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# Heterogeneous Expectations, Market Dynamics and Social Welfare

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ABSTRACT

This paper explores the extent to which the lack of rationality of economic agents has affected the economic fluctuations and the social welfare of the U.S. hog market. A group of articles has ascribed the business cycles of the hog market, observed by economists as early as the last century, mainly to the lack of rationality (cobweb expectations) of economic agents. In contrast, others, assuming the full rationality of economic agents, have ascribed them to production lags and external shocks. These two streams of thought are reconciled in this paper by adopting the mechanics of conventional rational expectations models and assuming heterogeneity in expectations. The dynamic model presented in this paper assumes two types of economic agents. One (rational agent) has rational expectations and the other (boundedly rational agent) has cobweb expectations. The fraction of boundedly rational agents is estimated along with other deep parameters of the model using the actual data of the market. Empirical test results indicate that some fraction of economic agents in the U.S. hog market are boundedly rational. Based on the model and the estimation results, simulation experiments can be performed to investigate how the presence of boundedly rational economic agents has affected the volatility of the economic variables and the social welfare of the market. This paper briefly describes the methodologies. The simulation results will be reported in the revised version.

JEL Classification: C61, C62, E32, E37

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## I. Introduction

The purpose of this paper is to investigate the extent to which the lack of rationality of economic agents has affected the economic fluctuations and the social welfare of the U.S. hog market. A group of articles[1] has ascribed the business cycles of the agricultural markets, observed by economists as early as the last century, mainly to the lack of rationality (cobweb expectations) of economic agents. In contrast, others, assuming the full rationality of economic agents, have ascribed them to production lags and external shocks.[2] These two streams of thought are reconciled in this paper by adopting the mechanics of conventional rational expectations models and assuming heterogeneity in expectations.

The bounded rationality model presented in this paper includes two types of economic agents. One is fully rational and the other is boundedly rational compared with his rational counterpart. If the fraction of boundedly rational economic agents is zero, the model collapses to a rational expectations model. Therefore, a rational expectations model can be regarded as a special case of this kind of bounded rationality model.

The fraction of boundedly rational agents is estimated along with other deep parameters of the model using the actual data of the market. The current version of the paper is mainly devoted to illustrating how to construct, solve and estimate the model of the hog market which assumes two types of expectations. In addition, it shows the estimation results.

Section 2 presents the models and solves them. The rational expectations model is presented first because the bounded rationality model of this paper is a generalized version of it. The rational expectations model is reformatted into a social planner's problem, then the social planning problem is converted into an optimal regulator problem. Once the transformation is done, the model is easily solved by the well-established optimal control theory.[3]

The bounded rationality model presented in this paper cannot be solved by the same procedures as shown above. Since there are two types of agents whose expectations are different from each other in a competitive market, the bounded rationality model cannot be formed as a representative's problem. Therefore, it can not be reformatted as a social planner's problem. However, Baak (1999) shows that in spite of the presence of boundedly rational economic agents, the bounded rationality model can be transformed into a fictional social planner's problem with

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[1] Examples are Ezekiel (1938), Harlow (1960), Talpaz (1974), and Hayes and Schmitz (1987).
[2] See, for example, Rosen, Murphy and Scheinkman (1994).
[3] This is illustrated in Anderson, Hansen, McGrattan and Sargent (1995, AHMS hereafter).

side equilibrium constraints. Therefore, it can be reformatted as a linear quadratic optimal regulator problem with distortions. The methodology of solving a distorted optimal regulator problem illustrated by McGrattan (1994) and AHMS is used to solve this model. If we assume that the fraction of boundedly rational agents is zero, the side constraints of the fictional social planner's problem are no longer binding. As a result, the distortions of the optimal regulator problem disappear. In this case, the rational expectations model and the bounded rationality model produce the same solution.

Once the solution is obtained, the fraction of boundedly rational agents along with other deep parameters of the model can be estimated by MLE. The methods of state space representations and innovations representations using the Kalman filter, illustrated by AHMS and Hansen and Sargent (1996), are useful in this step. The methodologies and results of empirical tests are shown in section 3. Empirical test results indicate that some fraction of economic agents in the U.S. hog market are boundedly rational.

After estimating the fraction of boundedly rational economic agents in the market, simulation experiments can be performed to investigate how the presence of boundedly rational economic agents has affected the volatility of the economic variables and the social welfare of the market. In particular, several sets of artificial data can be generated by the model using the estimated parameter values, while changing the fraction of the boundedly rational agents from zero to one. Each set of artificial data is related to a certain fraction of boundedly rational agents. Then, the variances of the quantity and price variables can be computed using the actual and artificial data and then compared. According to preliminary simulation experiments, the higher the fraction of boundedly rational agents is, the more volatile the economic variables are.

The simulations, however, are not contained in this version of the paper, since it requires some verification. The social welfare will also be measured and compared in the way illustrated above, and then will be reported in the revised version of the paper.

## 2. The Models

The models presented in this paper have the form of a dynamic model of renewable resources management in which optimal decision making is incorporated with animal population dynamics.[4]

[4] Such models can be found in Chavas and Klemme (1988), Rosen, Murphy and Scheinkman (1994), Chavas (1995) and Baak (1999).

The final output of the hog industry at time $t$ is the sum of surviving adult animals from the previous year and the one-year old animals joining the adult stock.[5]

(2.1) $$y_t = (1-\delta)k_{t-1} + gk_{t-1}$$

where $y_t$ is the final output, $k_{t-1}$ is the adult stock at time $t-1$[6], $\delta$ is the natural death rate, and $g$ is the birth (fertility) rate per adult animal. The death rate is assumed to be non-negative and the birth rate, positive. The final output is either marketed or held as an investment for future production (that is held for breeding). In this industry, the amount of capital (breeding) stock at time $t$ equals the amount of investment at time $t$.

(2-2) $$y_t = i_t + c_t$$

(2-3) $$k_t = i_t$$

where $i_t$ is investment, $c_t$ consumption (or sales), and $k_t$ breeding (capital) stock at time t.

