Applying Disequilibrium Growth Theory: Debt Effects and Debt Deflation*

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Abstract

In this paper we consider two polar dynamical models in which firms use debt (loans) to finance their investment expenditure, a three-dimensional supply driven one and a sophisticated 20D Keynesian growth model. In the first type of model debt accumulation of firms interacts with income distribution and resulting capital stock and employment growth patterns. In the second high dimensional model we have in particular sluggishly adjusting prices and quantities, Keynesian demand rationing and fluctuating capacity utilization for both labor and capital with all budget equations specified and a balanced growth reference path. These two polar growth perspectives are brought together in an intermediate 4D dynamics where debt accumulation of the simple model is combined with the possibility for deflationary processes of the general model. This intermediate case allows us to formulate and investigate, both analytically and numerically, situations of debt deflation in a demand constrained setup which augments the insights obtained from simple model and illustrates an important destabilizing feedback chain of the general 20D dynamics.

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JEL Classification: E12, E32.

*This is a shortened version of the working paper Chiarella and Flaschel (1999x) which contains more expalanations of the economic background, mathematical details and further numerical simulations.

In the recent public debate on problems of the world economy, 'deflation' or more specific 'debt deflation' is surely one of the expressions that has been consistently recurring.[1] Not only has its possible role in triggering the Great Depression of the 1930s come back into mind, but it has also been observed that there are similarities between recent global trends and the conditions of that episode, basically based on the joint occurrence of high levels of debt and falling prices ('the dangerous downside to cheaper cars and TVs'). Debt deflation thus concerns the interaction of high nominal debt of firms with falling profitability due to falling output prices (increasing real debt). Countries that have been frequently discussed in this respect are Japan, Russia and recently Brazil. The behavior of firms relying on rounds of downsizing and cost-cutting from the perspective of short-run profitability solely (short-term maximizers), thereby demolishing their productivity over the medium run, has been noted as a dangerous strategy caused by their dependence on financial markets.

Some commentators have also criticized the single-minded preoccupation of certain central banks and the IMF with inflation, and the word reflation has been coined in order to stress that providing some room for inflation should be of help in preventing global economic crisis. The viewpoint of the FED and of the government in the USA has of course received particular attention in this respect and the chairman of the FED, Alan Greenspan, was quoted with passages such as:[2]

> Deflationary forces that emerged a year ago were expanding "and there's no evidence of which I am aware which suggests that the process ... has stabilized."

Global growth strategies and the elements they should contain continue to be discussed (but not implemented in the form of coordinated interest rate cuts for example). The need for a fundamental restructuring of the IMF and of financial markets is continually stressed based of the judgment that the world is currently facing its biggest financial challenge since the 1930's. Debt deflation and its destabilizing potential therefore appears to be an important problem the world economy is currently facing.

Modern macroeconomic theory, as it has evolved since the Second World War, has paid scant attention to debt deflation. No doubt this is due to the fact that during that time the major economies in the world experienced a long period of growth followed by a long period of inflation from which we have just emerged. The classic study of debt deflation remains Fisher (1933), though Minsky (1982) in his writings on the financial instability hypothesis continued to warn of the dangers of another great depression. There is therefore an urgent need for economists to model the process of debt deflation in their interaction with monetary and fiscal policies that may stop the process of rising debt, falling prices and collapse into depression.

In this paper we embed the process of debt accumulation and debt deflation in a fully integrated and consistent (with respect to budget constraints) disequilibrium growth model of an open economy. The model contains a sufficient number of agents and markets to capture the essential dynamic features of modern macroeconomies, and stresses the dynamic interaction between the main feedback loops of capital accumulation, debt accumulation, price / wage inflation / deflation, exchange rate appreciation / depreciation, inventory accumulation and government monetary and fiscal policies. Our modeling framework relies on the disequilibrium growth model developed in Chiarella, Flaschel and Zhu (1998a) which is the fundamental special case of the general disequilibrium growth model of Chiarella and Flaschel (1998a,b,c) and Chiarella, Flaschel, Groh, Köper and Semmler (1998a,b), the essential difference being that here we focus on debt-financed investment of firms in the place of the pure equity financing considered in those earlier papers. We will thus add a further important feedback loop missing in our earlier approach to disequilibrium growth, namely the

[1]The following observations summarize some discussions in 'The Sydney Morning Herald' in September 1998 and The International Herald Tribune, January 30-31, 1999.

[2]The Sydney Morning Herald, September 21, 1998.

phases of capital accumulation arising from the creditor – debtor relationship between asset owning households and firms.

Keen (1999) has recently investigated the Fisher debt effect (between firms and financial intermediaries) in the context of an augmented Goodwin growth cycle model and has found that it may imply local asymptotic stability for the overshooting mechanism of this growth cycle, but can lead to instability (for high debt) outside a corridor around the steady state of the model. In addition he provides an interesting discussion of Fisher's vision of the interaction of over-indebtedness and deflation and also of Minsky's financial instability hypothesis. He extends the proposed model of the interaction of indebted firms and income distribution to also include the role of government behavior in such an environment and of nominal adjustment processes in the place of the real ones of the Goodwin model. Details of his approach to debt deflation will be discussed in the following sections of the paper where we will compare his modeling of this process with its embedding into our general disequilibrium model.

In order to avoid the complication of modeling how firms split their financial needs into equity supply and loan demand we assume that firms finance their investment decisions (fixed business investment as well as involuntary inventory investment) exclusively via loans. Introducing debt financing and removing equity financing from the general approach of Chiarella and Flaschel (1998a,b,c) has the further implication that there are now fluctuations in the income of firms that go beyond the windfall losses or profits caused by disappointed or over-satisfied sales expectations. There are now also pure profits (or losses) to be considered as they will result from systematic deviations of actual (or expected) sales from the factor costs of firms now including interest payments besides wage and import costs. The budget equations and financing behavior of firms therefore have to be reformulated in an appropriate way in order to take account of this deviation between total factor costs and the total proceeds of firms and the retained earnings based on them.

In the next section we briefly present the changes to the model of Chiarella and Flaschel (1998a,b,c) that are needed for a discussion of debt deflation from the perspective of national accounting. Section 3 then provides the new equations of the debt deflation model on the extensive form level and discusses these changes in comparison to the extensive form model considered in Chiarella and Flaschel (1998a) and also their relationships to the work of Keen (1999). Section 4 then gives a short description of the interior steady state of the model, its 20 laws of motion for its intensive form state variables (based on the core dynamics considered in Chiarella, Flaschel and Zhu (1998a)), including various algebraic equations that supplement these dynamical laws. Section 5, the core section of this paper, first reassesses the basic 3D model of Keen (1999) which only allows for debt accumulation, but not yet for deflationary processes, by presenting some propositions on this starting situation. We then consider a 4D subcase of our 20D model which extends the 3D model by including nominal price dynamics. We derive certain propositions on this special subcase. A next subsection considers the 4D and the general 20D dynamics numerically and probes the robustness of the propositions obtained from the 4D subcase. Our principal conclusion is that the model is indeed prone to the accelerating downward instability caused by the over-indebtedness of firms and declining prices for their output, if not stopped by floors to deflation of by appropriate actions of the government. Section 6 provides some conclusions. The Appendix to the paper provide details on the notation.

