Experimentation and Learning in Rational Addiction Models with Multiple Addictive Goods

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1 Introduction

The purpose of this paper is to study the consumption pattern of potentially addictive substances in a dynamic model of rational addiction under uncertainty. Stigler and Becker (1977) and Becker and Murphy (1988) have forcefully argued that addiction can be modeled as the outcome of rational behavior of forward looking individuals with stable preferences. The basic hypothesis of the rational addiction model is that individuals are aware of both costs and benefits of addictive consumption. Individuals choose an optimal consumption path considering all current and future consequences of additive behavior. This theory differs from earlier approaches that attributed addictive behavior to either myopic or irrational behavior.

One of the main drawbacks in most of the previous studies is the assumption that individuals have perfect foresight and hence operate in an environment without uncertainty. There are a number of reasons why uncertainty matters when dealing with addictive consumption and its consequences. The perfect foresight framework offers no scope for the regret observed among many addicts. Individuals are never fooled or get hooked into addiction. Orphanides and Zervos (1995) argue that uncertainty and learning through experimentation need to be incorporated into the rational addiction framework in order to account for "involuntary" addiction. In their framework, addiction results from a time-consistent expect-utility maximizing plan. Addiction is unintentional and results from experimentation with addictive goods that are known to provide a certain instantaneous pleasure and only probabilistic future harm.

While the analysis of Orphanides and Zervos (1995) provides some new insights into theory of consumption of addictive substances, it is subject to a number of simplifications. The empirical evidence suggests that individuals typically consume a bundle of different addictive substances. For example, individuals who are smoking are also more likely to consume larger amounts of alcohol and/or engage in binge drinking. This suggests that there is a close relationship between the consumption of multiple addictive substances.

It is important to understand the substitution patterns between different addictive goods in order to design meaningful public policies. For example, a tax increase on one addictive good may just lead to an increase in consumption of a close substitute, leaving the overall

consumption level of addictive and harmful substances almost unchanged. Alternatively, if two addictive goods are complements, then a policy aimed at reducing consumption of one substance may have positive spill-over effects since it also leads to a decrease of consumption of the second substance.

It is also important to understand the substitution patterns between multiple addictive goods that differ in their degree of harmfulness. For example, an individual may start experimenting with less dangerous substances and learn valuable information about his overall addictive tendencies. After learning that he does not seem to have addictive tendencies by consuming the less dangerous good, he may switch to more harmful substances. Therefore a policy which discourages the use of less harmful addictive substances may have large positive effects because it also prevents learning. As a result, consumption of the more harmful substances will decrease in the long run.

Adding multiple addictive goods to the rational addiction framework increases the computational complexity of the model. Even a simple dynamic model has four continuous state variables which is inherently hard to solve. One of the main problems encountered in the computational analysis is the approximation of the value function. Approximation methods are increasingly important in numerical dynamic programming since they allow researchers to solve models with large state spaces. In this paper, we focus on a new approximation method which has been recently developed by Coppejans (2000) in the context of nonparametric regression estimation.

The basic idea of this approach is to represent a function of several variables as superpositions of functions of one variable. These one-dimensional functions are approximated by B-splines, which have nice computational properties. This method allows us to parameterize the value function by a one-dimensional object which alleviates the curse of dimensionality typically encountered in these type of problems.

The rest of the paper is organized as follows. Section 2 presents a dynamic model of rational addiction under uncertainty. Section 3 discusses new techniques that can be used to approximate value functions. Section 4 presents the results of the computational experiments. Section 5 outlines some future work for this study.

2 Dynamic Model

2.1 The Extended Orphanides-Zervos Model

Let y_t denote income of an individual and c_t consumption of a numeraire good at date t. Define a_{1t} and a_{2t} as the consumption of potentially addictive goods like alcohol and tobacco. The stock of addictive consumption, s_{it}, evolves according to the following law of motion:

$$s_{it+1} \;=\; \delta_i \, s_{it} \;+\; a_{it} \qquad i = 1, 2. \tag{2.1}$$

We assume that consumers rank alternatives according to the following utility function:

$$U_t \;=\; u(c_t,\; a_{1t},\; a_{2t}) \;+\; \eta_t \, \theta \, v(a_{1t},\; a_{2t},\; s_{1t},\; s_{2t}), \tag{2.2}$$

where θ is equal to 1 if the individual has addictive tendencies or zero otherwise.

