* (4)*, factors are paid their marginal products according to* (14)*, and all markets clear.*

3 The policy selection.

Once the behaviour of private agents has been described, we shall turn to the policy selection. First, the policy under full-commitment will be analysed. Next, we compute the optimal policy under some restrictions and discuss whether debt-restructuring can make this policy time-consistent. The resulting policy will be named the policy under debt-commitment. Analytically, both policies are characterised in the short and long run. Nevertheless, welfare and growth comparisons are intractable. Given this difficulty, we resort to numerical solution methods. Thus, both policies under full-commitment and debt-restructuring are solved numerically and compared.

3.1 The full-commitment policy.

For the time being, we assume that future governments do commit to the optimal tax policy chosen by the current government. This assumption would be equivalent to the existence of a full-commitment that makes the optimal policy planned at date 0 sustainable. This policy is such that once the government at 0 selects a fiscal plan for $t \geq 0$, successive governments will be bound to set the policy that is the continuation of the original plan chosen at 0. In this policy scheme, tax rates do not need to be constant over time and may take positive or negative values. But we shall restrict this policy by an upper bound on capital tax rates set equal to unity.[8] This restriction translates into following equation:[9]

$$U_{c_t} \geq \beta U_{c_{t+1}} \{1 - \delta_k\} . \tag{15}$$

In a second best problem, the government chooses an allocation among the different possible competitive equilibria in order to maximise the welfare of the representative individual. This allocation together with the conditions for a competitive equilibrium provide the optimal tax policy that supports this allocation. A competitive equilibrium allocation is characterised by the upper bound on capital tax rates (15), the accumulation rule for public capital (8), the transversality conditions, and two restrictions: the resource constraint (13), and the implementability condition

$$\sum_{t=0}^{\infty} \beta^t \left[\left(c_t - {}_0b_t^c\right) U_c\left(c_t, 1 - l_t\right) - \left(l_t + {}_0b_t^w\right) U_x\left(c_t, 1 - l_t\right) \right] \leq W_0 U_c\left(c_0, 1 - l_0\right), \tag{16}$$

where W_0 is the individual's initial capital income, that is $W_0 = R_0 k_0$. The implementability condition (16) results from adding the budget constraint of the individual (3) over time and introducing the first order conditions, (5) – (7) and (14), into the resulting expression. Note also that if equations (13) and (16) hold, then the intertemporal government budget constraint (9) is satisfied. Finally, given that all valuable assets will be rather consumed over the life-time, the following transversality conditions must hold:

$$\begin{gathered} \lim_{t\to\infty} \sum_{s=t}^{\infty} \beta^s U_{c_s}\ {}_tb_s^c = 0, \ \lim_{t\to\infty} \sum_{s=t}^{\infty} \beta^s U_{c_s} \left(1 - \tau_s^l\right) f_{l_s}\ {}_tb_s^w = 0, \\ \lim_{t\to\infty} \beta^t U_{c_t} k_{t+1} = 0, \ \lim_{t\to\infty} \beta^t U_{c_t} g_{t+1} = 0. \end{gathered} \tag{17}$$

[8] This upper bound could be justified by means of limited-liability, there is a limit to the capital income that the individual can be taxed.

[9] This constraint results from combining $\tau_t^k \leq 1$ and the first order condition for capital (6).

Now, let us define the government optimisation program. This problem can be expressed as choosing the initial tax rate on capital income τ_0^k, and the sequences $\{c_t, l_t, k_{t+1}, g_{t+1}\}_{t=0}^{\infty}$ that maximise the welfare of the representative individual (1) subject to the resource constraint (13), the implementability condition (16), the upper bound on capital tax rates (15), the accumulation rule for public capital (8), and the transversality conditions on debt and on private and public capital (17), given initial values for debt $\{\ {}_0b_s^c,\ {}_0b_s^w\}_{s=0}^{\infty}$, private k_0 and public capital g_0.

A solution to this problem is characterised by the constraints (13) , (16) and (15) together with the following first order conditions for consumption, labour, private and public capital respectively:

$$\begin{aligned} \mu_{0t} &= W_c\left(c_t, l_t,\ {}_0b_t^c,\ {}_0b_t^w, \Theta_t, \lambda_0\right), &(18)\\ f_{l_t}\mu_{0t} &= W_x\left(c_t, l_t,\ {}_0b_t^c,\ {}_0b_t^w, \Theta_t, \lambda_0\right), &(19)\\ \mu_{0t} &= \beta\mu_{0t+1}\left\{1 + f_{k_{t+1}} - \delta_k\right\}, &(20)\\ \mu_{0t} &= \beta\mu_{0t+1}\left\{1 + f_{g_{t+1}} - \delta_g\right\}, &(21) \end{aligned}$$

with

$$\begin{aligned} W_{c_t} &= (1+\lambda_0)\, U_{c_t} + \lambda_0\left[U_{c_tc_t}\left(c_t - {}_0b_t^c + \Theta_t\right) - U_{c_tx_t}\left(l_t + {}_0b_t^w\right)\right], \\ W_{x_t} &= (1+\lambda_0)\, U_{x_t} + \lambda_0\left[U_{x_tc_t}\left(c_t - {}_0b_t^c + \Theta_t\right) - U_{x_tx_t}\left(l_t + {}_0b_t^w\right)\right], \end{aligned}$$

and

$$\Theta_t = \begin{cases} -W_0 + \frac{\phi_{00}}{\lambda_0}, \text{ for } t = 0, \\ \frac{1}{\lambda_0}\left(\phi_{0t} - \{1-\delta_k\}\, \phi_{0t-1}\right), \text{ for } t > 0, \end{cases}$$

where μ_{0t}, λ_0, ϕ_{0t} are the multipliers associated to constraints (13), (16) and (15) respectively.[10]

The transitional dynamics of this model can be attributed to a number of factors, namely, the individual's initial wealth, the restriction on capital taxation and the initial private to public capital ratio. The first two factors are clearly reflected by the first order conditions (18) – (21) . The initial capital income affects indirectly

[10] As pointed out by Lucas and Stokey (1983), second order conditions are not clearly satisfied because they involve third and second derivatives of the utility function. Then, let us assume that an optimal solution exists and that this solution is interior.

the whole problem and directly decisions that involve variables at 0. Besides, the initial debt structure affects directly all periods. In addition, economic decisions are taken differently depending on whether the restriction on capital taxation is binding. The last factor comes from the specific production function (12) and from the fact that both types of capital follow a different accumulation rule. Under these assumptions, the ratio private to public capital cannot adjust instantaneously to its steady-state value. Let us characterise the transition. At the initial date, given that initial capital revenues constitute pure rents, the optimal tax rate on capital income reaches obviously the upper bound, that is the unity. For $t \geq 1$, the optimal tax policy results from combining the optimal allocation and the competitive equilibrium conditions. However, the high non-linearity of these conditions makes an analytical characterisation require a number of simplifying assumptions. In the following proposition, the evolution of the consumption growth rate γ_{c_t}, the dynamics of the labour income tax rate, and those of labour to their corresponding steady state values, marked with an ss, are described:

Proposition 1 *Let* $\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}$ *and* $\left\{g_{t+1}, \tau_t^l, \tau_t^k\right\}_{t=0}^{\infty}$ *be the optimal allocation and optimal policy under full-commitment, respectively. Then,* $\gamma_{c_t} \leq \gamma_{c_{ss}}$ *for all* $t > 1$ *where* $\tau_t^k = 1$.