2 Reformulating the structure of the economy

The following tables and accounting identities provide a brief survey of the changes we make in this paper with respect to the structure of the 18D core disequilibrium growth dynamics investigated in Chiarella and Flaschel (1998a,b,c), Chiarella, Flaschel and Zhu (1998a). These changes basically concern the financing conditions of firms. We thus do not particular of the real part of the economy, but simply refer the reader to those earlier papers for the full details.[3]

We first reformulate the financial part of the economy.[4] Note that we here switch from pure equity financing to pure loan financing as far as the external fund raising of firms is concerned and that therefore the expected returns of firms are no longer distributed to households (but retained) in this revision of the 18D core model of Chiarella and Flaschel (1998a,b,c), Chiarella, Flaschel and Zhu (1998a) in order to allow us to concentrate on the effects of debt financing of firms'.

	Short-term Bonds of the Government	Long-term Bonds of the Government	**Loans to Firms**	Foreign Bonds of the Foreign Government
Workers	$\dot{B}_w$	–	–	–
Asset holders	$\dot{B}_c$	$\dot{B}^l_1$	$\mathbf{\dot{D}_f = \dot{D}^b_f - \delta_d D_f}$	$\dot{B}^l_2$
Firms	–	–	$\mathbf{\dot{D}_f = \dot{D}^b_f - \delta_d D_f}$	–
Government	$\dot{B}$	$\dot{B}^l$	–	–
Prices	$1\ [r]$	$p_b = 1/r_l$	$\mathbf{1\ [r_d]}$	$ep^*_b = e \cdot 1/r^*_l$
Expectations	–	$\pi_b = \hat{p}^e_b$	–	$\epsilon = \hat{e}^e$
Stocks	$B = B_w + B_c$	$B^l = B^l_1 + B^{l*}_1$	$\mathbf{D_f}$	B^l_2
Growth	$\hat{B}, \hat{B}_w, \hat{B}_c$	$\hat{B}^l, \hat{B}^l_1$	$\mathbf{\hat{D}_f = \dot{D}^b_f / D_f - \delta_d}$	$\hat{B}^l_2$

Table 1: *The financial part of the economy (Foreign country data: r^*_l).*

Table 1 shows that firms now use loans in the place of equities as instrument to finance (part of) their investment expenditures. These loans are supplied by pure asset holders in the gross amount $\dot{D}^b_f$ following the loan demand of firms. Loans are just an amount of money lent to firms (with a price of unity) and exhibit a variable rate of interest r_d applied to all loans (old and new, $D_f, \dot{D}^b_f$) in a uniform manner so that there is no term structure of interest rates on loans. Furthermore we assume that a certain fraction δ_d of the stock of existing loans D_f is repaid at each moment of time, and that only net amounts of new debt $\dot{D}_f = \dot{D}^b_f - \delta_d D_f$ need to be considered as far as budget equations and asset accumulation are concerned. Note that, following Chiarella and Flaschel (1998a), money is not treated as an asset, due to specific assumptions made there (where 'money' is treated as a pure medium of account).

Here, we consider briefly the production accounts, income accounts, accumulation accounts and financial accounts of two of the four agents of our economy, firms and asset holders, whose relationship to each other is changed by the introduction of loans from asset holders to firms (in the place of the equities and the dividend payments assumed in the original version of the model). These accounts provide basic information on changes compared to Chiarella and Flaschel (1998a).

[3]Note that we will ignore value added taxes on consumption goods and thus set the parameter $\tau_v = 0$ in the original 18D dynamics.

[4]Note that all additions made with respect to the general framework presented in Chiarella and Flaschel (1998a) are marked by bold letters.

Y, employment L_f^d of their workforce L_f^w and gross business fixed investment I and which use loans D_f and expected retained earnings (plus windfall profits) as financing instruments for their desired net investment. There are import taxes τ_m on imported commodities and payroll taxes τ_p (with respect to hours worked L_f^d in the sector of firms). There are no subsidies and no value added taxes on the consumption goods produced by firms. Note that all accounts are expressed in terms of the domestic currency.

Firms use up all imports J^d as intermediate goods which thereby become part of the unique homogeneous good Y that is produced for domestic purposes. They have replacement costs with respect to their capital stock, pay import taxes and wages including payroll taxes, and, as a new item, have to pay interest $r_d D_f$ on their stock of loans D_f. Their accounting profit is therefore equal to actual pure profits $\rho^a p_y K$ (based on actual sales) and notional income gone into actual inventory changes (besides the depreciation fund for capital stock replacement). Note that firms have sales expectations that follow actual sales in an adaptive fashion. They therefore experience (unexpected) windfall profits (or losses) for the financing of their fixed investment when their actual inventory changes are smaller than (larger than) their desired ones. Firms save all the income they receive and spend it on net fixed investment and on inventories of finished goods. The accumulation account is therefore self-explanatory as is the financial account which only repeats our earlier statements that the financial deficit of firms is financed by new loans from pure asset holders. Note that the amount $\delta_d D_f$ of existing loans must be repaid to asset holders (and replaced by new loans by assumption on credit market contracts) at each moment of time. Thus the sum of all new loans $\dot{D}_f^b$ must be diminished by this magnitude in order to arrive at the rate of change of the stock of loans $\dot{D}_f$. There are no direct (capital) taxes in the sector of firms, neither on property nor on profits. Note finally that the accumulation account of firms is based on realized magnitudes and thus refers to their intended inventories only implicitly.

Turning next to the sector of asset-holders, as in Chiarella and Flaschel (1998a), investment in housing as well as the supply of housing services is exclusively allocated to this sector. The production account of asset holders therefore shows the actual sale (not the potential sale) of housing services (set equal to the demand for housing services by assumption) which is divided into replacement costs and actual earnings or profits on the uses side of the production account.

Income of asset holders comes from various sources: interest payments on short- and long-term domestic bonds and on long-term foreign bonds (net of tax payments paid abroad), interest income on loans to firms and profits from housing rents. All domestic profit income is subject to tax payments at the rate τ_c and after tax income is by definition divided into the consumption of domestic commodities (including houses, but not housing services) and the nominal savings of asset owners.

Production Account of Firms:

——————— Uses ———————-	——————— Resources ———————
Depreciation $p_y \delta K$	Consumption $p_y C_w + p_y C_c + p_y G$
Imports including import taxes $p_m J^d$	Durables (Dwellings) $p_y I_h$
Wages (including payroll taxes) $w^b L_f^d$	Exports $p_x X$
Interest on loans $\mathbf{r_d D_f}$	Gross Fixed Business Investment $p_y I$
Profits $\Pi = \rho^a p_y K + p_y \dot{N}$	Actual Inventory Investment $p_y \dot{N}$

Income Account of Firms:

——————— Uses ———————-	——————— Resources ———————
Savings S_f^n	Profits Π

Accumulation Account of Firms:

——————— Uses ———————-	——————— Resources ———————
Gross Fixed Investment $p_y I$	Depreciation $p_y \delta K$
Inventory Investment $p_y \dot{N}$	Savings S_f^n
–	Financial Deficit FD

Financial Account of Firms:

——————— Uses ———————-	——————— Resources ———————
Financial Deficit FD	**Loans** $\mathbf{\dot{D}_f = \dot{D}_f^b - \delta_d D_f}$

The accumulation account shows the sources for gross investment of asset-holders in the housing sector, namely depreciation and savings, the excess of which (over housing investment) is then invested into financial assets. Note here that short-term bonds are fixed price bonds $p_b = 1$ (which are perfectly liquid), while loans as well as long-term bonds are not perfectly liquid, the latter since they have the variable price $p_b = 1/r_l$ so they are like consols or perpetuities (ditto for imported foreign bonds).[5]