Following Orphanides and Zervos (1995), η_t is a random variable that is equal to one with probability $\pi(s_{1t}, s_{2t})$ and zero otherwise. Its inclusion introduces a probabilistic occurrence of harmful side effects into the model. We assume that $\theta\eta_t$ is observed at the end of each period. Individuals maximize intertemporal utility,

$$E\left(\sum_{t=0}^{\infty} \beta^t \left[u(c_t,\; a_{1t},\; a_{2t}) \;+\; \eta_t \, \theta \, v(a_{1t},\; a_{2t},\; s_{1t},\; s_{2t})\right]\right), \tag{2.3}$$

subject to a sequence of budget constraints,

$$c_t \;+\; \sum_{i=1}^{2} p_{it} \, a_{it} \;=\; y_t. \tag{2.4}$$

An implicit assumption of this specification is that individuals do not save. This is a plausible assumption for young individuals who are likely to engage in experimentation with addictive substances.

When θ is unknown, it is assumed that individuals have beliefs given by $P_t = \text{prob}\{\theta = 0\}$. Given an initial prior, P_0, we assume that individuals optimally update their beliefs about θ using Bayes' Rule. Hence beliefs evolve according to

$$P_{t+1} \;=\; \begin{cases} \frac{P_t}{P_t + (1-P_t)(1-\pi(s_{1t}, s_{2t}))}, & \text{if} \qquad \eta_t \, \theta = 0, \\ 0, & \text{if} \qquad \eta_t \, \theta > 0. \end{cases} \tag{2.5}$$

Furthermore, we assume that there is income uncertainty in the model. Individuals have rational expectations and income transitions are characterized by the transition probability $p(y_{t+1} \mid y_t)$.

2.2 Optimal Decision Rules

Since we abstract from saving decisions, we can simplify the decision problem of the individuals by substituting the budget constraint into the utility function. Define

$$w(y_t, a_{1t},\ a_{2t}) \ = \ u((y_t \ - \ \sum_{i=1}^{2} p_{it} a_{it}),\ a_{1t},\ a_{2t}), \tag{2.6}$$

and

$$\Pi_t \ = \ \Pi_t(P_t, s_{1t}, s_{2t}) \ = \ \pi(s_{1t}, s_{2t})\,(1 - P_t). \tag{2.7}$$

Substituting the laws of motion of the stocks into the value function, we can express the dynamic programming problem faced by the individuals as follows:

$$\begin{aligned} V(y, s_1, s_2, P) \ &= \ \max_{a_1, a_2} \ w(y, a_1,\ a_2) + \Pi v(a_1, a_2, s_1, s_2) \\ &+ \ \beta \int \Big[\Pi\ V(y', a_1 + \delta_1 s_1, a_2 + \delta_2 s_2, 0) \\ &\qquad +(1 - \Pi)\ V(y', a_1 + \delta_1 s_1, a_2 + \delta_2 s_2, P/(1 - \Pi))\Big]\ p(y'|y)\, dy. \end{aligned} \tag{2.8}$$

2.3 A Parameterization of the Model

Since we can only solve the model above numerically, we need to parameterize the utility functions and the transition probabilities. We assume that the first component of the utility function is given by

$$u(c_t, a_{1t}, a_{2t}) \ = \ log(c_t) \ + \ \alpha\ log(a_t), \tag{2.9}$$

where

$$a_t \ = \ \Big[a_{1t}^{\rho_1} + \rho_2\ a_{2t}^{\rho_1}\Big]^{1/\rho_1}. \tag{2.10}$$

Note that this specification assumes that the numeraire good, c_t, and the aggregator, a_t, are substitutes. The specification of the aggregation function allows us to treat the two addictive substances as either complements or substitutes, depending on the choice of the substitution elasticity ρ_1.