Moreover, let $\left(\frac{k_0}{g_0}\right)$ *be close enough to its steady state value and* ${}_0b_t^w = \kappa_w$ *and* ${}_0b_t^c = 0$ *for all* $t > 1$*, then*

(i) $l_t \leq l_{ss}$, $\tau_t^l \leq \tau_{ss}^l$ *when* $\left(\frac{k_t}{g_t}\right) \geq \left(\frac{k}{g}\right)_{ss}$, *and* $l_t \geq l_{ss}$ *when* $\left(\frac{k_t}{g_t}\right) \leq \left(\frac{k}{g}\right)_{ss}$, *provided* $1 \geq \delta_g > \delta_k \geq 0$.

(ii) $\tau_t^l \leq \tau_{ss}^l$ *when* $l_t \leq l_{ss}$, *and* $\left(\frac{k_t}{g_t}\right) = \frac{\alpha}{1-\alpha}$ *for any* l_t, *provided* $1 \geq \delta_g = \delta_k \geq 0$.

(iii) $l_t \leq l_{ss}$, $\tau_t^l \leq \tau_{ss}^l$ *when* $\left(\frac{k_t}{g_t}\right) \leq \left(\frac{k}{g}\right)_{ss}$, *and* $l_t \geq l_{ss}$ *when* $\left(\frac{k_t}{g_t}\right) \geq \left(\frac{k}{g}\right)_{ss}$, *provided* $1 \geq \delta_k > \delta_g \geq 0$.

Proof. *See the appendix.* ■

Proposition 1 characterises the dynamics of the allocation and policy that solve the full-commitment model under a number of assumptions. These requirements reduce the sources of transition to the restriction on capital taxation. During the first periods of transition, the restriction on capital taxation will be binding. Observe that our economy exhibits a lower growth rate in this time interval. Given that lag values of ϕ_{0t} enter into the necessary conditions, (18) – (21), the transition will also include some periods in which this upper bound is non-binding. In this case, the evolution of the labour income tax rate and of labour are described. However, neither the sign of the capital tax rate nor the dynamics of the consumption growth rate can be determined for those periods. In a BGP, the optimal tax policy is characterised by the following statement:

Proposition 2 *If* $\left\{\tau_t^k, \tau_t^l\right\}_{t=0}^{\infty}$ *is the optimal tax policy under full-commitment, then* $\tau_{ss}^k = 0$ *and*

$$\tau_{ss}^l = \frac{\lambda_0 \left[\frac{1+\ {}_0b_{ss}^w}{1-l_{ss}} - \left(\frac{{}_0b^c}{c}\right)_{ss}\right]}{1+\lambda_0\left[\frac{1+\ {}_0b_{ss}^w}{1-l_{ss}}\right]}$$

in a BGP.

Proof. *See the appendix.* ■

Proposition 2 states Chamley's (1986) result. Current capital income should be taxed heavily, but in the long run it should not be taxed. Under full-commitment, this optimal policy is sustainable independently of the debt structure. Indeed, one-period consumption-indexed debt suffice for that purpose. But, in the absence of such a commitment, the government should reconsider its actions in period 1. Future governments would choose an optimal fiscal policy different from the one chosen at 0 for those future dates and, hence, the latter policy is time-inconsistent.

The incentives to select an allocation different from the one chosen for that date by previous governments come from different forces: the possibility of defaulting, "devaluing" debt and changing public spending, and capital and labour income tax rates. We assume that governments commit to honouring debt, but the two remaining forces are still active. The incentives to "devalue" debt come from the fact that an unexpected change in fiscal policy can lower the present value of outstanding debt. These incentives could be neutralised through debt restructuring. However, the change in the distortionary nature that capital taxation suffers cannot be modified by this method. Therefore, the optimal fiscal policy cannot be made time-consistent through debt restructuring because of the capital levy problem.

3.2 The policy under debt commitment.

From this section on, we assume that future governments can reconsider both taxation and spending plans, but commit to honouring debt and are free to redesign the public debt that will be inherited by next period government. In this context, we investigate under which conditions an optimal policy can be made time-consistent. A possible and natural solution would be to restrict our analysis to a zero-tax rate on capital income and study whether it can be made sustainable. This restriction on capital taxation implies that the following equation should hold:[11]

$$U_{c_t} = \beta U_{c_{t+1}} \left\{1 + f_{k_{t+1}} - \delta_k\right\} \tag{22}$$

The optimisation program solved by the government consists in choosing the sequences $\{c_t, l_t, k_{t+1}, g_{t+1}\}_{t=0}^{\infty}$ that maximise the welfare of the representative individual (1) subject to the resource constraint (13), the implementability condition (16), the zero-tax rate constraint on capital (22), the accumulation rule for public capital (8), and the transversality conditions (17), given initial values for debt, private and public capital. First order conditions for this problem are

$$\begin{aligned}
\mu_{0t} &= W_c\left(c_t, l_t, {}_0b_t^c, {}_0b_t^w, \Theta_{c_t}, \lambda_0\right), && (23)\\
f_{l_t}\mu_{0t} &= W_x\left(c_t, l_t, {}_0b_t^c, {}_0b_t^w, \Theta_{x_t}, \lambda_0\right), && (24)\\
\mu_{0t} &= \beta\mu_{0t+1}\left\{1 + f_{k_{t+1}} - \delta_k\right\} - \beta\xi_{0t} f_{k_{t+1}k_{t+1}} U_{c_{t+1}}, && (25)\\
\mu_{0t} &= \beta\mu_{0t+1}\left\{1 + f_{g_{t+1}} - \delta_g\right\} - \beta\xi_{0t} f_{k_{t+1}g_{t+1}} U_{c_{t+1}}, && (26)
\end{aligned}$$

where

$$\begin{aligned}
W_{c_t} &= (1+\lambda_0) U_{c_t} + \lambda_0 \left[U_{c_t c_t}\left(c_t - {}_0b_t^c + \Theta_{c_t}\right) - U_{c_t x_t}\left(l_t + {}_0b_t^w\right)\right],\\
W_{x_t} &= (1+\lambda_0) U_{x_t} + \lambda_0 \left[U_{x_t c_t}\left(c_t - {}_0b_t^c + \Theta_{x_t}\right) - U_{x_t x_t}\left(l_t + {}_0b_t^w\right)\right],
\end{aligned}$$

with

$$\Theta_{c_t} = \begin{cases} -W_0 + \frac{\xi_{00}}{\lambda_0}, \text{ for } t = 0,\\ \frac{1}{\lambda_0}\left(\xi_{0t} - \left\{1 + f_{k_t} - \delta_k\right\}\xi_{0t-1}\right), \text{ for } t > 0, \end{cases}$$

and

$$\Theta_{x_t} = \begin{cases} -W_0 + \frac{\xi_{00}}{\lambda_0} - f_{kl_0}\frac{U_{c_t}}{U_{c_t x_t}} k_0, \text{ for } t = 0,\\ \frac{1}{\lambda_0}\left(\xi_{0t} - \left\{1 + f_{k_t} - \delta_k\right\}\xi_{0t-1} - f_{k_t l_t}\frac{U_{c_t}}{U_{c_t x_t}}\xi_{0t-1}\right), \text{ for } t > 0, \end{cases}$$