The goal of a producer (or a supplier) in this industry is assumed to be to maximize the expected present discounted value of current and future profits over an infinite horizon of time.

### 2.1 The Rational Expectations Model

In section 2.1.1, a dynamic optimizing model of a representative rational hog producer in a competitive economy will be presented. Then, in section 2.1.2 there will be a demonstration of how the model is transformed into a social planner's problem of the Hansen and Sargent (1996) type (HS type social planning problem, hereafter). In section 2.1.3 it will be shown that the social planner's problem is equivalent to a discounted stochastic optimal regulator problem. As AHMS show, these transformations enable us to solve the rational expectations model more quickly and to perform MLE more efficiently.

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[5] Piglets become fully grown adult pigs at 1.5~2 years of age. But, they can be marketed at 6-7 months and join the breeding stock at 8 months. Therefore, it is reasonable to assume that they become adults after one period. See Chavas (1995).

### 2.1.1 The rational expectations model in a competitive hog market

The objective function a rational producer intends to maximize is the following:

$$(2.1.1)\quad \max_{i_t} E_0\left\{\sum_{t=1}^{\infty}\beta^t\left[p_t c_t - m_t c_t - h_t\left(k_t + \gamma_0 g k_t\right) - \frac{\psi_c}{2}c_t^2 - \frac{\psi_0}{2}k_t^2\right]\right\}$$

$$\text{s.t.}\quad (2.1.2)\qquad c_t = (1-\delta)k_{t-1} + g k_{t-1} - i_t \text{ [7]}$$

$$(2.1.3)\qquad k_t = i_t$$

where $p_t$ is the market price of an adult animal, $m_t$ is the marketing cost of preparing an adult animal for slaughter, and $h_t$, $\gamma_0 h_t$ are the one-period holding costs for a breeding animal and a piglet, respectively. The quadratic term $\frac{\psi_0}{2}k_t^2$ captures the increasing costs of holding adult animals and $\frac{\psi_c}{2}c_t^2$ captures the increasing costs of preparing for slaughter. The discount factor $\beta$ is assumed to be positive yet less than one. The parameters $\psi_c$ and $\gamma_0$ are assumed to be positive.

The costs $\{h_t, m_t\}_{t=0}^{\infty}$ are exogenous state variables, while the price stream $\{p_t\}_{t=0}^{\infty}$ is determined by the competitive market equilibrium. The exogenous state variables are assumed to be AR(1) processes.

(2.1.4) $h_t = (1-\rho_h)h^* + \rho_h h_{t-1} + \sigma_h \varepsilon_t^h$, where $h^* > 0, 0 < \rho_h < 1, 0 < \sigma_h$, $\varepsilon_t^h$ ~ white noise

(2.1.5) $m_t = (1-\rho_m)m^* + \rho_m m_{t-1} + \sigma_m \varepsilon_t^m$, where $m^* > 0, 0 < \rho_m < 1, 0 < \sigma_m$, $\varepsilon_t^m$ ~ white noise

It is assumed that the producers are able to observe the current values of endogenous and exogenous state variables before they make decisions at each period. In the meantime, the demand for pigs is assumed to be a linear function of the market price.

(2.1.6) $c_t = \alpha_0 - \alpha_1 p_t + d_t$, where $\alpha_0 > 0$, $\alpha_1 > 0$

(2.1.7) $d_t = \rho_d d_{t-1} + \varepsilon_t^d$, where $0 < \rho_d < 1$, $\varepsilon_t^d$ ~ white noise

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[6] $k_{t-1}$ is also breeding stock at time t-1. Breeding stock are regarded as capital in that they are being used for production.

[7] Equation (2.1) and (2.2) generate the constraint (2.1.2).

The first order condition (that is, the Euler equation) of the dynamic optimization problem is

$$(2.1.8)\quad E_t\left[\begin{array}{l} -p_t + m_t + \psi_c c_t - h_t(1+\gamma_0 g) \\ +\beta\left(p_{t+1}(1-\delta+g) - m_{t+1}(1-\delta+g) - \psi_c c_{t+1}(1-\delta+g)\right) \end{array}\middle| \Omega_t \right] = 0$$

where $\Omega_t$ is the information set which contains all information available to a producer at time t, such as, the current and past values of endogenous and exogenous state variables.

### 2.1.2 The social planner's problem

The objective function of a HS type social planner is the following:

$$(2.1.10)\qquad \underset{i_t}{Max}\, E_t\left(-\frac{1}{2}\left\{\sum_{t=0}^{\infty}\beta^t\left[\left(\mathbf{s}_t-\mathbf{b}_t\right)'\left(\mathbf{s}_t-\mathbf{b}_t\right)+\mathbf{g}_t{}'\mathbf{g}_t\right]\right\}\right)$$

$$\text{s.t. } \phi_g \mathbf{g}_t = -\phi_c c_t - \phi_i i_t + \Gamma \mathbf{k}_{t-1} + \mathbf{d}_t$$

$$\mathbf{k}_t = \Delta_k \mathbf{k}_{t-1} + \Theta_k i_t$$

$$\mathbf{h}_t = \Delta_h \mathbf{h}_{t-1} + \Theta_h c_t$$

$$\mathbf{s}_t = \Lambda \mathbf{h}_{t-1} + \Pi c_t$$

where $\mathbf{b}_t = U_b \mathbf{z}_t$, $\mathbf{d}_t = U_d \mathbf{z}_t$, $\mathbf{z}_t = \begin{bmatrix}1 & h_t & m_t & d_t\end{bmatrix}'$, $\mathbf{g}_t = \begin{bmatrix}g_{1,t} & g_{2,t}\end{bmatrix}'$, and $\mathbf{k}_{t-1} = k_{t-1}$. The vector of exogenous state variables $\mathbf{z}_t$ is assumed to have the following transition dynamics: $\mathbf{z}_{t+1} = A_{22}\mathbf{z}_t + C_2\varepsilon_{t+1}$, where $\varepsilon_{t+1}$ is a vector white noise.