Production Account of Households (Asset Owners):

——————— Uses ———————-	——————— Resources ———————
Depreciation $p_y \delta_h K_h$	Rent $p_h C_h^d$
Rent Earnings Π_h	

Income Account of Households (Asset Owners):

——————— Uses ———————-	——————— Resources ———————
Tax payment $\tau_c r B_c + \tau_c B_1^l$	Interest payment $r B_c + B_1^l$
Taxes $\tau_c(p_h C_h^d - p_y \delta_h K_h)$	Interest payment $e(1-\tau_c^*)B_2^l$
Tax payment $\boldsymbol{\tau_c} \mathbf{r_d D_f}$	**Interest on loans** $\mathbf{r_d D_f}$
Consumption $p_y C_c$	Rent Earnings Π_h
Savings S_c^n	

[5]Due to the assumption of a given nominal rate of interest on foreign bonds, these bonds can be liquidated if this is desired by domestic residents, but they are of course subject to exchange rate risks. Note that foreign bonds purchases by domestic residents are treated as residual in the wealth accumulation decisions of the asset holders of the model of this paper.

——————————— Uses ————————————-	——————————— Resources ———————————
Gross Investment $p_y I_h$	Depreciation $p_y \delta_h K_h$
Financial Surplus FS	Savings S_c^n

Financial Account of Households (Asset Owners):

——————————— Uses ————————————-	——————————— Resources ———————————
Short-term bonds $\dot{B}_c$	Financial Surplus FS
Long-term bonds $p_b \dot{B}_1^l$	
Foreign Bonds $e p_b^* \dot{B}_2^l$	
Loans $\mathbf{\dot{D}_f = \dot{D}_f^b - \delta_d D_f}$	

There is no taxation of financial wealth (held or transferred) in the household sector and also no real property tax. Furthermore, though asset holders will consider expected net rates of return on financial markets in their investment decision, there is no taxation of capital gains on these markets which descriptively seems realistic. We do not present the accounts of the worker households here as there is no change in their treatment in Chiarella and Flaschel (1998a). We also do not present the foreign account, the balance of payments, here , as there is also no change in this single account, representing trade in goods, in capital and interest payments.

There are finally the accounts of the fiscal and the monetary authority which are slightly altered through the above additions to the accounts of firms and asset holders. the only change in this regard is that the term $\tau_c r_d D_f$ has now to be added to the resources side of the income account of the government (and dividend payments to be removed).

Having presented the model from the ex post point of view concerning its new elements as compared to Chiarella and Flaschel (1998a,b,c), Chiarella, Flaschel and Zhu (1998a) we now turn to the structural form of the model. We present in the following section its technological and behavioral relationships, various definitions and the budget equations of the agents of the domestic economy, and finally also the laws of motion for quantities, prices and expectations that change due to the modifications of the model described above.

3 The augmented 18+2D system: Investment, debt and price level dynamics

We start with the structural equations of the advanced model of disequilibrium growth of Chiarella and Flaschel (1998a) which we reconsider here only with respect to the changes needed for a treatment of debt deflation.[6] We will compare these changes with the building blocks of the Keen (1999) model.

Module 1. of the model provides definitions of important rates of return ρ^e, ρ^a, an expected one based on sales expectations Y^e of firms and the actual one based on the actual sales Y^d of firms. Note that actual production Y exceeds expected sales by planned inventory

[6]Note that the $\beta's$ always denote speeds of adjustment in the following.

the currently expected and the actual rate of profit (net of depreciation $p_y\delta K$ and of interest payments r_dD_f) are both based actual exports $X = x_yY$, actual imports $J^d = j_yY$, and the actual employment $L^d = l_yY$ of the workforce of the firms. We are assuming a technology with fixed input / output coefficients ($Y^p = y^pK$ the potential output of firms) where export supply is in fixed proportion[7] to actual output Y, as is import demand and labor demand (the latter coefficient however subject to Harrod neutral technological change: $\hat{l}_y = n_l$).[8]

1. Definitions (Rates of Return and Real Growth):

$$\rho^e = \frac{p_yY^e + p_xx_yY - \mathbf{r_d D_f} - w^bl_yY - p_mj_yY - p_y\delta K}{p_yK} \tag{1}$$

$$\rho^a = \frac{p_yY^d + p_xx_yY - \mathbf{r_d D_f} - w^bl_yY - p_mj_yY - p_y\delta K}{p_yK} \tag{2}$$

$$\gamma = n + n_l \quad \text{all given magnitudes} \tag{3}$$

The two rates of profits used in Chiarella and Flaschel (1998a) are now defined by subtracting the interest payments of firms to asset holders based on the amount of loans D_f they have accumulated over the past. Furthermore trend growth in the world economy is given by the rate γ which is identified with the natural rate of growth n of the domestic working population and the given rate of Harrod neutral technological progress n_l.

Keen (1999, p. XXX) considers a prototype 3D model of classical growth (in a closed economy) where besides the direct investment of capitalists (who own the firms and who reinvest all of their profit income, based on the profits of firms) there is also only pure loan financing of the remaining investment expenditure of the firms. These loans are supplied by financial asset holders (called banks) which are to be treated explicitly if his approach is to be compared with the one we present. Keen has no demand constraints on the market for goods which implies that his (uniquely determined) measure of the profitability $\rho = \rho^e = \rho^a$ of the firms' activities is based on actual = potential output $Y^p = y^pK$ throughout. This gives as actual (= expected) rate of profit, used to describe the investment behavior of firms in his paper, a rate of the type:

$$\rho = \frac{p_yY^p - r_dD_f - wl_yY - p_y\delta K}{p_yK}$$

These profits accrue directly to the real capital owning households who do not consume, but only invest (and are therefore named firms by Keen (1999).

Module 2. provides the equations that concern the household sector where two types of households are distinguished, workers and pure asset or wealth owners. There is no change in the behavioral equations of worker households, as compared to Chiarella and Flaschel (1998a), which are therefore not repeated in the present paper. In Keen (1999) workers spend what they get: $p_yC_w = wL^d$ (as in the original Goodwin (1967) growth cycle model) and wage income wL^d is not taxed since there is no government sector in the basic form of his model.[9]

The other type of household of our model, the asset owners, desire to consume C_c (goods and houses as supplied by firms through their domestic production Y) at an amount that is growing exogenously at the rate γ which is therefore independent of their current nominal disposable income Y_c^{Dn}. The consumption decision is thus not treated here as an important

[7]The case of neoclassical smooth input and output substitution is considered in detail in Powell and Murphy (1997) and Chiarella, Flaschel, Groh, Köper and Semmler (1998a,b).

[8]For the details of all the notation as well as further explanations of the equations used in this model the reader is referred to Chiarella and Flaschel (1998a) and an appendix of this paper. Note again that the changes made to the model of Chiarella and Flaschel (1998a) are represented by bold letters in the following.

[9]We denote by V the rate of employment L^d/L and assume for labor supply natural growth at a given rate $n = \hat{L}$.

diminished by the nominal value of their consumption $p_y C_c$ is then spent on the purchase of financial assets as well as on investment in housing services supply (for worker households). Note here that our one good representation of the production of domestic commodities entails consumption goods proper and houses so that asset holders buy houses for their consumption as well as for investment purposes.