[11]Introduce $\tau_t^k = 0$ and equation (14) into the first order condition for capital (6).

where μ_{0t}, λ_0, ξ_{0t} are the multipliers associated to the constraints (13), (16) and (22) respectively.[12]

These conditions, (23)-(26), together with (13) , (16) and (22) describe the optimal allocation. The optimal tax policy is obtained through the competitive equilibrium conditions. The transition is driven by the effect of the initial wealth, the zero-tax rate constraint on capital income and the private to public capital ratio. Next proposition describes the transitional dynamics of this policy and allocation:

Proposition 3 *Let* $\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}$ *and* $\left\{g_{t+1}, \tau_t^l\right\}_{t=0}^{\infty}$ *be the optimal allocation and optimal policy under debt-commitment respectively, then close enough to a BGP it holds that*

(i) $\tau_t^l \leq \tau_{ss}^l$, $\gamma_{c_t} \leq \gamma_{c_{ss}}$ *when* $l_t \leq l_{ss}$ *and* $\left(\frac{k_t}{g_t}\right) \leq \left(\frac{k}{g}\right)_{ss}$, *and* $.l_t \geq l_{ss}$, $\gamma_{c_t} \geq \gamma_{c_{ss}}$ *when* $\left(\frac{k_t}{g_t}\right) \geq \left(\frac{k}{g}\right)_{ss}$, *provided* $1 \geq \delta_g > \delta_k \geq 0$.

(ii) $\tau_t^l \leq \tau_{ss}^l$ *when* $l_t \leq l_{ss}$, *provided* $1 \geq \delta_g = \delta_k \geq 0$.

(iii) $l_t \leq l_{ss}$, $\tau_t^l \leq \tau_{ss}^l$ *when* $\left(\frac{k_t}{g_t}\right) \leq \left(\frac{k}{g}\right)_{ss}$ *provided* $1 \geq \delta_k > \delta_g \geq 0$.

Proof. *See the appendix.* ■

Proposition (3) characterises the dynamics of the debt-commitment model. By restricting our analysis to the dynamics around a BGP, the sources of transition amount to the zero-tax rate restriction on capital income. Observe that the evolution of the tax rate on labour income and those of labour are similar to the transition that the model under full-commitment displays. In contrast, we cannot conclude whether the economy will exhibit a lower growth rate during the transition. Likewise, in a BGP we can state the following:

Proposition 4 *If* $\left\{\tau_t^l\right\}_{t=0}^{\infty}$ *is the optimal tax policy under a zero-tax rate on capital income, then*

$$\tau_{ss}^l = \frac{\lambda_0 \left[\frac{1+ \ {}_0b_{ss}^w}{1-l_{ss}} - \left(\frac{{}_0b^c}{c}\right)_{ss}\right]}{1 + \lambda_0 \left[\frac{1+ \ {}_0b_{ss}^w}{1-l_{ss}}\right]}$$

in a BGP and, moreover, the zero-tax rate constraint on capital income does not restrict the allocation and policy selection in a BGP, that is, $\xi_{ss} = 0$.

[12] As in the previous section, we assume that an optimal solution that is interior exists.

Proof. *See the appendix.* ■

Let us turn now to the time-inconsistency problem. First, we have assumed that governments commit to honouring debt. Moreover, by restricting our analysis to a zero-tax rate on capital income, the capital levy problem disappears. Nevertheless, the incentives to lower the present value of debt and change the public spending and the labour income tax rate persist. In the absence of full-commitment, next period government would reconsider the spending and taxation plans, and the policy plan at 0 will be time-inconsistent. This time-inconsistency implies that the allocation and policy described in this section would not take place and that the final result would involve a welfare reduction. In order to prevent this welfare loss, we should study whether debt restructuring can make this policy time-consistent.

Proposition 5 *If the sequences* $\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}$ *and* $\left\{g_{t+1}, \tau_t^l\right\}_{t=0}^{\infty}$ *are respectively the optimal allocation and optimal policy under a zero-tax rate on capital income, then it is always possible to choose* $\{\ {}_1b_t^c,\ {}_1b_t^w\}_{t=1}^{\infty}$ *at market prices* (7) *such that the continuation* $\{c_t, l_t, k_{t+1}\}_{t=1}^{\infty}$ *and* $\left\{g_{t+1}, \tau_t^l\right\}_{t=1}^{\infty}$ *of the same allocation and policy are a solution for the government problem when it is reconsidered at date* 1*. This could be done through the following debt-structure:*

$$ {}_1b_t^c - {}_0b_t^c = \left[\frac{\lambda_0}{\lambda_1} - 1\right] {}_0b_t^c + \Gamma_t^c, \tag{27}$$

$$ {}_1b_t^w - {}_0b_t^w = \left[\frac{\lambda_0}{\lambda_1} - 1\right] ({}_0b_t^w + 1) + \Gamma_t^w, \tag{28}$$

where

$$\Gamma_t^c = \begin{cases} \left[\frac{\xi_{00} - \lambda_1 k_1}{\lambda_1}\right] \left(1 + f_{k_t} - \delta_k\right), \textit{ for } t = 1, \\ 0, \textit{ for } t > 1, \end{cases}$$

and

$$\Gamma_t^w = \begin{cases} -\left[\frac{\xi_{00} - \lambda_1 k_1}{\lambda_1}\right] \left(\frac{\theta}{1-\theta}\right) \frac{(1-l_t)^2}{c_t} f_{k_t l_t}, \textit{ for } t = 1. \\ 0, \textit{ for } t > 1. \end{cases}$$

By induction, the same is true for all later periods.

Proof. *See the appendix.* ■

The present value of the new debt issues at date 0 is determined by the government budget constraint (11) and there are infinite pairs of sequences $\{\ {}_1b_t^c,\ {}_1b_t^w\}_{t=1}^{\infty}$ that satisfy the restriction, but just one that enables the government to make the optimal policy time-consistent. Proposition 5 guarantees that under that debt structure the allocation and policy chosen for $t \geq 1$ by the government at 0, and described by propositions 3 and 4, will be sustainable. Hence, in economies with private capital, the optimal fiscal policy under a zero-tax rate constraint on capital income can be made time-consistent through debt restructuring.

How is the debt structure that enables the government to make the optimal policy time-consistent? In our model, this debt structure is difficult to characterise because the sign of $(\xi_{00} - \lambda_1 k_1)$ is unknown. Nevertheless, it can be seen through (27) and (28) that debt indexed to consumption and to after-tax-wage maturing at date $t > 1$ should follow a constant pattern with respect to the inherited debt. On the other hand, debt maturing at date 1 should follow a different pattern. This asymmetric behaviour reflects the existing asymmetry in the set of first-order conditions $(23)-(26)$. In all previous studies, the debt structure that ensured time-consistency had always the same pattern independently of the maturing date. This different behaviour in the debt maturing at date 1 is a distinctive property of this paper and shows that in economies with private capital the debt structure that ensures time-consistency must take into account the effect of the initial capital income.