The second component of the utility function is given by

$$v(a_{1t}, a_{2t}, s_{1t}, s_{2t}) \ = \ s_{1t}^{\gamma_{1,1}}(\gamma_{2,1} \ + \gamma_{3,1} a_{1t}) \ + \ s_{2t}^{\gamma_{1,2}}(\gamma_{2,2} \ + \gamma_{3,2} a_{2t}). \tag{2.11}$$

We assume that the probability distribution, $\pi(s_{1t}, s_{2t})$, is exponential,

$$\pi(s_{1t}, s_{2t}) \;=\; 1 \;-\; \exp(-\zeta_1 s_{1t} \;-\; \zeta_2 s_{2t}). \tag{2.12}$$

Finally, the transition density for income is a finite state Markov Chain model that is an approximation to some log-normal regression model.

3 Computation

Smooth approximation methods estimate the value function, $V_t(\cdot)$, by a smooth function of the state variables, x, rather than by calculating the value function on finite grid of points.[1] The basic idea is to parameterize the value function and choose the parameter vector such that the approximation is as close as possible to the correct value function. More formally, consider the standard problem of solving a dynamic problem using backward recursion. Suppose we already have computed a smoothed version of the value function in $t+1$, which we denote by $V_{t+1}(x)$. Hence we can compute estimates of $V_t(x)$ for any finite grid $\{x_1, ..., x_N\}$ using backward recursion. The main idea of smooth approximation is to estimate $V_t(x)$ only on a small grid and use a clever imputation algorithm to impute the values for $V_t(x)$ which are not in the grid.

Another way of interpreting smooth approximation is to think of it as an estimation problem. The vector $\{V_t(x_1), ..., V_t(x_N)\}$ can then be interpreted as the data. The problem is to estimate the unknown function $V_t(x)$ by projecting it on class of nicely behaved functions. The "error" in the estimation is the approximation error. Once we have estimated the value function, we can use our estimates to predict the value function outside our sample (e.g. the grid). Following this analogy, we need to pick a class of functions $V_t(x|\theta)$ which effectively parameterize the value function. We then estimate θ using our data $\{V_t(x_1), ..., V_t(x_N)\}$ and standard optimization algorithms.

The main problem encountered in this approach is to chose a nicely behaved class of functions, and hence an approximation method, that satisfies a number of properties. First, the approximated value function should be close to the true value function on the set of grid points, i.e. the approximation error should be small. Second, the approximation algorithm

[1] The first part of this section follows Rust (1995) and Judd (1998).

must be computationally efficient and robust. Finally, parameterization should be "tight", allowing us to approximate high dimensional functions.

Unfortunately, approximating multi-dimensional functions is difficult because of the well known curse of dimensionality. For example, let $x \in X \subset \Re^d$ with $X = [a_j, b_j]^d$, $-\infty < a_j < b_j < \infty$, $j = 1, \ldots, d$. For some given $\epsilon > 0$, we will typically have to approximate $V_t(\cdot)$ at around $O(1/\epsilon^{d/2})$ number of points, assuming that $V(\cdot)$ is twice continuously differentiable, in order for the approximation error to be no greater than ϵ, $\sup_{x \in X} |V_t(x) - V_t(x|\theta)| \leq \epsilon$. An example of $V_t(x|\theta)$ in this case is a quadratic tensor-product B-spline.[2] Observe that $1/\epsilon^{2/d}$ grows exponentially in terms of d. Hence the computational time required to solve moderately high dimensional problems can be quite burdensome.