2. Households (Asset-Holders):

$$Y_c^{Dn} = (1-\tau_c)[\mathbf{r_d D_f} + rB_c + B_1^l + p_h C_h^d - p_y \delta_h K_h)] + e(1-\tau_c^*)B_2^l \tag{4}$$

$$\hat{C}_c = \gamma \tag{5}$$

$$S_c^n = Y_c^{Dn} - p_v C_c = \dot{B}_c + \frac{\dot{B}_1^l}{r_l} + \frac{e\dot{B}_2^l}{r^{l*}} + \mathbf{\dot{D}_f} + p_y(I_h - \delta_h K_h), \quad \dot{B}_c = \dot{B} - \dot{B}_w \tag{6}$$

Changes in this sector of the economy are again quite small, since we only have to add interest income (from loans) to the income of asset holders (and to remove dividend payments as there are now no equities) and to describe later on how much of their savings goes into new loans $\dot{D}_f$ to firms. Note that the term $e\dot{B}_2^l/r^{l*}$ adjust residually to the other changes in the wealth composition of asset holders.

Keen (1999) does not consider explicitly the agents that supply credit to firms in the forms of loans (his 'banks'). Yet there must be a budget equation for these agents, since their interest income will generally differ from their supply of new loans. Thus something like the following must be assumed in his approach:

$$r_d D_f = \dot{D}_f + p_y C_c,$$

since he has no demand constraint for the supply of output, which can only be true if the budget constraints for the three types of agents imply that the demand for goods is always equal to their supply: $Y^p = C_w + C_c + I$. The budget equation just shown together with the one that has been assumed above for workers (workers spend what they get), and below for firms – as in Keen (1999) – indeed just guarantee this outcome. Note that the consumption of these 'credit institutions' may become negative in the Keen (1999) model, if they lend more than they get as interest rate income, in which case they must be considered as supplying commodities to the goods market (from their stock of goods). Note finally that debt accumulation in Keen (1999) as well as in the present model does not consider debt repayments explicitly but rather considers only their net effect.

In the module 3. we describe the sector of firms, whose planned gross investment demand I is assumed to be always fulfilled, just as all consumption. We thus assume for the short-run of the model, see Chiarella and Flaschel (1998a) for the details, that it is always of a Keynesian nature, i.e., aggregate goods demand is never rationed, due to the existence of excess capacities, inventories, overtime work and other buffers.[10] There is thus only one regime possible, the Keynesian one, for the short-run of the model, while supply side forces surface only in the medium and the long run (Keen's (1999) model by contrast is completely supply side based). Note that we only display the investment relationships of the model, since there is no change in the description of technology, the employment policy of the firms and the like from Chiarella and Flaschel (1998a).

3. Firms (Investment Behavior):

$$g_k = \alpha_1^k(\rho^e - r_d) + \alpha_2^k(r_l - r) + \alpha_3^k(U - \bar{U}) + \gamma + \delta \tag{7}$$

$$\dot{D}_f = p_y(I - \delta K) + p_y(\dot{N} - \mathcal{I}) - \rho^e p_y K \tag{8}$$

$$\hat{K} = I/K - \delta = g_k - \delta \tag{9}$$

The rate of capacity utilization $U = Y/Y^p$ is defined on the basis of the concept of potential output (discussed earlier) and will receive importance when describing the investment

[10] Certainly these exist in real market economies.

supplies labor effort of amount L_f^d as determined by the present state of sales expectations Y^e (plus voluntary inventory production). This labor force of firms is adjusted in a direction that reduces the excess or deficit in the utilization of the employed labor force, $L_f^d - L_f^w$, which means that firms intend to return to the normal usage of their labor force thereby. An additional growth term for the employed labor force takes account of the trend growth γ of domestic output, but is diminished by the effect of Harrod neutral technical change, n_l, which working in isolation would reduce the workforce of the firms.

Explicitly presented above is the formulation of the desired gross rate of capital stock accumulation of firms, $g_k = I/K$, which depends on relative profitability, measured by the deviation of the expected rate of profit, ρ^e, from the interest rate, r_d, firms have to pay on their debt, on the interest rate spread $r_l - r$, between long and short-term government debt, representing the tightness of monetary policy and its believed effects on economic activity, on the rate of capacity utilization U of the capital stock of firms in its deviation from the desired rate of capacity utilization, $\bar{U}$, which is given exogenously, and on trend growth γ and the rate of depreciation δ of business fixed investment. When comparing the rates ρ^e, r_d in their investment decision firms decide to increase their investment projects via additional debt if $\rho^e - r_d > 0$ holds (and vice versa). They do not pay attention here to (expected) inflation and the implied real rate of interest on their loans when making this decision. This would make the considered dynamics much more involved and we expect that its inclusion would add further momentum to the debt effects investigated in later sections.

The budget equation (8) says that firms have to finance net investment and all inventory changes $\dot{N}$ (unintended inventory changes $\mathcal{I}$) by the profits that are based on actual output Y (expected sales Y^e, respectively) or by making new loans. Note here that unintended inventory disinvestment gives rise to windfall profits to firms which are retained and thus used to finance part of the fixed business investment if $\dot{N} - \mathcal{I} < 0$ holds true. The last equation of the above module of the model finally defines the growth rate of the capital stock which is determined by the net rate of capital accumulation planned by firms (due to the Keynesian nature of the short run of the model).

Keen (1999) assumes as budget equation of firms (owned by capitalists) the equation

$$\dot{D}_f = p_y(I - \delta K) - \rho p_y K,$$

where ρ is the actual rate of profit. Firms therefore finance net investment $I - \delta K$ by means of profits $\rho p_y K$ (which are always re-invested) and new debt, the latter being determined residually. There are no (unintended) inventory changes, but there is full capacity growth with goods demand always equal to goods supply. Furthermore, he assumes that gross investment is driven by the rate of profit (net of interest) $\rho = \rho^e$ solely which gives $I/K = \alpha^k(\rho - \rho_{min}) + \delta$ in the place of his equation $I/Y = \alpha^k(\rho)$, and hence $\hat{K} = \alpha^k(\rho - \rho_{min})$ in our notation (We neglect the nonlinearity in Keen's investment equation). These two equations constitute two of the three laws of motions of his basic model of debt accumulation and wage inflation or deflation.

The next equation describes the tax income of the government which compared to Chiarella and Flaschel (1998a) where taxes on the interest paid for loans are added (and taxes on dividends now ignored).

4. Government (Fiscal and Monetary Authority):

$$\begin{aligned} T^n &= \tau_w[wL^d + w^u(L - L^w) + w^r\alpha_l L_2] + \tau_p w L^d \\ &+ \tau_c[\mathbf{r_d D_f} + rB + B^l + p_h C_h^d - \delta_h K_h] + \tau_m e p_m^* J^d \end{aligned} \tag{10}$$

Keen (1999) considers the government sector but based on dynamic government expenditure and taxation rules that differ from the ones of our approach, see Chiarella and Flaschel (1998a) for details. This module of the model may be used as in Keen (1999) to consider the issue of automatic stabilizers.

and Flaschel (1999x). We however here present briefly the two Phillips curves that describe money wage and price level dynamics since these equations are of crucial importance when the problem of debt accumulation is considered in a deflationary or inflationary) environment.