The objective function (2.1.10) can represent the preceding competitive equilibrium model by defining the parameters and the matrices of the parameters such as

$$\Lambda = \Delta_h = \Theta_h = 0; \quad \Pi = \tfrac{1}{\sqrt{\alpha_1}}; \quad U_b = \begin{bmatrix}\tfrac{\alpha_0}{\sqrt{\alpha_1}} & 0 & 0 & \tfrac{1}{\sqrt{\alpha_1}}\end{bmatrix};$$

$$\phi_c = \begin{bmatrix}1\\0\\-f_7\end{bmatrix} \quad \phi_i = \begin{bmatrix}1\\-f_5\\0\end{bmatrix} \quad \phi_g = \begin{bmatrix}0 & 0\\1 & 0\\0 & 1\end{bmatrix} \quad \Gamma = \begin{bmatrix}(1-\delta+g)\\0\\0\end{bmatrix} \quad U_d = \begin{bmatrix}0 & 0 & 0 & 0\\0 & f_6 & 0 & 0\\0 & 0 & f_8 & 0\end{bmatrix}$$

$$\Delta_k = 0 \qquad \Theta_k = 1 \qquad A_{22} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ (1-\rho_h)h^* & \rho_h & 0 & 0 \\ (1-\rho_m)m^* & 0 & \rho_m & 0 \\ (1-\rho_d)d^* & 0 & 0 & \rho_d \end{bmatrix} \qquad C_2 = \begin{bmatrix} 0 & 0 & 0 \\ \sigma_h & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_d \end{bmatrix}$$

$$f_5^2 = \psi_0,\; f_7^2 = \psi_c;\;\; f_5 f_6 = \gamma_0 g,\; f_7 f_8 = 1$$

With the definitions of the variables and the parameters illustrated above, the objective function (2.1.10) generates the same first order condition as equation (2.1.8) with $p_t$ replaced by the market equilibrium condition (2.1.6), the same constraints as (2.1.2) and (2.1.3), and the same processes for exogenous state variables as equations (2.1.4), (2.1.5) and (2.1.7).

### 2.1.3 The optimal regulator problem

The social planning problem presented above can be transformed into a discounted linear regulator problem of the following form:

(2.1.11)
$$\underset{u_t}{Max}\; E_t\left( -\sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \mathbf{x}_t & u_t \end{bmatrix} \begin{bmatrix} R & W \\ W' & Q \end{bmatrix} \begin{bmatrix} \mathbf{x}_t \\ u_t \end{bmatrix} \right)$$
$$s.t.\;\; \mathbf{x}_{t+1} = A\mathbf{x}_t + Bu_t + C\varepsilon_{t+1}$$

where $\mathbf{x}_t$ is a vector of state variables, and $u_t$ is a vector of control variables.

Hansen and Sargent (1996) illustrate that an HS type social planning problem is transformed into an optimal linear regulator problem by defining the vectors and matrices in the regulator problem (2.1.11) as follows:

$\mathbf{x}_t = [\mathbf{h}_{t-1}'\; \mathbf{k}_{t-1}'\; \mathbf{z}_t']',\; u_t = i_t,$

$$A = \begin{bmatrix} \Delta_h & \theta_h U_c\begin{bmatrix}\phi_c & \phi_g\end{bmatrix}^{-1}\Gamma & \theta_h U_c\begin{bmatrix}\phi_c & \phi_g\end{bmatrix}^{-1} U_d \\ 0 & \Delta_k & 0 \\ 0 & 0 & A_{22} \end{bmatrix} \qquad B = \begin{bmatrix} -\theta_h U_c\begin{bmatrix}\phi_c & \phi_g\end{bmatrix}^{-1}\phi_i \\ \Theta_k \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 \\ 0 \\ C_2 \end{bmatrix}$$

$$H_s = \begin{bmatrix} \Lambda & \Pi U_c\begin{bmatrix}\phi_c & \phi_g\end{bmatrix}^{-1}\Gamma & \Pi U_c\begin{bmatrix}\phi_c & \phi_g\end{bmatrix}^{-1}U_d - U_b \end{bmatrix} \qquad H_c = \begin{bmatrix} -\Pi U_c\begin{bmatrix}\phi_c & \phi_g\end{bmatrix}^{-1}\phi_i \end{bmatrix}$$

$$G_s = \left[0 \quad U_g\left[\phi_c \quad \phi_g\right]^{-1}\Gamma \quad U_g\left[\phi_c \quad \phi_g\right]^{-1}U_d\right] \qquad G_c = \left[-U_g\left[\phi_c \quad \phi_g\right]^{-1}\phi_i\right]$$

$$U_c = \begin{bmatrix}1 & 0 & 0 & 0 & 0\end{bmatrix} \qquad U_g = \begin{bmatrix}0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

$$R = -\tfrac{1}{2}\left(H_s' H_s + G_s' G_s\right) \qquad Q = -\tfrac{1}{2}\left(H_c' H_c + G_c' G_c\right) \qquad W = -\tfrac{1}{2}\left(H_c' H_s + G_c' G_s\right)$$

The solution of the optimal control problem (2.1.11) is given by $u_t = -F\mathbf{x}_t$, where $F = (Q + \beta B' PB)^{-1}(\beta B' PA + W)$ and $P = R + \beta A' P' A - (\beta A' PB + W')(Q + \beta B' PB)^{-1}(\beta B' PA + W)$.

AHMS and Hansen McGrattan and Sargent (1994) provide several algorithms to search for the value of $P$.[8] Once the value of $P$ is known, the solution of the model can be obtained by computing $F$.

The following section explores how we can solve a bounded rationality model using optimal control theory.