In this section, a restricted policy has been made time-consistent. However, it is not known yet how desirable this policy is. In a model with exogenous public spending and no growth, Chari, Christiano and Kehoe (1994) compared a Ramsey system with and without a zero-tax rate restriction on capital income. The model without the constraint exhibits high initial capital tax rates, followed by a zero-tax rate from then on. They conclude that most of welfare gains come from capital tax revenues. In view of this result, they also argue that since the temptation to renege on the future capital tax policy, i.e. a zero-tax rate, is so large, the time-inconsistency problem of capital taxation is quantitatively severe. In this paper, our policy does not make use of capital taxation. This may lead us to think that our restricted tax policy, though time-consistent, could imply high welfare losses.

3.3 Numerical solution for both models.

In order to compare this policy with the one under full-commitment, we use a numerical solution method for non-linear rational expectations models, in particular the eigenvalue-eigenvector decomposition method that is suggested by Novales et al. (1999) and based on Sims (1998). This method consists of the following. First, the necessary conditions are obtained and transformed so that are functions of either ratios or variables that are constant in a BGP.[13] Their steady state values are found, as table 1 shows. Then, these conditions are linearised around the steady state in order to find the stability conditions. They are obtained by imposing orthogonality between each eigenvector associated with an unstable eigenvalue of the linear system and the variables of the system. These stability conditions are imposed into the original non-linear model to compute a numerical solution.

[Insert Table 1 about here.]

In order to simulate the series, some realistic parameter values are chosen. The discount rate β is 0.99. The coefficient A in the production function equals 0.48. The parameter α is 0.25 as in Barro (1990). Depreciation rates for private δ_k, and public capital δ_g, are 0.025 and 0.03 respectively.[14] The preference parameter θ is 0.3 so as to have reasonable values for leisure. Initial values for private and public capital are respectively 15 and 45. Initial debt values are zero for all periods. Both models share the same set of parameter values and depart from the same initial state.

Series for both policies, under full-commitment and with debt restructuring, are simulated.[15] We report the main results. As figure 1 shows, the policy under debt-commitment yields a higher growth rate in the short run. In the long run, the growth rate under debt restructuring approaches from below the rate attained under full-commitment. In the BGP, growth rates under full-commitment and under debt restructuring are respectively 3.83 and 3.84 per cent.

[Insert Figure 1 about here.]

[13] Note that all variables that are not constant grow at the same rate in a BGP.

[14] Since public capital is provided without charge, it is expected to suffer a faster depreciation.

[15] All simulations are carried out with the program GAUSS-386.

This similarity between long run growth rates may reflect that the differences in welfare may not be so dramatic. Numerically, the welfare attained under full-commitment and under debt restructuring take values of 76.3204 and 73.4250 respectively; then, this debt restructuring policy involves a welfare reduction of 3.79%. Therefore, the debt-commitment policy is quite close both in terms of growth and welfare to the policy under full-commitment.[16] One could argue that these results could hinge on the upper bound on capital tax rates in the full-commitment policy. Nevertheless, under the same parameter values and initial conditions, an unrestricted full-commitment policy yields a 4.27% growth rate and welfare sized by 84.3657. Additionally, the first best policy would allows us to test whether this numerical closeness is significant. Under the same circumstances, the first best policy yields a 13.87% growth rate and a welfare value of 249.2070. This result allows us to confirm that the policy under debt commitment and the full-commitment policy are very close both in growth and in welfare terms.

Our results contrast with Chari, Christiano and Kehoe's (1994) findings. This difference may come from two facts: first, our model allows for endogenous growth, so future allocations play a more important role for welfare; second, our taxes finance a productive public investment rather than an exogenous stream of government spending. In comparison with lump-sum taxation, our tax structure distorts the individual decisions and reduces welfare. The zero-tax rate restriction on capital income makes the existing tax structure more distortionary. If the amount that the government needs to raise through these taxes is endogenous rather than exogenous, the final tax structure seems intuitively less distortionary. Moreover, in the present model, public spending may be more important for welfare than the way of financing it. Under both policies, full-commitment and debt restructuring, the stream of public investment behaves similarly; in the short run, the public capital rate of growth is higher under full-commitment, this inequality reverses in the medium term, and both become quite similar in the long run as figure 2 shows.

[Insert Figure 2 about here.]

[16] These results hold under different changes in parameter and initial values.

The way of financing this spending differs in both models. Under full-commitment, tax rates on capital income are high in the first periods and zero thereafter. On the other hand, tax rates on labour income are negative in the initial period and positive from then on. In comparison with the policy under debt restructuring, the tax rate on labour income is smaller in the short under full-commitment and both become quite similar in the long run. In order to spread the excess of burden, debt is issued in an equivalent way in both models as figure 5 shows. In the short run, the government incurs in cash flow surpluses, whereas in the medium and in the long run this surplus vanishes. The main difference is that under full-commitment the first period involves a cash flow deficit that becomes surplus after few periods.

[Insert Figures 3, 4, 5, 6, and 7 about here.]

Finally, it would be interesting to see how the debt-structure that ensures time-consistency behaves. As we have just mentioned, under debt-commitment, the government incurs in a surplus at date 0. How is this surplus distributed over time? Observe that since we depart from zero initial debt holdings, a surplus implies that negative claims are issued. In our numerical solution, positive debt indexed to after-tax-wage is issued for all t. This debt evolves smoothly along t, but it takes a higher value at date 1. Hence, this surplus is carried out through negative issues of debt indexed to consumption. These debt issues take a very negative at date 1, and zero thereafter. Given the way that the initial capital income and the debt indexed to consumption enter into the implementability condition of government at date 1, equation (39), these negative issues are intended to compensate the wealth effect.

4 Conclusions.

We have investigated the time-inconsistency problem of optimal fiscal policy in a model with private capital and endogenous growth achieved via public capital. A policy restricted to a zero-tax on capital income has been made time-consistent through debt restructuring. The Modigliani-Miller theorem breaks down and debt serves as an incentive device to implement the selected allocation and optimal policy. We conclude that the tax policy under debt restructuring is quite close both in growth and in welfare terms to the optimal policy under full-commitment. In this sense, we can also argue that the time-inconsistency of capital taxation is not quantitatively so severe.

In economies with privately-owned capital, it is well known that the debt restructuring method cannot make the optimal policy plan time-consistent. We have reconsidered this issue and we have established that in economies with private capital, a time-consistent policy plan requires both a debt-commitment and a rule on capital taxation. In this paper, we have chosen a zero-tax rate constraint on capital income. However, if this tax-rate rule were to be chosen by the government, one could wonder whether this rule would be set different from zero.

A criticism that often emerges in this literature is that such a rich debt structure in debt-variety and maturity calendar has no clear counterpart in actual economies. One can argue that nowadays financial products indexed to very different economic variables are present in financial markets. Besides, as pointed out by Faig (1994), future social security pensions form an example that resembles debt indexed to after-tax wage. Nevertheless, it would be interesting to study what the government could better do without such a rich debt structure. Then, it may be the case that the optimal policy cannot be made time-consistent. Thus, policy selection should be restricted to sustainable policies. In the choice of the best sustainable policy, it would be interesting to see whether debt restructuring of the remaining instruments still matters and how much. The economy performance would provide a measure for the importance of a rich public financial structure.