5. Wage-Price Adjustment Equations, Expectations:

$$\hat{w}^b = \beta_{w_1}(V-\bar{V}) + \beta_{w_2}(V_f^w - \bar{V}_f^w) + \kappa_w(\hat{p}_y + n_l) + (1-\kappa_w)(\pi^l + n_l) \quad (11)$$

$$\hat{p}_y = \beta_p(U-\bar{U}) + \kappa_p(\hat{w}^b - n_l) + (1-\kappa_p)\pi^l \quad (12)$$

These two equations describe the dynamics of nominal (gross) wages as dependent on demand pressure terms, specifically the outside and the inside employment of workers, $V-\bar{V}, V_f^w-\bar{V}_f^w$, measured as deviation from NAIRU levels, and on cost pressure terms, here the short-term actual and the medium term expected rate of price inflation, $\hat{p}_y, \pi^l$, augmented by the rate of productivity growth, n_l. We shall set $\bar{V}_f^w$ equal to 1 which means that each employed worker provides one unit of labor if there is no over- or underemployment within firms. Similarly, price inflation depends on demand pressure in the market for goods, here solely measured by the rate of capacity utilization, $U-\bar{U}$, in its deviation from the NAIRU rate of capacity utilization,[11] and on wage cost pressure, diminished by productivity growth. These equations are explained in more detail in Chiarella and Flaschel (1998a).

Keen (1999) considers a money wage Phillips curve, $\hat{w} = \beta_w(V)$ based on demand pressure on the (outside) labor market solely, and assumes that the price level p_y is given ($\equiv 1$ implying of course $\pi^l = 0$). His third law of motion of the considered growing economy can thus be obtained from equation (11) by assuming $\beta_{w_2} = 0, \kappa_w = 1$, yielding for the dynamics of the share of wages u in national income:

$$\hat{u} = \beta_{w_1}(V-\bar{V}) - n_l,$$

where we again use a linear form.[12]

The module of asset price dynamics of Chiarella and Flaschel (1998a) needs to be augmented in the present context by just one equation describing the dynamics of the interest rate on loans (while all other adjustment processes in these markets remain as before):

6. Asset Prices, Expectations and Interest Rate Adjustments:

$$\dot{\mathbf{r}}_{\mathbf{d}} = \beta_{\mathbf{r_d}}(\mathbf{r_l} - \mathbf{r_d}) \quad (13)$$

We here simply assume that the rate of interest on loans follows the rate of interest on long-term government bonds with some time delay, measured by the speed of adjustment β_{r_d}, similar to the other delayed interest rate parity conditions used in our model. In Keen (1999) the rate of interest r_d on loans is a given magnitude, so that there is no need there to formulate a law of motion for this interest rate variable. Note that it may take considerable time until the steering of the short-term rate of interest r by the central bank (via the Taylor interest rate policy rule) can actually exercise a significant effect on the interest rate on loans governing the firms investment decision in the present model.

Summing up the dynamics of the core model of Keen (1999) builds on a money wage dynamics of type $\hat{w} = \beta_w(V-\bar{V}), V = L^d/L$ with only demand pressure influences (since the price level is still kept constant), on an investment-demand driven growth path $\alpha^k(\rho - \rho_{min})$ that is partly financed by loans (at a given rate of interest) and on the budget equation of firms $\dot{D}_f = p_y(I-\delta K) - \rho p_y K$ where ρ is given by $\frac{p_y Y^p - r_d D_f - w l_y Y^p - p_y \delta K}{p_y K}$. These three dynamic laws operate in a fixed proportions technological environment (exhibiting Harrod neutral

[11] The term NAIRU is used in an extended way in this paper and should be read as Non-Accelerating Inflation Rate of Utilization. Note here also that a second term in the price Phillips curve could be given by the deviation of desired inventory holdings from actual inventory holdings.

[12] In a final section, he briefly considers prices for consumption and capital goods separately, but does not yet represent their dynamics by way of formalized laws of motion.

demand constraint on the market for goods. We shall reconsider this fundamental approach to debt-financed economic growth in intensive form, and also its implications, in the next section.

We have added to Keen's model in particular an endogenous determination of the price level and of the rate of interest paid on loans, and also a Keynesian demand constraint. Furthermore, as shown in Chiarella and Flaschel (1998a), there are detailed descriptions of the behavior of the fiscal and the monetary authority in our extension of this model, more advanced types of structural relationships for consumption, investment and financial wealth accumulation (still without feedback effect on the real side of the economy due to the lack of wealth effects in consumption) and also a detailed treatment of asset markets and their dynamics with heterogeneous expectations formation on these markets as well as with respect to wage and price formation. Full details of those buildings blocks not represented and explained in the present paper are provided in Chiarella and Flaschel (1998a).

4 Intensive form representation of the 20D dynamics

In this section we present our modification of the 18D core model of Chiarella, Flaschel and Zhu (1998a) allowing for the consideration of debt financing undertaken by firms as discussed in the previous section. To simplify the model slightly we assume that $C_c \equiv 0$ holds and thus neglect the consumption of asset holders. We stress that the resulting dynamics on the state variable or intensive form level are then of dimension 20, due to the additional laws of motion formulated in the preceding section for the accumulation of debt by firms and for the interest rate on their loans. We start with a compact presentation of these 20 laws of motion (with the new state variables $d_f = D_f/(p_y K)$ and r_d), all in per unit of capital terms, and will present thereafter their unique interior steady state solution.

As first group we consider the quantity adjustment mechanisms on the market for goods, concerning sales expectations y^e and actual inventories ν, and for labor, concerning the employment policy of firms, l_f^{we}, the evolution of full employment labor intensity l^e (all employment is measured in efficiency units) and of the stock of housing:

$$\begin{aligned} \dot{y}^e &= \beta_{y^e}(y^d - y^e) + (\gamma - (g_k - \delta))y^e && (1)\\ \dot{\nu} &= y - y^d - (g_k - \delta)\nu && (2)\\ \dot{l}_f^{we} &= \beta_l(l_f^{de} - l_f^{we}) + [\gamma - (g_k - \delta)]l_f^{we} && (3)\\ \hat{l}^e &= \gamma - (g_k - \delta) && (4)\\ \hat{k}_h &= g_h - \delta_h - (g_k - \delta) && (5) \end{aligned}$$

The first of these five laws for quantity movements describes the adjustment of sales expectations y^e in view of the observed expectational error $y^d - y^e$ based on currently realized sales y^d, augmented by a term that takes account of the fact that this adjustment is occurring in a growing economy. Next, inventories ν change according to the gap between actual output y and actual sales y^d, again reformulated to take care of the growth of the capital stock. Employment of firms, l_f^{we}, is changed in order to reduce the discrepancy that currently exist between the actual employment l^{de} of the employed and their normal employment, here measured by l_f^{we}. The growth rate of the factor endowment ratio l^e is simply given by the difference between the natural rate of growth (including Harrod neutral technical change) and the growth rate of the capital stock $g_k - \delta$. Similarly, the growth rate of the housing stock is simply given by the difference of the corresponding accumulation rates.