## 2.2 The Bounded Rationality Model

The bounded rationality model presented in this section assumes two types of producers who have different expectations for the future prices of pig. One type (a rational producer) is assumed to know that the market price of pig is endogenously determined in the market and purposefully acquires and uses all available information to predict future prices.

The other type (a boundedly rational producer) is assumed to treat hog price as an exogenous variable whose value is determined independently from market dynamics.[9] However, with the exception of his approach to hog prices, this type is assumed to be as rational as the rational agent.

[8] I assume that the regulatory conditions illustrated in AHMS are satisfied. These conditions are needed to guarantee a unique stationary solution. If the conditions are violated, the algorithms do not work, thus the computer programs which implement the algorithms yield error messages. In practice, the conditions are checked during the computations.

[9] While rational economic agents know that the value of the variable is endogenously determined in the market, some other agents may not know this. The differences in knowledge between these two types can be explained by differences in education and/or experience, as Chavas (1995) claims. On the other hand, some economic agents may treat an endogenous variable like an exogenous variable when they predict the future value of the variable, even though they understand it is endogenous because of deliberation and/or information costs, as Conlisk (1996) and Brock and Hommes (1996) explain. This paper does not investigate the reason for heterogeneity in beliefs.

The only difference between the two types of hog producers is in their predictions of future pig prices.

The integral sum of all producers is assumed to be unity without loss of generality. The fraction of boundedly rational producers is denoted by $n$. By definition, $n$ is between zero and one. The objective function of a producer, whether rational or boundedly rational, is assumed to be the same as in section 2.1. The economic environment, such as the dynamics of the exogenous variables, is also assumed to be the same.

### 2.2.1 The bounded rationality model in a competitive market

The objective function of a hog producer is

$$\text{(2.2.1)} \quad \max_{i_{j,t}} E_{j,0}\left\{\sum_{t=1}^{\infty}\beta^{t}\left[p_t c_{j,t} - m_t c_{j,t} - h_t\left(k_{j,t} + \gamma_0 g k_{j,t}\right) - \frac{\psi_c}{2}c_{j,t}^{2} - \frac{\psi_0}{2}k_{j,t}^{2}\right]\right\}$$

$$\text{s.t.} \quad c_{j,t} = (1-\delta)k_{j,t-1} + g k_{j,t-1} - i_{j,t}$$

$$k_{j,t} = i_{j,t}$$

The above objective function and constraints are the same as in section 2.1, with the exception of subscript j in the endogenous state variables $c$, $k$, and the control variable $i$, which denotes that the variables are associated with a type j (j=1 or 2) producer. Hereafter, subscripts 1 and 2 represent rational and boundedly rational economic agents, respectively. For example, $c_{1,t}$ and $c_{2,t}$ are the quantities supplied by a rational and boundedly rational producer, respectively.

Even though these two types of producers have the same objective function, they have different Euler equations because they predict future hog prices differently. The Euler equation of a rational producer is

$$\text{(2.2.2)} \quad E_t\left[\begin{array}{l} -p_t + m_t + \psi_c c_{1,t} - h_t(1+\gamma_0 g) \\ +\beta\left(p_{t+1}(1-\delta+g) - m_{t+1}(1-\delta+g) - \psi_c c_{1,t+1}(1-\delta+g)\right) \end{array}\right] = 0$$

Equation (2.2.2) is the same as equation (2.1.8) which was presented as the Euler equation of a rational producer in section 2.1.

On the other hand, the Euler equation of a boundedly rational producer is

(2.2.3)
$$E_t\begin{bmatrix} -p_t + m_t + \psi_c c_{2,t} - h_t(1+\gamma_0 g) \\ +\beta\left(-m_{t+1}(1-\delta+g) - \psi_c c_{2,t+1}(1-\delta+g)\right) + E_{2,t}\left(\beta p_{t+1}(1-\delta+g)\right) \end{bmatrix} = 0$$

where $E_{2,t}$ denotes the expectations operator of a boundedly rational agent. In this paper it is assumed that boundedly rational producers believes that hog prices follow AR(1) processes. That is, $E_2(p_{t+1}) = a_0 + a_1 p_t$.

The aggregate (market) Euler equation (2.2.5) is obtained by the summation of equation (2.2.2) and (2.2.3) after multiplying the fraction of rational producers, $(1-n)$, to equation (2.2.2) and the fraction of boundedly rational producers, $n$, to equation (2.2.3).

(2.2.5)
$$E_t\begin{bmatrix} -(1-n)p_t + m_t + \psi_c c_{1,t} - h_t(1+\gamma_0 g) \\ +\beta\left((1-n)p_{t+1}(1-\delta+g) - m_{t+1}(1-\delta+g) - \psi_c c_{1,t+1}(1-\delta+g)\right) \\ -np_t + n\beta(1-\delta+g)(a_0 + a_1 p_t) \end{bmatrix} = 0$$

The same law of motion as in equation (2.1.9) is also obtained by aggregating the constraints of each type of producer.

The market demand equation (2.1.6) and the dynamics of exogenous state variables (2.1.4), (2.1.5) and (2.1.7) are still assumed. Now the market as a whole is described by the Euler equation (2.2.5), the law of motion (2.1.9), the demand equation (2.1.6) and the dynamic processes of the exogenous variables (2.1.4), (2.1.5) and (2.1.7). As a matter of fact, the bounded rationality model is different from the rational expectations model only in the case of the Euler equation.

The deep parameters of the model can be estimated with these six equations using GMM. Chavas (1995) adopts this methodology. In this paper, however, the deep parameters are estimated with the solutions of the model using MLE following the line of AHMS. Among the merits of the AHMS methodology are that it uses all the available hog data sets whether or not they are shown in the model, and that it does not require proxies for exogenous variables. In fact, three quantity data sets and one price data set are available in the U.S. pork market, and the empirical research in section 3 uses all and only these four data sets. In addition, the methodology adopted in this paper automatically checks the stationarity condition of the model. In the case of linear quadratic models, the stationarity of the model should be carefully checked to ensure that the model is meaningful both theoretically and empirically.