To overcome this type of problem, Coppejans (2000) has proposed the following estimator for $V_t(\cdot)$

$$\sum_{k=1}^{2d+1} g_k \left(\lambda_{1,k} \phi_k(x_1) + \cdots + \lambda_{d,k} \phi_k(x_d) \right),$$

where g_k and ϕ_k are estimated by univariate cubic B-splines, ϕ_k is restricted to be non-decreasing, and $\lambda_{j,k} > 0$ is a scalar with $\sum_{j=1}^{d} \lambda_j = 1$. In Coppejans (2000), this estimator is shown to work well at estimating high-dimensional problems. Most important, under suitable assumptions, we only need $O(1/\epsilon^{1/2})$ number of evaluating points to get an approximation error of ϵ. The key is that, unlike above, $1/\epsilon^{1/2}$ no longer depends on d.

[2] Denote uniformly placed knots on the interior of $[a_j, b_j]$ as $\{\xi_{l,j}\}$, $l = 1, \ldots, L_j$. In the construction of the univariate B-spline, additional knots are also placed at a_j and b_j; for example, a quadratic (third order) B-spline has three additional knots at each endpoint, and a cubic (fourth order) B-spline has four additional knots. A basis for the univariate B-spline of rth order, $\{B_{l,j,r}(x_j)\}_{l=1}^{L_j+3}$, is defined recursively as in de Boor (1978),

$$\begin{aligned} B_{l,j,r}(x_j) &= \frac{x_j - \xi_{l,j}}{\xi_{l+r-1,j} - \xi_{l,j}} B_{l,j,r-1}(x_j) + \frac{\xi_{l+r,j} - x_j}{\xi_{l+r,j} - \xi_{l+1,j}} B_{l+1,j,r-1}(x_j), \quad \text{if } r \geq 2, \\ B_{l,j,1} &= \begin{cases} 1, & \text{if } x_j \in [\xi_{l,j}, \xi_{l+1,j}) \\ 0, & \text{if otherwise.} \end{cases} \end{aligned}$$

Given any $x \in X$, a basis for the tensor-product B-spline of order r is (e.g. see Schumaker, 1981)

$$\sum_{m_1=k_1}^{k_1+r-1} \cdots \sum_{m_d=k_d}^{k_d+r-1} B_{m_1,1,r}(x_1) \cdots B_{m_d,d,r}(x_d)$$

where k_j is the smallest integer, $l+1$, such that $x_j \geq \zeta_{l,j}$. The tensor-product B-spline coefficients are calculated by least squares minimization.

To further speed up compilation, we propose a variant of the above estimator,

$$\mathcal{P}(x) + \sum_{k=1}^{l<2d+1} g_k \left(\lambda_{1,k}\phi_k(x_1) + \cdots + \lambda_{d,k}\phi_k(x_d)\right), \tag{3.1}$$

where $\mathcal{P}(x)$ is a quadratic polynomial of total order, and g_k and ϕ_k are estimated by quadratic B-splines.[3] The estimator in (3.1) will be referred to as the *reduced B-spline*. We still require that ϕ_k is monotonic, which is easily imposed in this case by requiring that the analogous B-spline coefficients are monotonic.

4 Experiment

The goal of this section is to show that the estimator proposed in the last section works reasonably well. For simplicity, we will treat income, y, as static, fixing its value at one. We will begin with an example similar to that in Orphanides and Zervos (1995), where there is just one addictive good ($d = 2$). Let $\delta = 0.5$, $\beta = 0.9$, $p = 1$, and

$$\begin{aligned} u(c,a) &= \log(c) + \log(a), \\ v(a,s) &= \sqrt{s}(-100 + 99a), \\ \pi(s) &= 1 - \exp(0.1s). \end{aligned}$$

In the case $d = 2$, the dynamic programming problem can be solved relatively quickly using standard methods; however, this is not the case when $d = 3$, as in our second example where we look at two potentially addictive drugs. Extending the above case, let $\delta_1 = \delta_2 = 0.5$, $p_1 = p_2 = 1$,

$$\begin{aligned} u(c,a) &= \log(c) + \log(a), \\ a &= \left(a_1^{0.35} + a_2^{0.35}\right)^{1/0.35}, \\ v(a,s) &= \sqrt{s_1}(-100 + 99a_1) \ + \ \sqrt{s_2}(-100 + 99a_2), \\ \pi(s) &= 1 - \exp(0.1s_1 + 0.1s_2). \end{aligned}$$