Next we consider the nominal dynamics in the real sector of the economy which are described by four dynamical laws.[13] Note here that the laws of motion for wages, w^e, net of payroll taxes

[13] where κ denotes the expression $1/(1 - \kappa_w \kappa_p) \in (0, \infty)$.

show that reduced form Phillips curves (exhibiting only one rate of inflation) are generally not as simple as is often assumed in the literature.[14]

$$\hat{w}^e = \pi^l + \kappa[\beta_{w_1}(l^{we}/l^e - \bar{V}) + \beta_{w_2}(l_f^{de}/l_f^{we} - 1) + \kappa_w \beta_p(y/y^p - \bar{U})] \quad (6)$$
$$\hat{p}_y = \pi^l + \kappa[\kappa_p(\beta_{w_1}(l^{we}/l^e - \bar{V}) + \beta_{w_2}(l_f^{de}/l_f^{we} - 1)) + \beta_p(y/y^p - \bar{U})] \quad (7)$$
$$\dot{\pi}^l = \beta_{\pi^l}(\alpha_{\pi^l}(\hat{p}_y - \pi^l) + (1 - \alpha_{\pi^l})(0 - \pi^l)) \quad (8)$$
$$\hat{p}_h = \beta_h(\frac{c_h^w}{k_h} - \bar{U}_h) + \kappa_h \hat{p}_y + (1 - \kappa_h)\pi^l \quad (9)$$

As already noted we now use reduced form Phillips curves for wage inflation $\hat{w}^e$ and price inflation $\hat{p}_y$ which both depend on the demand pressures in the markets for labor (external and internal ones: $l^{we}/l^e - \bar{V}, l_f^{de}/l_f^{we} - 1$) as well as for goods, $y/y^p - \bar{U}$. The change of the rate of inflation expected over the medium run, π^l, is determined as a weighted average of adaptively formed expectations and regressive ones (which realize that the steady state rate of inflation is zero in the present model). Finally, the inflation rate for housing services depends on the demand pressure term $\frac{c_h^w}{k_h} - \bar{U}_h$ in the market for these services,[15] and on actual and perceived cost push expressions, here simply based on a weighted average concerning the inflation rate of domestic output.

Next follow the dynamical laws for long-term bond price dynamics and exchange rate dynamics (including expectations) which basically formulate a somewhat delayed adjustment towards interest rate parity conditions and are supplemented by heterogeneous expectations formation of partially adaptive and partially perfect type. Note that perfect foresight, concerning the proportion $1-\alpha_s$ of market participants, does not appear explicitly, see Chiarella and Flaschel (1998a,b,c) for details):

$$\hat{p}_b = \frac{\beta_{p_b}}{1 - \beta_{p_b}(1 - \alpha_s)}[(1 - \tau_c)r_l + \alpha_s \pi_{bs} - (1 - \tau_c)r], \quad r_l = 1/p_b \quad (10)$$
$$\dot{\pi}_{bs} = \beta_{\pi_{bs}}(\hat{p}_b - \pi_{bs}) \quad (11)$$
$$\hat{e} = \frac{\beta_e}{1 - \beta_e(1 - \alpha_s)}[(1 - \tau_c)r_l^* + \alpha_s \epsilon_s - ((1 - \tau_c)r_l + \pi_b)], \quad r_l = 1/p_b \quad (12)$$
$$\dot{\epsilon}_s = \beta_{\epsilon_s}(\hat{e} - \epsilon_s) \quad (13)$$

Note that the literature generally only considers the border case where $\alpha_s = 0$ is used in conjunction with infinite adjustment speeds on the two considered markets. This gives rise to two interest parity conditions coupled with myopic perfect foresight on bond price and exchange rate movements, a situation of knife edge instability, which is stabilized by means of the so-called jump variable technique.

The next set of dynamical laws concerns the evolution of short-term and long-term debt of the government (the issuing of which is here governed by fixed propositions $\alpha_b^g, 1 - \alpha_b^g$), its wage and import taxation policy and the interest rate policy of the central bank.

$$\dot{b} = \alpha_b^g[gy^e + rb + b^l - t^a - t^c + w^a] - (\hat{p}_y + g_k - \delta)b \quad (14)$$

[14]Such disentangled laws of motion for nominal prices and wages are obtained from their originally interdependent presentation, see the preceding section, by solving the two linear equations of this section with respect to the variables $\hat{w}^e - \pi_l, \hat{p}_y - \pi_l$ giving the expressions in equations (6) and (7), which both make use of our measures of demand pressure on the market for labor *and* for the goods market (and on expected medium-run inflation). It is intuitively obvious that the removal of wage or price inflation cost-push pressure, $\hat{w}^e, \hat{p}_y$ from price or wage dynamics must imply that both the goods and the labor market expressions will be present in the resulting disentangled Phillips curves which thus are in a significant way more general than the ones usually considered in the theoretical or applied literature on price Phillips curves (unless one assumes – as some kind of Okun's law – that all demand pressures variables used are positive multiples of each other).

[15]where $\frac{c_h^w}{k_h}$ represents the rate of capacity utilization demanded on this market and $\bar{U}_h$ its NAIRU level.

$$\hat{\tau}_w = \alpha_{\tau_{w1}}(\frac{d_g}{\bar{d}_g} - 1), \quad d_g = \frac{b + p_b b^l}{y^e} \tag{16}$$

$$\hat{\tau}_m = \alpha_{\tau_m}\frac{p_x^* x - (1+\tau_m)p_m^* j^d}{p_x^* x}, \quad x = x_y y, j^d = j_y y \tag{17}$$

$$\dot{r} = -\beta_{r_1}(r - r_l^*) + \beta_{r_2}\hat{p}_y + \beta_{r_3}(y/y^p - \bar{U}) \tag{18}$$

Since these laws of motion, apart from the interest rate policy rule, are not of central interest in the following analysis we here only briefly state that the first two are immediate consequences of the government budget constraint (based in particular on various sources of tax income), that wage taxation is adjusted in the direction of a target ratio of government debt, $\bar{d}_g$, and that import taxes are adjusted to ensure a balanced trade account in the steady state (which greatly simplifies the calculation of the steady state). The interest rate policy rule (18) is important since it could be of help to counteract accelerating debt deflation, by lowering nominal interest rates in situations of depressed output and price deflation.

There remain the two dynamical laws that are new to the model:

$$\dot{\mathbf{d}}_\mathbf{f} = \mathbf{g}_\mathbf{k} - \delta + \mathbf{y} - \mathbf{y}^\mathbf{d} - \beta_\mathbf{n}(\beta_{\mathbf{n}^\mathbf{d}}\mathbf{y}^\mathbf{e} - \nu) - \gamma\beta_{\mathbf{n}^\mathbf{d}}\mathbf{y}^\mathbf{e} - \rho^\mathbf{e} - (\hat{\mathbf{p}}_\mathbf{y} + \mathbf{g}_\mathbf{k} - \delta)\mathbf{d}_\mathbf{f} \tag{19}$$

$$\dot{\mathbf{r}}_\mathbf{d} = \beta_{\mathbf{r}_\mathbf{d}}(\mathbf{r}_\mathbf{l} - \mathbf{r}_\mathbf{d}) \tag{20}$$

Though the dynamical law for absolute debt accumulation considered in the preceding section is by and large a simple one, its representation on the intensive form level is somewhat complicated due to the fact that unintended inventory changes are involved (and expressed in intensive form) besides the rate of capital accumulation $g_k - \delta$ and due to the fact that debt is now calculated in per value unit of capital (divided by $p_y K$) which transformed to growth rates gives rise to $-(\hat{p}_y + g_k - \delta)d_f$. By contrast, there is no change needed in the law of motion for interest on loans since it only involves state variable of the model right from the start. We shall consider in the next section the evolution of the ratio d_f in situations of increasing generality, at first only coupled with laws of motion for nominal wage adjustment and the evolution of labor intensity in a supply side growth model. Thereafter we include a static simplification of the quantity adjustment processes on the goods market considered above (leading to a demand driven growth model) and add the price level dynamics (9) to not only allow for debt accumulation, but also for goods price deflation in situations of depressed rates of capacity utilization. Finally we will also study more general cases where quantity and price adjustment processes interact (yet still without much stress on fiscal policies, asset markets, all kinds of expectations, the housing sector and the openness of the economy.