### 2.2.2 The social planning problem with side constraints.

The bounded rationality model presented in section 2.2.1 cannot be represented by a social planner's problem because the model violates the first welfare theorem by assuming heterogeneity in beliefs. The economic agents are no longer homogeneous in the bounded rationality model.

Competitive equilibrium models with some distortions such as externalities and taxes also violate the first welfare theorem. However, in the case of those models, the dynamic optimization problem of a representative economic agent can be replaced by a social planning problem with some side equilibrium constraints. That is, for those models, there can be a fictional social planning problem whose first-order conditions are the same as those of the competitive solution.[10]

In the meantime, if we examine the Euler equation (2.2.5), it looks like the Euler equation of a representative economic agent who faces some distortions such as externalities and taxes. The first two lines of the Euler equation are the same as in the Euler equation of a rational rancher if $p_t$ in the latter is replaced by $(1-n)p_t$. The third line is an extra element in the Euler equation of a rational rancher, and it can be treated as being imposed by an equilibrium condition associated with externalities. Specifically, if we include an extra term, $q_t i_t$, in the objective function and if $q_t$ should be equal to the third line of the Euler equation (2.2.5) in the equilibrium,[11] the aggregate Euler equation (2.2.5) can be regarded as the first-order condition of a representative agent with $p_t$ replaced by $(1-n)p_t$. The variable $q_t$ is exogenous to the fictional agent but endogenous to the model.

Based on this background, the bounded rationality model presented in section 2.2.1 can be reformatted into a HS type social planning problem with side constraints. The dynamic optimization problem of the fictional social planner is

[10] For example, see Becker (1985) and McGrattan (1994).

[11] The fictional agent is assumed to accept this side equilibrium condition as given, just like the demand equation. Therefore, when we solve the objective function, $q_t$ is treated as an exogenous variable. After obtaining the first order condition of the objective function, the equilibrium condition of $q_t$ is plugged in. This is much like the "big" $K_t$ in the versions of Lucas and Prescott's (1971) investment model analyzed by Kydland and Prescott (1977) and Whiteman (1983).

$$\text{(2.2.6)} \qquad \underset{i_t}{Max}\, E_t\left(-\frac{1}{2}\left\{\sum_{t=0}^{\infty}\beta^t\left[\left(\mathbf{s}_t-\mathbf{b}_t\right)'\left(\mathbf{s}_t-\mathbf{b}_t\right)+\mathbf{g}_t{}'\mathbf{g}_t-2i_tq_t\right]\right\}\right)$$

$$\text{s.t. } \phi_g\mathbf{g}_t=-\phi_c c_t-\phi_i i_t+\Gamma\mathbf{k}_{t-1}+\mathbf{d}_t$$

$$\mathbf{k}_t=\Delta_k\mathbf{k}_{t-1}+\Theta_k i_t$$

$$\mathbf{h}_t=\Delta_h\mathbf{h}_{t-1}+\Theta_h c_t$$

$$\mathbf{s}_t=\Lambda\mathbf{h}_{t-1}+\Pi c_t$$

where, $\Pi=\frac{\sqrt{1-n}}{\sqrt{\alpha_1}}$, $U_b=\left[\frac{\alpha_0\sqrt{1-n}}{\sqrt{\alpha_1}}\ 0\ 0\ \frac{\sqrt{1-n}}{\sqrt{\alpha_1}}\right]$. The other variables and parameters are defined in the same way as in section 2.1.2.

The objective function (2.2.6) contains a new term, $i_tq_t$, which was not a part of the objective function (2.1.10) in the rational expectations model. The variable $q_t$ should satisfy the following side constraint. As stated above, $q_t$ is exogenous to the fictional social planner, but endogenous to the model.

$$\text{(2.2.7)} \qquad q_t=B_0+B_1k_{t-1}-A_1i_t-A_1d_t$$

where $B_0=A_0-\alpha_0A_1$, $B_1=A_1(1-\delta+g)$, $A_0=n\beta(1-\delta+g)a_0$,
$A_1=\frac{n}{\alpha_1}(1-\beta(1-\delta+g)a_1)$

The right hand side of the constraint (2.2.7) is the same as the third line of the Euler equation (2.2.5). To make the third line serve as a function of state and control variables, I substituted the demand equation (2.1.6) and the law of motion ( 2.1.9).

In the following section, we will see how to transform the objective function (2.2.6) into an optimal regulator problem with distortions (side equilibrium conditions).

### 2.2.3 The optimal regulator problem

The dynamic optimization problem of a fictional social planner (2.2.6) can be transformed into the following discounted optimal linear regulator problem with side equilibrium conditions:

(2.2.8)
$$\underset{u_t}{Max}\, E_t\left( -\sum_{t=0}^{\infty} \beta^t \left( \begin{bmatrix} \mathbf{x}_t & q_t \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}_t \\ q_t \end{bmatrix} + u_t{}' R u_t + 2\begin{bmatrix} \mathbf{x}_t & q_t \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} u_t \right) \right)$$
$$s.t.\ \ \mathbf{x}_{t+1} = A_x \mathbf{x}_t + A_q q_t + B u_t + C\varepsilon_{t+1}$$

where $Q_{11} = Q$, $Q_{12} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]'$, $Q_{21} = Q_{12}{}'$, $Q_{22} = 0$, $W_1 = W$, $W_2 = -\tfrac{1}{2}$, $A_x = A$, $A_z = 0$. The vectors and matrices ($\mathbf{x}_t$, $u_t$, $Q, R, W, A, B, C$) are defined in the same way as in section 2.1.13.[12] The side equilibrium condition (2.2.7) is imposed in the following form:

(2.2.9)
$$q_t = \Theta \mathbf{x}_t + \Psi u_t$$

where $\Theta = \begin{bmatrix} 0 & B_1 & B_2 & B_3 & B_4 & B_0 & 0 & 0 & C_0 & C_1 \end{bmatrix}$ and $\Psi = C_0$.