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Appendix A.

Proof of Proposition 1.

Equate the RHS of (20) and (21), and solve for labour

$$l_t = \left[\frac{\delta_g - \delta_k}{A}\right]^{\frac{1}{1-\alpha}} * X_t^{-\frac{1}{1-\alpha}},$$

with

$$X_t = \left[(1-\alpha)\left(\frac{k_t}{g_t}\right)^{\alpha} - \alpha\left(\frac{k_t}{g_t}\right)^{-(1-\alpha)}\right]. \tag{29}$$

If $\delta_g > (<)\ \delta_k$, it can be seen that $\frac{\partial X_t}{\partial\left(\frac{k_t}{g_t}\right)} > 0$ and $\frac{\partial l_t}{\partial X_t} < (>)\ 0$. Hence, by the chain rule $\frac{\partial l_t}{\partial\left(\frac{k_t}{g_t}\right)} < (>)\ 0$. Besides, if $\delta_g = \delta_k$, $\frac{k_t}{g_t} = \frac{\alpha}{1-\alpha}$. Finally, combining equations (18) and (19) with (5), the labour income tax rate is

$$\tau_t^l = \frac{\lambda_0\left[\frac{1}{1-l_t} + \frac{{}_0b_t^w}{1-l_t} - \frac{{}_0b_t^c}{c_t}\right] - \left[\frac{1}{\beta}\frac{\phi_{ot-1}}{c_{t-1}} - \frac{\phi_{ot}}{c_t}\right]}{1 + \lambda_0\left[\frac{1}{1-l_t} + \frac{{}_0b_t^w}{1-l_t}\right]}, \tag{30}$$

which becomes

$$\tau_t^l = \frac{\lambda_0\left[\frac{1}{1-l_t} + \frac{\kappa_w}{1-l_t}\right] - \left[\frac{1}{\beta}\frac{\phi_{ot-1}}{c_{t-1}} - \frac{\phi_{ot}}{c_t}\right]}{1 + \lambda_0\left[\frac{1}{1-l_t} + \frac{\kappa_w}{1-l_t}\right]},$$

where $\left[\frac{1}{\beta}\frac{\phi_{ot-1}}{c_{t-1}} - \frac{\phi_{ot}}{c_t}\right] \geq 0$ given the initial debt structure and that $\frac{\phi_{ot}}{c_t}$ approaches zero from above. Then it can be seen that when l_t approaches from below its steady state value so does τ_t^l. Finally, by simple inspection of equation (15), if $\tau_t^k = 1$ holds, we have that $\gamma_{c_t} < \gamma_{c_{ss}}$.■

Proof of Proposition 2.

First, let us demonstrate that the capital tax rate is zero in a BGP. Considering (2) and (18), we can write the government first order condition for capital (20) as

$$W_{c_t} = \beta W_{c_{t+1}} \left\{1 + f_{kt+1} - \delta_k\right\}, \tag{31}$$

where $W_{c_t} = U_{c_t} H_{c_t}$,with

$$H_{c_t} = 1 - \lambda_0 \left[\frac{{}_0b_t^c}{c_t} - \frac{1}{\lambda_0}\left(\frac{\phi_{0t}}{c_t} - \left\{1 - \delta_k\right\} \frac{\phi_{0t-1}}{c_{t-1}} \frac{c_{t-1}}{c_t} \right) \right]. \tag{32}$$

All terms in (32) are either variables or ratios that are constant in a BGP. Therefore, $H_{c_t} = H_c$, and thus, $\frac{W_{c_t}}{W_{c_{t+1}}} = \frac{U_{c_t} H_c}{U_{c_{t+1}} H_c} = \frac{U_{c_t}}{U_{c_{t+1}}}$ in the long run. Comparing (6) and (31), it is obvious that the tax rate on capital income is zero in a BGP. Note also that as the capital tax rate becomes zero, the upper bound on capital tax rates will not be binding and the multiplier ϕ_{0t} will be zero. Finally, given that $\frac{\phi_{ot}}{c_t}$ is zero in the long run, the labour income tax rate (30) becomes

$$\tau^l_{ss} = \frac{\lambda_0 \left[\frac{1+\ {}_0b^w_{ss}}{1-l_{ss}} - \left(\frac{{}_0b^c}{c}\right)_{ss} \right]}{1 + \lambda_0 \left[\frac{1+\ {}_0b^w_{ss}}{1-l_{ss}} \right]}. \blacksquare \tag{33}$$

Proof of Proposition 3.

Equate (25) and (26) and solve for labour

$$l_{t+1} = \left[\frac{\delta_g - \delta_k}{A} \right]^{\frac{1}{1-\alpha}} * Q_t^{-\frac{1}{1-\alpha}},$$

where

$$Q_t = X_t - \frac{\frac{\xi_{0t-1}}{c_{t-1}}}{\left(1 + \frac{1}{\beta}\frac{\xi_{0t-1}}{c_{t-1}} - \frac{\xi_{0t}}{c_t}\right)} \left[\alpha\left(1-\alpha\right) \frac{c_{t-1}}{c_t} \frac{c_t}{k_t} \left(\left(\frac{k_t}{g_t}\right)^{\alpha} + \left(\frac{k_t}{g_t}\right)^{-(1-\alpha)} \right) \right].$$

From equation (29), we know that $\frac{\partial X_t}{\partial \left[\frac{k_t}{g_t}\right]} > 0$. Since $\frac{\xi_{0t}}{c_t}$ approaches zero from above, we can guarantee that around a BGP, if $\delta_g > (<)\ \delta_k$ and $\left(\frac{k_t}{g_t}\right) \leq \left(\frac{k}{g}\right)_{ss}$, then $Q_t \leq Q_{ss}$ and $l_t \geq (\leq)\ l_{ss}$. From (22), if $\delta_g > \delta_k$, one can see that $\frac{\partial \gamma_{c_t}}{\partial\left(\frac{k_t}{g_t}\right)} < 0$. When $\delta_g = \delta_k$, the dynamics of $\left(\frac{k_t}{g_t}\right)$ cannot be determined. Finally, combining equations (23) and (24) with (5), the labour income tax rate is

$$\tau^l_t = \frac{\lambda_0 \left[\frac{1+\ {}_0b^w_t}{1-l_t} - \frac{{}_0b^c_t}{c_t} \right] - \left[\frac{1}{\beta}\frac{\xi_{ot-1}}{c_{t-1}} - \frac{\xi_{ot}}{c_t} \right]}{1 + \lambda_0 \left[\frac{1+\ {}_0b^w_t}{1-l_t} \right] - \frac{\xi_{ot-1}}{c_{t-1}} \frac{c_{t-1}}{c_t} \left(\frac{\theta}{1-\theta}\right) \left(1-l_t\right) \alpha\left(1-\alpha\right) A \left(\frac{k_t}{g_t}\right)^{\alpha-1} l_t^{-\alpha}}, \tag{34}$$

which, under the long run assumptions about inherited debt, becomes

$$\tau_t^l = \frac{\lambda_0 \left[\frac{1+\kappa_w}{1-l_t}\right] - \left[\frac{1}{\beta}\frac{\xi_{ot-1}}{c_{t-1}} - \frac{\xi_{ot}}{c_t}\right]}{1 + \lambda_0 \left[\frac{1+\ \kappa_w}{1-l_t}\right] - \frac{\xi_{ot-1}}{c_{t-1}}\frac{c_{t-1}}{c_t}\left(\frac{\theta}{1-\theta}\right)(1-l_t)\,\alpha\,(1-\alpha)\,A\left(\frac{k_t}{g_t}\right)^{\alpha-1} l_t^{-\alpha}},$$

where, given that $\frac{\xi_{ot}}{c_t}$ approaches zero from above, $\left[\frac{1}{\beta}\frac{\xi_{ot-1}}{c_{t-1}} - \frac{\xi_{ot}}{c_t}\right] \geq 0$. Obviously, when l_t approaches from below its steady state value so does τ_t^l. ■

Proof of Proposition 4.