These laws of motion make use of the following algebraic relationships, see Chiarella and Flaschel (1998a,b,c) for details:

$$\begin{aligned}
y &= y^e + \beta_n(\beta_{n^d}y^e - \nu) + \gamma\beta_{n^d}y^e \\
l_f^{de} &= l_y^e y, \quad l_y^e \quad \text{the labor coefficient in efficiency units} \\
l_g^{de} &= l_g^{we} = \alpha_g g y^e, \quad l^{de} = l_f^{de} + l_g^{de}, \quad l^{we} = l_f^{we} + l_g^{we} \\
y_{w1} &= w^e[l^{de} + \alpha^u(l^e - l^{we}) + \alpha^r\frac{L_2(0)}{L_1(0)}l^e]/p_y \\
c_g^w &= c_1(1-\tau_w)y_{w1}, \quad c_h^w = p_y c_2(1-\tau_w)y_{w1}/p_h \\
\rho^e &= y^e - \delta + (ep_x^*/p_y)x_y y - \mathbf{r}_\mathbf{d}\mathbf{d}_\mathbf{f} - ((1+\tau_p)w^e/p_y)l_f^{de} - ((1+\tau_m)ep_m^*/p_y)j_y y \\
g_k &= \alpha_1^k(\rho^e - r_d) + \alpha_2^k(r_l - r) + \alpha_3^k(y/y^p - \bar{U}) + \gamma + \delta, \quad r_l = 1/p_b \\
g_h &= \alpha_1^h((1-\tau_c)((p_h/p_y)c_h^w/k_h - \delta_h) - ((1-\tau_c)r_l - \pi^l)) + \alpha_2^h(r_l - r) \\
&+ \alpha_3^h(\frac{c_h^w}{k_h} - \bar{U}_h) + \gamma + \delta_h, \quad r_l = 1/p_b
\end{aligned}$$

$$
\begin{aligned}
\pi_b &= \alpha_s \pi_{bs} + (1-\alpha_s)\hat{p}_b \\
t^a &= \tau_w w^e [l^{de} + \alpha^u (l^e - l^{we}) + \alpha^r \frac{L_2(0)}{L_1(0)} l^e]/p_y + \tau_p w^e l^{de}/p_y + \tau_m e p_m^* j_y y/p_y \\
t^c &= \tau_c [\mathbf{r_d d_f} + rb + b^l + (p_h/p_y)c_h^w - \delta_h k_h] \\
w^a &= w^e [\alpha^u (l^e - l^{we}) + \alpha^r \frac{L_2(0)}{L_1(0)} l^e + (1+\tau_p) l_g^{de}]/p_y
\end{aligned}
$$

Inserting these equations into the above 20 laws of motion gives an explicit system of twenty autonomous nonlinear differential equations in the 20 state variables of the model shown in eq.s (1) - (20) of this section. Note that we have to supply as initial conditions the relative magnitude $\frac{L_2(0)}{L_1(0)}$ and that the evolution of price levels is subject to zero-root hysteresis, see Chiarella and Flaschel (1998a) for details.

We present next the 20 steady state values of the model. Note that we have now debt of firms and of the government in the model and denote their actual and steady debt - capital ratios by d_f, d_g. Note finally that the steady state is parametrically dependent on a given output price level p_y which is not determined by the model (due to the Taylor type interest rate policy pursued by the central bank) and must be specified arbitrarily.

$$
\begin{aligned}
y^e &= \frac{y^p \bar{U}}{1+\gamma\beta_{n^d}} \quad [y = y^p \bar{U}] && (21) \\
\nu &= \beta_{n^d} y^e && (22) \\
l_f^{we} &= l_f^{de} = l_y^e y^p \bar{U} \quad [\text{total employment: } l^{we} = l_f^{we} + l_g^{we}, l_g^{we} = \alpha_g g y^e] && (23) \\
l^e &= (l_f^{we} + \alpha_g g y^e)/\bar{V} && (24) \\
k_h &= \frac{c_2(y^e(1-g) - (\gamma+\delta))}{c_1(r_l^* + \delta_h) + (\gamma+\delta_h)c_2} && (25) \\
w^e &= \frac{\omega^{be} p_y}{1+\tau_p}, \quad [\omega^{be} = \frac{y^e - \delta - \mathbf{r_l^* d_f} - r_l^*}{l_f^{we}}] && (26) \\
p_y &= \quad \text{arbitrary} && (27) \\
\pi^l &= 0 && (28) \\
p_h &= p_y(r_l^* + \delta_h)/\bar{U}_h && (29) \\
p_b &= 1/r_l^* && (30) \\
\pi_{bs} &= 0 && (31) \\
e &= \frac{s_o - [\tau_w y_{w1} + \tau_p \frac{w^e}{p_y} l^{we}]}{\tau_m p_m^* j_y y/p_y}, \quad \epsilon_s = 0 && (32) \\
b &= \alpha_b^g \bar{d}_g y^e, \quad b_l = r_l^*(1-\alpha_b^g)\bar{d}_g y^e && (33) \\
\tau_w &= 1 - \frac{p_h \bar{U}_h k_h}{c_2 p_y y_{w1}}, \quad \tau_m = \frac{p_x^* x_y - p_m^* j_y}{p_m^* j_y} && (34) \\
r &= r_l^* && (35) \\
\mathbf{d_f} &= \frac{\gamma - \mathbf{r_l^*}}{\gamma}, \quad \mathbf{r_d}[= \rho^{\mathbf{e}}] = \mathbf{r_l^*} && (36)
\end{aligned}
$$

The two equations for the wage tax rate τ_w and for the rate of exchange e also require the further defining expressions:

$$
\begin{aligned}
c_h^o &= \bar{U}_h k_h \\
t_o^c &= \tau_c [\mathbf{r_l^* d_f} + r_l^* b + b^l + (p_h/p_y)c_h^o - \delta_h k_h]
\end{aligned}
$$

$$s_o = gy + r_l b + b - t_o + \frac{\quad}{p_y}[\alpha (l - l \quad) + \alpha \frac{\quad}{L_1(0)} l \,] + (1 + \tau_p)\frac{\quad}{p_y}\alpha_g gy - \frac{\quad}{\alpha_b^g}$$

$$y_{w1} = w^e[l^{we} + \alpha^u(l^e - l^{we}) + \alpha^r \frac{L_2(0)}{L_1(0)} l^e]/p_y$$

Note that the parameters of the model have to be chosen such that k_h, τ_w, e are all positive in the steady state.[16] Note also that we require $\alpha_s > 1 - 1/\beta_x$ for $x = p_b, e, p_e$ in order to satisfy the restrictions established in Chiarella and Flaschel (1998c).

Equation 21 gives the steady state solution of expected sales y^e per unit of capital K (and also output y per K) and eq. 22 provides on this basis the steady inventory-capital ratio N/K. Eq. 23 represents the amount of workforce (per K) employed by firms which in the steady state is equal to the hours worked by this workforce. It also shows total employment (per K) where account is taken of the employment in the government sector in addition. Eq. 24 represents full employment labor intensity (in the steady state) while the last expression for the quantity side of the model, in eq. 25, provides the steady value of the housing capital stocks per unit of the capital stock of firms.