These models will be called, respectively, Model 1 and Model 2. We will solve the models by the two methods discussed in the previous section: the tensor-product B-spline

[3]For example, suppose $d = 2$. Then the basis for the quadratic polynomial of total oder is $1, x_1, x_2, x_1x_2, x_1^2, x_2^2, x_1x_2^2, x_1^2, x_2, x_1^2x_2^2$.

and the reduced B-spline. For both models, the stock variable takes values in $[0, 2]$ and the probability takes values in $[0, 1]$. The action space has been discretized with a bin width of 0.025 between adjacent points.

For the first method, the state space has been discretized so that the width between adjacent points is 0.025, implying that N is 1,326 and 67,626, respectively, when $d = 2$ and $d = 3$. The number of interior knots used for the stock and probability are, respectively, ten and five. For the second method, the width is 0.2 between adjacent points, which corresponds to an N of 66 and 726, respectively. The number of interior knots for each B-spline is set to five, and the number of superposition terms, l, used in each model are one and two, respectively.

For the first model, the tensor-product B-spline took about five minutes to compute, while the reduced B-spline took only a minute.[4] In the second model, the tensor-product B-spline took over five days to compute, while the other method took about two hours. Note that in the second model, not only has the dimension increased, but the number of actions (control variables) has doubled, which further increases the compilation time.

It is easiest to compare the results visually, and as a result, some plots of the policy function are provided in Figures 1-3. Overall, the reduced B-spline compares favorably. That is it estimates the general shapes and patterns of the policy function satisfactory. Note that it is not surprising that the tensor-product B-spline does a better job of approximation. In general, the cost of methods that circumvent the curse of dimensionality is that they will not be as accurate; the benefit is that they are much quicker to compute. Of course more experiments will be needed before general conclusion about the performance of the reduced B-spline can be made.

5 Future Work

The results in the last section show that methods like the reduced B-spline can work at solving these types of dynamic programming problems. However, the following model, because of its parsimonious parameterization, looks even more promising. Lorentz (1966) showed that

[4]The programs are written in Fortran and run on a Sun Ultra 2 workstation.

any continuous function, $f(\cdot)$, can be represented as

$$f(x_1, \ldots, x_d) = \sum_{k=1}^{2d+1} g\left(\lambda_1 \phi_k(x_1) + \cdots \lambda_d \phi_k(x_d)\right). \tag{5.1}$$

One of the fascinating features about this form is that the ϕ_k's are fixed; that is they do not depend on $f(\cdot)$. Also note that the $g(\cdot)$ function is the same across $k = 1, \cdots, 2d+1$. Given this, we propose first estimating the ϕ_k's by using the methods used in Lorentz's proofs. What is especially nice about this is that the estimates can be re-used for different value functions with the same number of state variables, d. Then $g(\cdot)$ can be approximated by a quadratic B-splines. Given that we have chosen k knots, there will only be $k+3$ parameters involved in estimating functions of the form on the right hand side of (5.1). This should make estimation and function evaluation very fast. This will be especially important when we solve the model using empirical data.