First, we prove that ξ_{0t} is zero in a BGP. Given (2) and (18), the first order condition for capital in the government problem (25) can be written as

$$W_{c_t} = \beta W_{c_{t+1}}\left\{1 + f_{kt+1} - \delta_k\right\} - \beta \xi_{0t} f_{k_{t+1}k_{t+1}} U_{c_{t+1}}, \tag{35}$$

where $W_{c_t} = U_{c_t} H_{c_t}$, with

$$H_{c_t} = 1 + \lambda_0 \left[\frac{{}_0b_t^c}{c_t} - \frac{1}{\lambda_0}\left(\frac{\xi_{0t}}{c_t} - \left\{1 + f_{k_t} - \delta_k\right\}\frac{\xi_{0t-1}}{c_{t-1}}\frac{c_{t-1}}{c_t}\right)\right]. \tag{36}$$

It can be checked that $H_{c_t} = H_c$ in a BGP. Then, equation (35) becomes

$$U_{c_t} = \beta U_{c_{t+1}}\left\{R_{t+1} + \frac{\xi_{0t}}{c_t}\frac{c_t}{c_{t+1}}\frac{c_{t+1}}{k_{t+1}}\alpha\,(1-\alpha)\,\frac{A}{H_c}\left(\frac{k_{t+1}}{g_{t+1}}\right)^{\alpha-1} l_{t+1}^{1-\alpha}\right\}. \tag{37}$$

Note that $\frac{c_{t+1}}{k_{t+1}}, \frac{c_t}{c_{t+1}}, \frac{k_{t+1}}{g_{t+1}}$, and l_{t+1} take values different from zero. Then, combining equations (22) and (37), the ratio $\frac{\xi_{0t}}{c_t}$ must be zero in a BGP. Furthermore, if $\frac{\xi_{0t}}{c_t} = 0$, the zero-tax rate constraint (22) is already satisfied through (37), hence ξ_{0t} is zero in a BGP.

Since $\frac{\xi_{0t}}{c_t}$ approaches zero, the labour income tax rate (34) in a BGP becomes

$$\tau_{ss}^l = \frac{\lambda_0\left[\frac{1+\ {}_0b_{ss}^w}{1-l_{ss}} - \left(\frac{{}_0b^c}{c}\right)_{ss}\right]}{1 + \lambda_0\left[\frac{1+\ {}_0b_{ss}^w}{1-l_{ss}}\right]}. \tag{38}$$

Note that since the debt-commitment policy and the full-commitment policy take different λ_0 and l_{ss}, then their corresponding steady-state tax rates on labour income, (38) and (33), differ under both policies. ■

Proof of Proposition 5.

We consider the policy plans under the governments at 0 and at period 1. If both can be solved for the same allocation, the solution can be generalised for all later periods, and hence the policy plan at period 0 is made time-consistent. The government at period 0 can make its policy plan time-consistent by selecting debt values such that the same allocation $\{c_t, l_t, k_{t+1}\}_{t=1}^{\infty}$ and policy $\left\{g_{t+1}, \tau_t^l\right\}_{t=1}^{\infty}$ would solve both optimisation problems at 0 and 1. Therefore, this allocation must solve the different sets of restrictions and first order conditions that describe each plan. Let us now present both sets of conditions. When the government at date 0 plans a policy for $t \geq 0$, the policy and allocation solve the set of constraints,

$$\sum_{t=0}^{\infty} \beta^t \left[U_{c_t} \left(c_t - \ {}_0b_t^c\right) - U_{x_t} \left[l_t + \ {}_0b_t^w\right]\right] \leq U_{c_0} W_0, \tag{39}$$

$$c_t + k_{t+1} + g_{t+1} \leq Ak_t^{\alpha} \left(l_t g_t\right)^{1-\alpha} + \left(1 - \delta_k\right) k_t + \left(1 - \delta_g\right) g_t, \text{ for all } t \geq 0, \tag{40}$$

$$U_{c_t} = \beta U_{c_{t+1}} \left\{1 + f_{k_{t+1}} - \delta_k\right\}, \text{ for all } t \geq 0, \tag{41}$$

the first order conditions for the individual,

$$U_{x_t} = \left(1 - \tau_t^l\right) U_{c_t} f_{l_t}, \text{ for all } t \geq 0, \tag{42}$$

and the government plan,

$$W_x \left(c_t, l_t, \ {}_0b_t^c, \ {}_0b_t^w, \Theta_{x_t}, \lambda_0\right) = f_{l_t} W_c \left(c_t, l_t, \ {}_0b_t^c, \ {}_0b_t^w, \Theta_{c_t}, \lambda_0\right), \text{ for all } t \geq 0, \tag{43}$$

$$\mu_{0t} = \beta \mu_{0t+1} \left\{1 + f_{k_{t+1}} - \delta_k\right\} - \beta \xi_{0t} f_{k_{t+1} k_{t+1}} U_{c_{t+1}}, \text{ for all } t \geq 0, \tag{44}$$

$$\mu_{0t} = \beta \mu_{0t+1} \left\{1 + f_{g_{t+1}} - \delta_g\right\} - \beta \xi_{0t} f_{k_{t+1} g_{t+1}} U_{c_{t+1}}, \text{ for all } t \geq 0. \tag{45}$$

The set of equations, (39)-(45), forms the system that government at date 0 solves at announcing its policy plan.