The dynamic optimization problem (2.2.8) includes a discount factor ($\beta$) and cross-product terms. McGrattan(1994) and AHMS suggest converting the problem to one without cross-products and discounting in order to simplify the analysis. Let $\tilde{\mathbf{x}}_t = \beta^{t/2}\mathbf{x}_t$, $\tilde{q}_t = \beta^{t/2} q_t$, $\tilde{u}_t = \beta^{t/2} u_t$, $\tilde{\varepsilon}_t = \beta^{t/2}\varepsilon_t$, $\tilde{Q}_{11} = Q_{11} - W_1 R^{-1} W_1{}'$, $\tilde{Q}_{12} = Q_{12} - W_1 R^{-1} W_2{}'$, $\tilde{Q}_{22} = Q_{22} - W_2 R^{-1} W_2{}'$, $\tilde{A}_x = \sqrt{\beta}\left(A_x - BR^{-1}W_1{}'\right)$, $\tilde{A}_q = \sqrt{\beta}\left(A_q - BR^{-1}W_2{}'\right)$, $\tilde{B} = \sqrt{\beta} B$, $\tilde{\Theta} = \left(I + \Psi R^{-1} W_2{}'\right)^{-1}\left(\Theta - \Psi R^{-1} W_1{}'\right)$, and $\tilde{\Psi} = \left(I + \Psi R^{-1} W_2{}'\right)^{-1}\Psi$.

With these definitions, we can restate the optimization problem as follows[13]

(2.2.10)
$$\underset{u_t}{Max}\, E_t\left( \sum_{t=0}^{\infty} \begin{bmatrix} \tilde{\mathbf{x}}_t & q_t \end{bmatrix} \begin{bmatrix} \tilde{Q}_{11} & \tilde{Q}_{12} \\ \tilde{Q}_{21} & \tilde{Q}_{22} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}}_t \\ q_t \end{bmatrix} + \tilde{u}_t{}' R \tilde{u}_t \right)$$
$$s.t.\ \ \tilde{\mathbf{x}}_{t+1} = \tilde{A}_x \tilde{\mathbf{x}}_t + \tilde{A}_q \tilde{q}_t + \tilde{B}\tilde{u}_t + C\tilde{\varepsilon}_{t+1}$$

[12] These objects are defined in the same way as in section 2.1.3, but they are not exactly the same as in section 2.1.3 because their components are defined differently in section 2.2. For example, the definition of $Q$ is $-\frac{1}{2}\left(H_c{}' H_c + G_c{}' G_c\right)$, and the definition of $H_c$ is $-\Pi U_c\left[\phi_c \ \ \phi_g\right]^{-1}\phi_i$ in both sections 2.1 and 2.2. However, the definitions of $\Pi$ are different in the two sections.

[13] See Appendix in McGrattan(1994), Appendix B.3 in AHMS.

Let $\hat{A} = \tilde{A}_x + \tilde{A}_q\tilde{\Theta}$, $\hat{Q} = \tilde{Q}_{11} + \tilde{Q}_{12}\tilde{\Theta}$, $\hat{B} = \tilde{B} + \tilde{A}_q\tilde{\Psi}$, and $\tilde{A} = \tilde{A}_x - \tilde{B}R^{-1}\tilde{\Psi}'\tilde{Q}_{12}{}'$. The solution of the problem is given by $\tilde{u}_t = -\tilde{F}\mathbf{x}_t$, where $\tilde{F} = \left(R + \tilde{B}'P\hat{B}\right)^{-1}\tilde{B}'P\hat{A}$, and $P$ satisfies $P = \hat{Q} + \tilde{A}'P\hat{A} - \tilde{A}'P\hat{B}(R + B'P\hat{B})^{-1}B'P\hat{A}$. The solution of the original problem is given by $F = \left(R + W_2{}'\Psi\right)^{-1}\left(R\tilde{F} + W_1{}' + W_2{}'\Theta\right)$ and the equilibrium law of motion for $\mathbf{x}_t$ is $\mathbf{x}_{t+1} = A^0\mathbf{x}_t$, $A^0 = A_x + A_q\Theta - A_q\Psi F - BF = \beta^{-1/2}(\hat{A} - \hat{B}F)$.[14]

The computer programs capable of searching for the solution for the bounded rationality model (the optimal linear regulator problem with distortions) of this paper were coded by modifying AHMS's computer programs used for solving their rational expectations model (the optimal linear regulator problem without distortions). As in the case of the computer programs for the rational expectations model, the computer programs for the bounded rationality model also yield error messages if the regulatory conditions for a unique stationary solution are violated. The conditions are checked during the computations.

Once the solution is found, the deep parameters of the model can be estimated using MLE. The following section briefly illustrates the methodology and presents the empirical test results.

## 3. The Empirical Tests

The fraction of boundedly rational producers in the U.S. pork market is estimated in this section by MLE using observed pork data. The same data set as Chavas (1995) is used. Yet, the empirical tests are performed with detrended data while Chavas (1995) uses non-detrended data. Chavas (1995) contains trend in his model. In contrast, the dynamic model employed in this paper does not include any trend. Therefore, the detrended data are used for empirical tests. The data are detrended using linear trends which are obtained by OLS.

The data set contains annual observations for the pig crop ($x_t$), the number consumed ($c_t$), the breeding stock ($k_t$), and the price of an adult animal ($p_t$) for the U.S. during the period 1945~1990. Some information on the data is presented in Table 1.

[14] See Appendix B.3 in AHMS and Appendix B in Hansen, McGrattan and Sargent (1994).

Section 3.1 briefly explains how observed prices and quantities can be used to estimate the deep parameters of the model. The key is to construct a Gaussian log-likelihood function using the observables: first, the observables are represented by functions of state variables of the model (state space representations); second, the state-space representations are converted into innovations representations using the Kalman filter; third, the innovations are computed by the innovations representations; fourth, a Gaussian likelihood function is computed by the innovations. The deep parameters can then be estimated by maximizing the likelihood function.