6 References

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Model 1

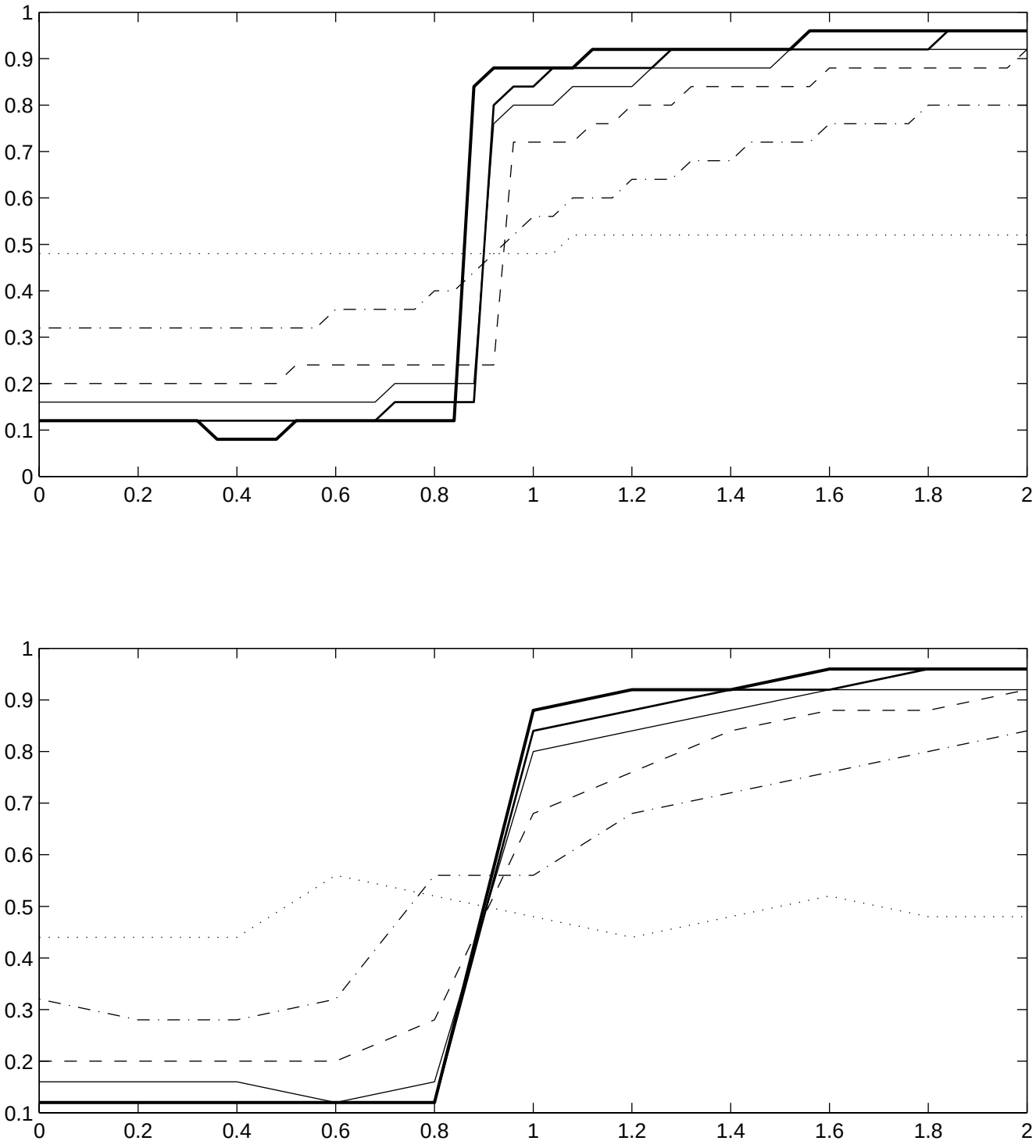

Figure 1. The lines, in descending order of thickness, are conditioned on probabilities of addictiveness, P, of 1.0, 0.8, 0.6, 0.4, 0.2, and 0.0. The vertical axis represents a and the horizontal axis represents s. The top plot is the tensor-product B-spline, and the bottom plot is the reduced B-spline.