When government at period 1 selects a policy for $t \geq 1$, this policy and the corresponding allocation satisfy the system of constraints,

$$\sum_{t=1}^{\infty} \beta^t \left[U_{c_t} \left(c_t - \ {}_1b_t^c\right) - U_{x_t} \left[l_t + \ {}_1b_t^w\right]\right] \leq U_{c_1} W_1, \tag{46}$$

$$c_t + k_{t+1} + g_{t+1} \leq Ak_t^{\alpha} \left(l_t g_t\right)^{1-\alpha} + \left(1 - \delta_k\right) k_t + \left(1 - \delta_g\right) g_t, \text{ for all } t \geq 1, \tag{47}$$

$$U_{c_t} = \beta U_{c_{t+1}} \left\{1 + f_{k_{t+1}} - \delta_k\right\}, \text{ for all } t \geq 1, \tag{48}$$

and the first order conditions for the individual,

$$U_{x_t} = \left(1 - \tau_t^l\right) U_{c_t} f_{l_t}, \text{ for all } t \geq 1, \tag{49}$$

and the government program,

$$W_x\left(c_t, l_t,\ {}_1b_t^c,\ {}_1b_t^w, \Theta_{x_t}, \lambda_1\right) = f_{l_t} W_c\left(c_t, l_t,\ {}_1b_t^c,\ {}_1b_t^w, \Theta_{c_t}, \lambda_1\right), \text{ for all } t \geq 1, \tag{50}$$

$$\mu_{1t} = \beta\mu_{1t+1}\left\{1 + f_{k_{t+1}} - \delta_k\right\} - \beta\xi_{1t} f_{k_{t+1}k_{t+1}} U_{c_{t+1}}, \text{ for all } t \geq 1, \tag{51}$$

$$\mu_{1t} = \beta\mu_{1t+1}\left\{1 + f_{g_{t+1}} - \delta_g\right\} - \beta\xi_{1t} f_{k_{t+1}g_{t+1}} U_{c_{t+1}}, \text{ for all } t \geq 1. \tag{52}$$

Equations, (46)-(52), form the system that the government at date 1 solves when computing its policy plan.

Now, let us prove that the allocation $\{c_t, l_t, k_{t+1}\}_{t=1}^{\infty}$ and policy $\left\{g_{t+1}, \tau_t^l\right\}_{t=1}^{\infty}$ that solve the policy plan at 0 for $t \geq 1$ can solve the policy plan at 1. Since the sequences $\left\{c_t, l_t, k_{t+1}, g_{t+1}, \tau_t^l\right\}_{t=1}^{\infty}$ solve the former, it is also a solution for equations $(47) - (49)$. First, to solve (50) for the same allocation, one debt instrument at each period is needed. Let us make (51) and (52) time-consistent. Equate both, ξ_t can be expressed as $\mu_{t+1}\left[\frac{R_{t+1} - \left(1 + f_{g_{t+1}} - \delta_g\right)}{U_{c_{t+1}}\left(f_{k_{t+1}k_{t+1}} - f_{k_{t+1}g_{t+1}}\right)}\right]$ and (51) and (52) can be written as

$$\begin{aligned}
\mu_t &= \beta\mu_{t+1}\left[R_{t+1} - f_{k_{t+1}k_{t+1}}\left[\frac{R_{t+1} - \left(1 + f_{g_{t+1}} - \delta_g\right)}{f_{k_{t+1}k_{t+1}} - f_{k_{t+1}g_{t+1}}}\right]\right], \\
\mu_t &= \beta\mu_{t+1}\left[R_{t+1} - f_{k_{t+1}g_{t+1}}\left[\frac{R_{t+1} - \left(1 + f_{g_{t+1}} - \delta_g\right)}{f_{k_{t+1}k_{t+1}} - f_{k_{t+1}g_{t+1}}}\right]\right].
\end{aligned}$$

Therefore, once μ_{1t} and μ_{0t} take the same value, the same allocation solves both equations. In order to make μ_{1t} take that value, an extra debt instrument for each date is needed. Note that when $\mu_{1t} = \mu_{0t}$, we have $\xi_{1t} = \xi_{0t}$ for the same allocation. So far, two debt instruments are needed in order to solve all equations, but (46). The path for this debt is function of λ_1. Once the government at 0 finds this function, it is imposed into the budget constraint (3), which leads to a specific debt structure. Then, by Walras' law, the implementability condition (46) holds. Thus, under that debt structure, the continuing allocation and policy planned at 0 solves the policy plan at 1.

Let us now find the debt structure that provides time consistency. Four types of debt need to be found: (i) the new inherited debt indexed to consumption for the first period; (ii) debt indexed to consumption for second period and on; (iii) the new inherited debt indexed to after-tax-wage for the first period; (iv) debt indexed to after-tax-wage for second period and on. Now, we find the evolution of debt indexed to after-tax-wage from the second period on. Let us consider the first order conditions for consumption and leisure under both plans (43) and (50),

$$\begin{aligned} W_x\left(c_t, l_t,\ {}_0b_t^c,\ {}_0b_t^w, \Theta_{x_t}, \lambda_0\right) - f_{l_t} W_c\left(c_t, l_t,\ {}_0b_t^c,\ {}_0b_t^w, \Theta_{c_t}, \lambda_0\right) &= 0. \quad (53)\\ W_x\left(c_t, l_t,\ {}_1b_t^c,\ {}_1b_t^w, \Theta_{x_t}, \lambda_1\right) - f_{l_t} W_c\left(c_t, l_t,\ {}_1b_t^c,\ {}_1b_t^w, \Theta_{c_t}, \lambda_1\right) &= 0. \quad (54) \end{aligned}$$

When (53) and (54) are satisfied, we can equate their LHS, and find

$$\begin{aligned} &[\lambda_0 - \lambda_1]\left(\left(U_{x_t} - f_{l_t} U_{c_t}\right) + \left(U_{c_t x_t} - f_{l_t} U_{c_t c_t}\right) c_t - \left(U_{x_t x_t} - f_{l_t} U_{c_t x_t}\right) l_t\right. \\ &- \left[\tfrac{\xi_{0t-1} - \xi_{1t-1}}{\lambda_0 - \lambda_1}\right] U_{c_t} f_{k_t l_t}\Big) = -\left(U_{c_t x_t} - f_l U_{c_t c_t}\right)\left(\lambda_1\ {}_1b_t^c - \lambda_0\ {}_0b_t^c + \left(\left(\xi_{0t} - \xi_{1t}\right)\right.\right. \quad (55)\\ &- \left(1 + f_{k_{t-1}} - \delta_k\right)\left(\xi_{0t-1} - \xi_{1t-1}\right)\Big)\Big) - \left(U_{x_t x_t} - f_{l_t} U_{c_t x_t}\right)\left[\lambda_1\ {}_1b_t^w - \lambda_0\ {}_0b_t^w\right]. \end{aligned}$$

Let us divide equation (55) by $-\left(U_{x_t x_t} - f_{l_t} U_{c_t x_t}\right)\lambda_1$, take into account that $\xi_{0t} = \xi_{1t}$ and then add ${}_0b_t^w\left[\frac{\lambda_0}{\lambda_1} - 1\right]$, and rearrange terms to obtain

$$\begin{aligned} &\left[\tfrac{U_{x_t x_t} - f_{l_t} U_{c_t x_t}}{U_{c_t x_t} - f_{l_t} U_{c_t c_t}}\right]\left\{\left[\tfrac{\lambda_0}{\lambda_1} - 1\right]\left[l_t +\ {}_0b_t^w - \tfrac{U_{x_t} - f_{l_t} U_{c_t}}{U_{x_t x_t} - f_{l_t} U_{c_t x_t}}\right] - \left[{}_1b_t^w -\ {}_0b_t^w\right]\right\} = \\ &\qquad \left[{}_1b_t^c - \tfrac{\lambda_0}{\lambda_1}\ {}_0b_t^c + \left[\tfrac{\lambda_0}{\lambda_1} - 1\right] c_t\right]. \end{aligned} \quad (56)$$