Section 3.2 presents the tests results.

### 3.1 The likelihood function

The state variables of the bounded rationality model have the following law of motion: [15]

(3.1.1) $$\mathbf{x}_{t+1} = A^0\mathbf{x}_t + C\varepsilon_{t+1}$$

In the meantime, the four observables can be expressed as linear functions of the state variables of the model. We know that $c_t = (1-\delta+g)k_{t-1} - i_t$ and $k_t = i_t$. The function for $p_t$ can be obtained from the function for $c_t$ and the demand function. The number of pig crops is determined by the birth rate and the breeding stock; $x_t = gk_t$. By equation (2.3), $x_t = gi_t$. The equilibrium solution is $i_t = -F\mathbf{x}_t$. If we add measurement errors to the observables, we can obtain the following:

(3.1.2) $$\mathbf{y}_t = G\mathbf{x}_t + v_t$$

where $\mathbf{y}_t = [x_t \;\; k_t \;\; c_t \;\; p_t]'$, $v_t$ is a martingale difference sequence of measurement errors that satisfies $Ev_tv_t' = \mathbf{R}$, $E\varepsilon_{t+1}v_s' = 0$ for all $t+1 \geq s$, and

[15] The formula for $A^0$ is given in section 2.2.3.

$$G=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & (1-\delta+g) & 0 & 0 & 0 & 0\\ 0 & \frac{-(1-+g)}{\alpha_1} & \frac{\alpha_0}{\alpha_1} & 0 & 0 & \frac{1}{\alpha_1}\end{bmatrix}+\begin{bmatrix}-\tilde{F}\\ -\tilde{F}\\ \tilde{F}\\ -\tilde{F}/\alpha_1\end{bmatrix}.$$

Equations (3.1.1) and (3.1.2) yield the following state space representation of the model:

(3.1.3)
$$\mathbf{x}_{t+1}=A^0\mathbf{x}_t+C\varepsilon_{t+1}$$
$$\mathbf{y}_{t+1}=GA^0\mathbf{x}_t+GC\varepsilon_{t+1}+v_{t+1}$$

Then, the time-invariant innovations representation for system (3.1.3) is

(3.1.4)
$$\hat{\mathbf{x}}_{t+1}=A^0\hat{\mathbf{x}}_t+Ka_t$$
$$\mathbf{y}_{t+1}=GA^0\hat{\mathbf{x}}_t+a_t$$

where $\hat{\mathbf{x}}_t=\hat{E}\left[\mathbf{x}_t\middle|\mathbf{y}_t,\mathbf{y}_{t-1},\cdots,\mathbf{y}_0,\hat{\mathbf{x}}_0\right]$, $a_t=\mathbf{y}_{t+1}-\hat{E}\left[\mathbf{y}_{t+1}\middle|\mathbf{y}_t,\mathbf{y}_{t-1},\cdots,\mathbf{y}_0,\hat{\mathbf{x}}_0\right]$, $K$ is the Kalman gain, and $\hat{E}$ is the linear least squares projection operator. The innovations to $\mathbf{y}_t$ (i.e., $a_t$) can be recursively computed by (3.1.4), if we assume $\hat{\mathbf{x}}_0=\mathbf{x}_0$ where $\mathbf{x}_0$ is the initial value of $\mathbf{x}_t$. That is, $a_0=\mathbf{y}_1-GA^0\hat{\mathbf{x}}_0$, $\hat{\mathbf{x}}_1=A^0\hat{\mathbf{x}}_0+Ka_0$, $a_1=\mathbf{y}_2-GA^0\hat{\mathbf{x}}_1$ and so on. Then, the log likelihood function for $\{\mathbf{y}_t\}$ is

(3.1.5)
$$L(\theta)=-\frac{1}{2}\sum_{t=0}^{T}\left\{\log\left|\Omega_t\right|+trace(\Omega_t^{-1}a_ta_t')\right\}$$

where $\Omega_t$ is the covariance matrix of $a_t$. AHMS shows that $\Omega_t$ equals $GA^0\Sigma(GA^0)'+\mathbf{R}$ where $\Sigma$ is the state covariance matrix associated with the Kalman filter. The Kalman gain and the state covariance matrix can be computed by a MATLAB function ***kfilter*** coded by Hansen and Sargent (1996).

AHMS illustrate the analytic derivatives of the log-likelihood function and the formula for the standard errors, respectively, in the Appendix and section 11 of their paper.

### 3.2 The estimations and results

In section 2, I assumed that boundedly rational hog producers believed that the hog price had an AR(1) process. Therefore, the first stage of the estimation requires that we determine the coefficients of the AR(1) process governing the expectation scheme of them. The coefficients in the AR(1) process are estimated by OLS using hog price data. The estimates (with standard errors in parenthesis) of the coefficients are shown in the following regression equation:

$$\text{(3.2.1)} \qquad p_{t+1} = \underset{(3.243)}{12.062} + \underset{(0.117)}{0.549}\, p_t, \quad R^2 = 0.351$$

Because the expectations of the boundedly rational agents are formulated from a time series model of the market price, it may be interpreted as a kind of quasi-rational expectations suggested by Nerlove, Grether and Carvalho (1979).

Some parameter values are set a priori to reduce the number of parameters to be estimated. The discount factor ($b$) is assmed to be 0.96 following some agricultural literature.[16] The death rate ($d$) is assumed to be 0.08. It is known that the natural death rate in the hog industry has been around that number. Other assumptions are made arbitrarily: $y_0 = 0.0001$, $y_c = 0.0001$, $h^* = 5$, $m^* = 5$. It should be noted that he empirical test results are not sensitive to these arbitrary initial assumptions within reasonable boundaries: $10^{-2} \le y_0 \le 10^{-5}$, $10^{-2} \le y_c \le 10^{-5}$, $1 \le h^* \le 15$, $1 \le m^* \le 15$.