Model 2: $s_2 = 0$

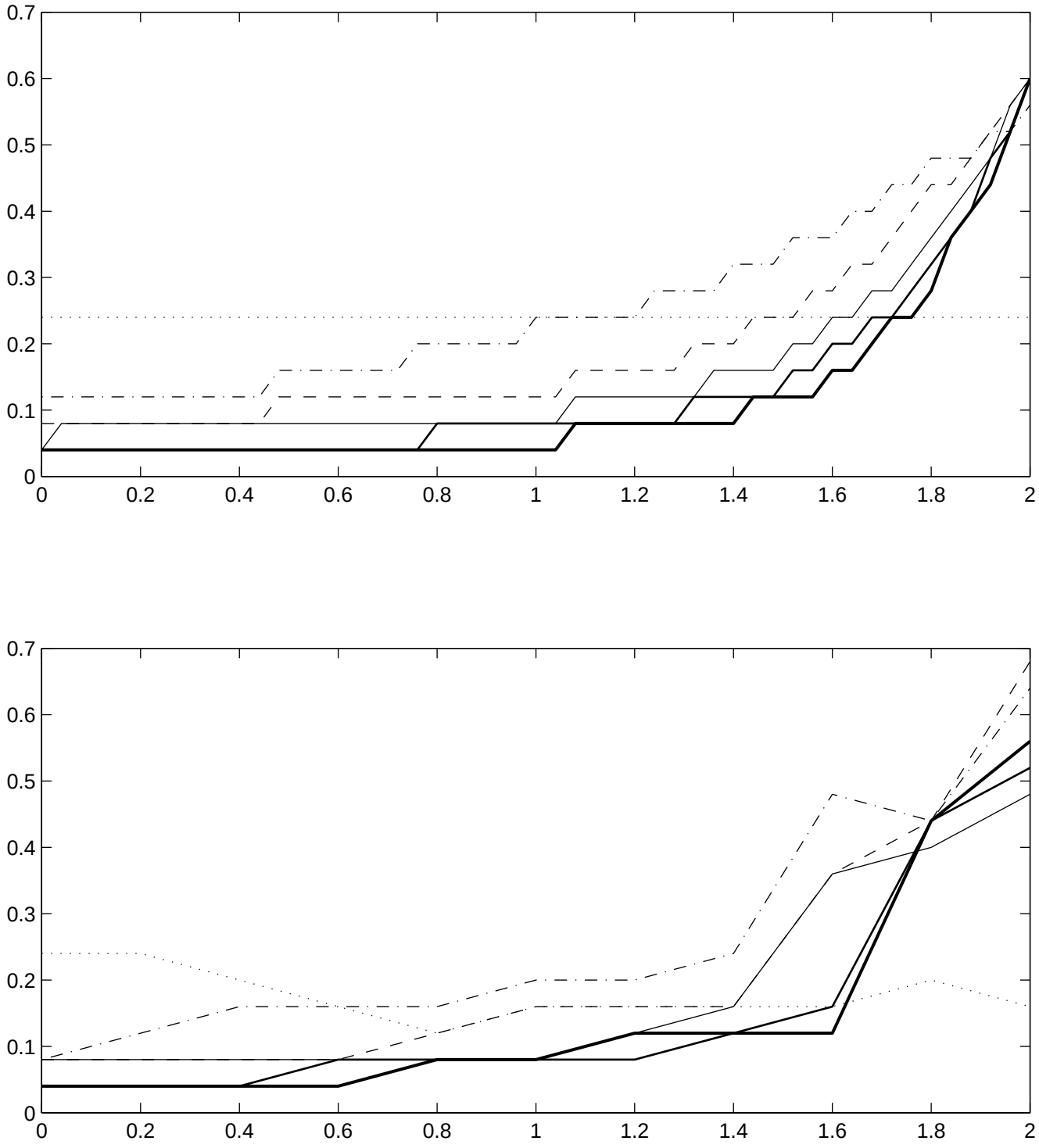

Figure 2. The lines, in descending order of thickness, are conditioned on probabilities of addictiveness, P, of 1.0, 0.8, 0.6, 0.4, 0.2, and 0.0. The vertical axis represents a_1 and the horizontal axis represents s_1. The top plot is the tensor-product B-spline, and the bottom plot is the reduced B-spline.