Now, let us find the equation for $\mu_{0t} = \mu_{1t}$. Substitute μ_t by its value, equation (23), divide by $-U_{x_t x_t}\lambda_1$, consider that $\xi_{0t} = \xi_{1t}$ and add ${}_0b_t^w\left[\frac{\lambda_0}{\lambda_1} - 1\right]$, we obtain

$$\begin{aligned} &\left[\tfrac{U_{x_t x_t}}{U_{c_t x_t}}\right]\left\{\left[\tfrac{\lambda_0}{\lambda_1} - 1\right]\left[l_t +\ {}_0b_t^w - \left[\tfrac{U_{x_t}}{U_{x_t x_t}}\right]\right] - \left[{}_1b_t^w -\ {}_0b_t^w\right]\right\} = \\ &\qquad \left[{}_1b_t^c - \tfrac{\lambda_0}{\lambda_1}\ {}_0b_t^c + \left[\tfrac{\lambda_0}{\lambda_1} - 1\right] c_t\right]. \end{aligned} \quad (57)$$

Equate the LHS of (56) and (57), rearrange terms and obtain

$${}_1b_t^w -\ {}_0b_t^w = \left[\frac{\lambda_0}{\lambda_1} - 1\right]\left[{}_0b_t^w + l_t - \left[\frac{U_{c_t x_t} U_{c_t} - U_{c_t c_t} U_{x_t}}{U_{c_t c_t} U_{x_t x_t} - \left(U_{c_t x_t}\right)^2}\right]\right].$$

Taking into account the specific utility function form (2), we have

$${}_1b_t^w -\ {}_0b_t^w = \left[\frac{\lambda_0}{\lambda_1} - 1\right]\left[{}_0b_t^w + 1\right],$$

which is the new inherited debt indexed to after-tax-wage for $t > 1$. This procedure needs to be replicated for the three remaining debt types. ■

Appendix B: Numerical Solution Method.

In this appendix, we outline a more precise explanation of the eigenvalue-eigenvector decomposition method.[17] This method consists of the following:

i) Some reasonable values for the initial state variables and for the parameters are selected.

ii) The model is solved for a steady state. The solution of the debt-commitment model, $\{c_t, l_t, g_{t+1}, k_{t+1}, \tau_t^l, \mu_{0t}, \xi_{0t}\}_{t\geq 0}$ and λ_0, is characterised by the set of necessary conditions, (5) and (23) – (26), and the restrictions, (13), (16) and (22). If the number of total periods were T, the system would have $T*4+(T-1)*3+1$ equations and the same number of unknown variables. However, our economy extends over an infinity of periods. For each period, there are four equations involving variables at that date, three equations that link future to current variables and one that is a function of variables in all dates. For the purpose of solving the system, the equations that link current and future variables need to be replaced by the stability conditions that depend on variables at the current date. In an endogenous growth model, steady-state levels of the variables change over time. These variables are transformed so as to take constant values in a steady state, for example, $w_t^{kg} = \frac{k_t}{g_t}$, $w_t^{cc} = \frac{c_t}{c_{t-1}}$, $w_t^{ck} = \frac{c_t}{k_t}$, and $w_t^{\xi c} = \frac{\xi_{0t}}{c_t}$. However, notice that to find the value of λ_0, that is independent of the moment of time, we need to know the whole series of variables and plugged them into (16). To solve this, a steady-state for a given value of λ_0 can be computed, and later, we search for the value of λ_0 that solves (16). Next, the conditions and restrictions, (5), (23) – (26), (13) and (22), are written as functions of the new set of variables. Finally, since all new variables are constant in a BGP, we can take away the t index, and find a steady-state.

iii) All restrictions and conditions are linearised around the steady state. These equations can be viewed as a function $f\left(w_t^{ck}, l_t, w_t^{kg}, w_t^{cc}, w_t^{\phi c}\right)$. One can define

[17] As an example, the equations that are included concern the debt-commitment policy.

$y_t = \left(w_t^{ck} - w_{ss}^{ck}, l_t - l_{ss}, w_t^{kg} - w_{ss}^{kg}, w_t^{cc} - w_{ss}^{cc}, w_t^{\phi c} - w_{ss}^{\phi c}\right)$ and do a first order Taylor approximation around the steady state

$$\left.\frac{\partial f}{\partial y_t}\right|_{ss} y_t + \left.\frac{\partial f}{\partial y_{t-1}}\right|_{ss} y_{t-1} = Ay_t + By_{t-1} = 0.$$

iv) The unstable eigenvalues of the linear system are computed. We find the set of eigenvalues and eigenvectors of the matrix $-(A^{-1}) * B$. An unstable eigenvector is defined as one that takes an absolute value greater than $\beta^{-\frac{1}{2}}$, this number is chosen so that the objective function is bounded.

v) The stability conditions are computed. They are obtained by imposing orthogonality between each eigenvector associated with an unstable eigenvalue and the variables of this system, that is,

$$Cy_t = 0,$$

where C is the matrix of eigenvectors associated to unstable eigenvalues. These stability conditions guarantee that the transversality conditions hold.

vi) The stability conditions are imposed into the original non-linear model. The equations that linked current to future variables are replaced by the stability conditions. A solution can be now computed. Notice that since the stability conditions are computed for the linearised system, the solution involves some numerical error.

For a more complete review, see Novales et al. (1999) and Sims (1998).■

Figures and tables.

Figure 1: Consumption growth rates.

Figure 2: Public capital growth rate.

Figure 3: Tax rates on capital income.

Figure 4: Tax rates on labour income.

Figure 5: Cash flow in present value.

Figure 6: Debt structure that makes the policy time-consistent.

Figure 7: Debt structure for $t \geq 1$. DC and DW stand for debt indexed to consumption and to after-tax-wage respectively.

TABLE 1. Summary of Results from the Numerical Solution Method.

Variables	**Policy 0**	**Policy 1**	**Policy 2**	**Policy 3**
$\left(\frac{c}{k}\right)_{ss}$	0.03956	0.03858	0.03838	0.03849
$\mathbf{l}_{ss}$	0.83333	0.29043	0.27352	0.27523
$\left(\frac{k}{g}\right)_{ss}$	0.34284	0.35462	0.35589	0.35578
$\boldsymbol{\tau}^{l}_{ss}$	–	0.83320	0.84134	0.84092
$\boldsymbol{\tau}^{k}_{ss}$	–	0	0	0
M. E. K.	2.49e-7	1.95e-9	4.94e-5	0.71765
M. E. G.	2.49e-7	3.45e-10	4.94e-5	0.70100
M. E. T.	–	–	0	0.03586
E. I.	–	-5.2e-10	-1.83e-7	-2.17e-6
Growth rate	13.87%	4.274%	3.838%	3.849%
Welfare	249.207	84.3657	76.3204	73.4250

Policy 0 is the first best policy.

Policy 1 is the policy under full-commitment.

Policy 2 is the policy under full-commitment with $\tau^k_t \leq 1$.

Policy 3 is the policy under debt-commitment with $\tau^k_t = 0$.

M.E.K. and M.E.G. stand for maximum error at satisfying the first order condition for private and public capital respectively in the government program.

M.E.T. stands for maximum error at satisfying the capital tax rate restriction.

E.I. stands for the error that is made at satisfying the implementability condition.