I also assume that there is error in measuring $x_t$ and $c_t$. In particular, the (1,1) element and (3,3) element of $\mathbf{R}$ (the covariance matrix of the measurement errors) are assumed to be equal to $s_x^2$ and $s_c^2$. All other elements of $\mathbf{R}$ are set to zero.

The remaining parameters are estimated based on the rational expectations and the bounded rationality model. In Table 2, the estimates and the standard errors of the parameters are reported.

[16] See Baak (1999) and AHMS.

The fraction of boundedly rational ranchers, $n$, is significantly estimated to be 0.281. The other parameter values are also significantly estimated by both models and turn out to be within reasonable boundaries.

The specification test of the two models using log-likelihood ratio supports the bounded rationality model over the rational expectations model. The null and alternative hypotheses of this test are:

$$H_0: \quad n = 0$$
$$H_A: \quad n \geq 0$$

In general, the specification test of two competing models (restricted and unrestricted) can be performed by computing the statistic, $2(L(\hat{\theta}) - L(\tilde{\theta}))$, since the statistic has a $\chi^2(r)$ distribution.[17] However, in the case presented in this paper, the statistic does not have a $\chi^2$ distribution because the alternative hypothesis has an inequality constraint. That is, the parameter space is restricted under the alternative hypothesis. Gourieroux, Holly and Monfort (1982) and Andrews (1996) show that the statistic of this case is distributed as a mixture of Chi-squared distributions. Specifically, under the null hypothesis, the distribution of the statistic is $\frac{1}{2}\chi^2(0) + \frac{1}{2}\chi^2(1)$. The critical value at the 5% significance level is 2.71 and the likelihood ratio exceeds the critical value with a large margin.

One-step ahead forecasting experiments are performed by the two models. Table 3 shows the average of squared prediction errors for each of the four observables. The bounded rationality model produces lower forecast errors associated with three out of the four observables.

## 4. Measuring Market Volatility and Social Welfare

In this paper, I presented a bounded rationality model of the U.S. hog industry and formed, based on the model, an estimation procedure which is capable of obtaining a point estimate of the fraction of economic agents in the industry.

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[17] $L(\hat{\theta})$ is the log-likelihood value of the unrestricted model (the bounded rationality model) and $L(\tilde{\theta})$ is the log-likelihood value of the restricted model (the rational expectations model). The degree of freedom, $r$, is the number of restrictions. In this paper, the restriction is $n = 0$ and thus the number of restrictions is 1.

Then, the fraction of boundedly rational economic agents was estimated based on the solution of the model using MLE. The empirical test results indicate that about 30% of the hog producers are boundedly rational. The point estimate of the fraction of boundedly rational agents is positive and significant and the log-likelihood ratio test rejects the null hypothesis of full rationality in favor of the bounded rationality model.

Based on the estimation results and the model, simulation experiments can be performed to investigate how the presence of boundedly rational economic agents has affected the volatility of the economic variables and the social welfare of the market. In particular, several sets of artificial data can be generated by the model using the estimated parameter values, while changing the fraction of the boundedly rational agents from zero to one. Each set of artificial data is related to a certain fraction of boundedly rational agents. Then, the variances of the quantity and price variables can be computed using the actual and artificial data and then compared. According to preliminary simulation experiments, the higher the fraction of boundedly rational agents is, the more volatile the economic variables are.

The simulations, however, are not contained in this version of the paper, since it requires some verification. The social welfare will also be measured and compared in the way illustrated above, and then will be reported in the revised version of the paper.

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[Table 1] Sample Means and Standard Deviations

| Variable | Mean | Std. Dev. |
|---|---|---|
| $x_t$ | 89.825 | 6.513 |
| $k_t$ | 9.714 | 2.102 |
| $c_t$ | 82.183 | 6.421 |
| $p_t$ | 20.936 | 5.916 |

[Table 2] MLE Test Results of the Two Models

| Parameters Estimated | Theoretical Benchmarks | Rational Expectations Model[a] ( let $n = 0$ ) | | Bounded Rationality Model $a_0 = 12.435,\ a_1 = 0.881$ | |
|---|---|---|---|---|---|
| | | Estimates | Std. Error | Estimates | Std. Error |
| $n$ | $0 \le n \le 1$ | n.a. | n.a. | 0.281 | 0.038 |
| $a_0$ | + | 105.47 | 0.107 | 101.84 | 0.116 |
| $a_1$ | + | 0.671 | 0.004 | 0.588 | 0.004 |
| $g$ | + | 6.802 | 0.004 | 6.684 | 0.004 |
| $r_h$ | $0 < r_h < 1$ | 0.732 | 0.011 | 0.970 | 0.013 |
| $r_m$ | $0 < r_m < 1$ | 0.632 | 0.021 | 0.55 | 0.222 |
| $s_h$ | + | 1.115 | 0.067 | 1.346 | 0.859 |
| $s_m$ | + | 1.0199 | 0.463 | 1.046 | 0.046 |
| $s_x$ | + | 2.2103 | 0.024 | 1.920 | 0.017 |
| $s_c$ | + | 62.546 | 0.000 | 92.654 | 0.000 |
| Log Likelihood | | -2174.02 | | -1749.41 | |

a. The test results of this column are produced by the computer programs for the rational expectations model. The computer programs for the bounded rationality model re-produce these results when we set $n = 0$.

[Table 3] Average of Squared Prediction Errors

| | $x_t$ (pig crop) | $k_t$ (breeding) | $c_t$ (slaughter) | $p_t$ (price) |
|---|---|---|---|---|
| R.E[a)] | 2.67 | $3.56\text{x}10^{-2}$ | $1.07\text{x}10^{2}$ | $3.11\text{x}10^{-1}$ |
| B.R[b)] | 3.45 | $8.51\text{x}10^{-3}$ | 8.21x10 | $1.79\text{x}10^{-4}$ |

a) R.E. = rational expectations model
b) B.R. = bounded rationality model