Model 2: $s_2 = 1$

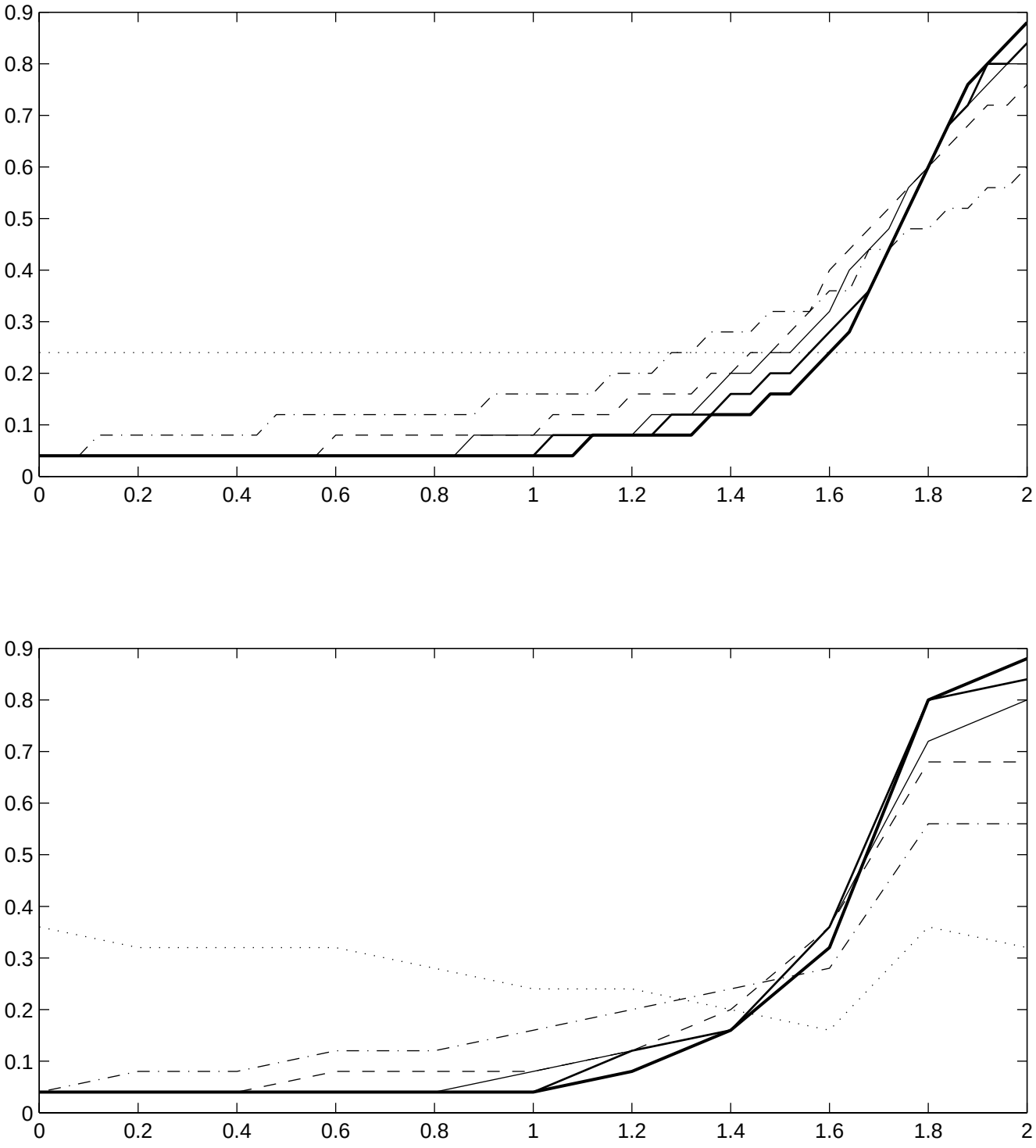

Figure 3. The lines, in descending order of thickness, are conditioned on probabilities of addictiveness, P, of 1.0, 0.8, 0.6, 0.4, 0.2, and 0.0. The vertical axis represents a_1 and the horizontal axis represents s_1. The top plot is the tensor-product B-spline, and the bottom plot is the reduced B-spline.