Eq. 26 concerns the nominal wage level (net of payroll taxes and in efficiency units) to be derived from the steady state value for gross real wages ω^{be}, which include payroll taxes, which depends on the amount of interest to be paid on the loans of firms. The steady state value of the price inflation rate expected to hold over the medium run is zero, see eq. 28, since the inflationary target of the central bank is zero in the present formulation of the model. This also implies that all nominal magnitudes (up to nominal wages) have no long run trend in them and that all expected rates of change, see eq.s 28, 31, 33, must be zero in the steady state. Again, in eq. 27, p_y can be any value due to the assumptions made on monetary policy and money balances. Note that all nominal magnitudes, up to the price for long term bonds p_b, see eq. 30, depend on p_y and thus change proportionally when this price level magnitude is changed. As remaining nominal magnitudes we have the price level p_h for housing rents (in eq. 29), to be calculated from the uniform rate of interest r_l^* of the economy in the steady state (provided by the world economy), and the nominal exchange rate, e, in eq. 32, which is given by a complicated expression to be obtained from the government budget constraint, due to the import taxation rule followed by the government.

There follows the steady state value of short-term government debt per unit of capital $b = B/(p_v K)$ as well as the one for long-term domestic bonds, in eq.s 34 and 35, which are both simple consequences of the debt adjustment rule of the government and the rigid proportions by which government splits its debt in short- and long-term components. The steady state value of the wage tax rate, see eq. 37, is obtained from wage income - spending relationships of worker households, here performed on the basis of the housing services demanded and supplied in the steady state,[17] while the steady value of the import tax rate, in eq. 37, just balances the trade account (when import taxes are included). The interest rate policy rule of the central bank, due to its formulation, implies that the interest rate on short-term government debt must settle down at the given foreign rate, r_l^*, in the steady state.

Again, the new equations are eq.s 39 and 40, where the steady debt - capital ratio of firms is easily obtained from the budget constraint of firms and is positive if and only if the world rate of interest is smaller than the natural rate of growth (including the rate of technical progress) of the domestic economy. Finally, the steady value of the rate of interest on loans, r_d settles down at $r = r_l^*$. This closes the presentation of the interior steady state solution of our 20D dynamical model.

The Keen (1999) model , discussed earlier, based on our linear behavioral assumptions, can be represented as 3D dynamical system in the state variables $u = wL^d/p_y Y^p$, the wage share, $V = L^d/L$, the rate of employment, and $d_f = D_f/p_y K$, the debt to capital ratio of the firms,

[16]There are further simple restrictions on the parameters of the model due to the economic meaning of the variables employed.

[17]making use of gross steady wage income y_{w1} and the marginal propensity to spend this income on housing services.

$$\begin{aligned}
\hat{u} &= \beta_w(V-\bar{V})-n_l && (37)\\
\hat{V} &= \alpha^k(\rho-\rho_{min})-(n+n_l) && (38)\\
\dot{d}_f &= \alpha^k(\rho-\rho_{min})(1-d_f)-\rho && (39)
\end{aligned}$$

where $\rho = y^p(1-u)-\delta-r_d d_f$ is the actual rate of profit in this supply driven approach to economic growth.[19]

There is not yet a foreign and a government sector in this form of the Goodwin growth cycle model, but only the interaction of firms (capitalists) and worker households. The first two equations of this model would in fact be identical to the original Goodwin (1967) growth cycle approach if debt were not in the formulation of the profit rate and if $\alpha^k = 1, \rho_{min}$ would hold, in which case capitalists would just invest all income not going into wages and thus would determine the rate of growth of the employment rate as the difference between capital stock growth $\hat{K} = \rho$ and effective labor supply growth $n+n_l$. But we assume $\alpha^k > 1$, so that investment must be financed to some extent via loans which, of course, then implies the redefinition of the rate of profit of firms as shown above.

The third equation of the 3D model is derived from the budget equation of firms[20]

$$\dot{D}_f = \alpha^k(\rho-\rho_{min})K - \rho K$$

by use of the definitional relationship $\dot{d}_f = \dot{D}_f/K - \hat{K}d_f,\ d_f = D_f/K$. We stress that the dynamics automatically guarantee that u, V stay positive when they start positive, but that $\rho = y^p(1-u)-\delta-r_d d_f \geq 0, V \leq 1$ need not be fulfilled at all times. Furthermore, we should have $\alpha^k(\rho-\rho_{min})+\delta \geq 0$ at all times, since disinvestment can by assumption at most occur at rate δ. Note that this last inequality can be used to argue that $\rho \geq 0$ is not really needed for the viability of the model under the assumed investment behavior. Referring in addition to overtime work when the labor market is exhausted we may argue that the constraint $\rho \geq \rho_{min} - \delta/\alpha^k$ is really the crucial one.

In the next section we investigate the 3D model analytically in order to see if it provides insights into the properties of the 20D dynamics. Conversely, these 20D dynamics provides us with the perspective of how to augment the 3D core case by price level dynamics in order to obtain a basic 4D case where debt accumulation and deflation can be investigated analytically, from the perspective of supply driven growth.

The 4D model of debt deflation is augmented in the 20D situation by Rose effects in the wage-price interaction (which say that either wage or price flexibility must be destabilizing with respect to the implied real wage adjustments), by Keynes-effects (which here are more direct than is usually the case due to the monetary policy rule assumed), by Mundell-effects (which state that the interaction between price inflation and expected price deflation must be destabilizing if the adaptive component of these expectations is operating with sufficient speed), by Metzler-effects (which imply accelerator-type instability of the inventory adjustment mechanism when it operates with sufficient speed) and by cumulative (destabilizing) effects in financial markets (if adjustments are fast) due to positive feedback loops between expected changes and resulting actual changes of financial variables in our delayed adjustment processes towards overall interest rate parity (uniform rates of return). All these effects are of course partial in nature and must be studied in their interaction in a full analysis of the 20D model. However, we will only consider in the next section effects that concern the real part of the economy in its interactions the debt accumulation of firms and thus leave the other markets in the financial sector of the economy for later investigations (by assuming low adjustment speeds in the market for long-term domestic and foreign bonds).

[18] Note again that the price level p_y is kept fixed ($\equiv 1$) in the core version of the Keen model and that the rate of interest r_d is also a given magnitude in this model.

[19] due to the assumption $Y^d \equiv Y \equiv Y^p$ in the Keen (1999) paper.

[20] Note here again that this model assumes $p_y = 1$.

Romer–Lucas type

In this appendix, we follow Schneider and Ziesemer (1994, p.17) and use as starting point of our extended representation for the technological underpinnings of the general model of disequilibrium growth of the main aprt of the paper the following two equations:

$$
\begin{aligned}
\widehat{A} &= \eta\left(\frac{L_2^d}{L^d}\right), \quad \eta^{'} > 0, \quad \text{the research unit of firms} && (40)\\
Y &= K^\beta (AL_1^d)^{1-\beta} A^\xi, \quad \text{the production unit of firms} && (41)
\end{aligned}
$$

In these equations we denote by $\widehat{A}$ the growth rate of the labor productivity as it is generated by the research unit of firms, through their decision to allocate L_2^d of their total employment $L^d = L_1^d + L_2^d$ to this unit. Since we have unemployment in our disequilibrium approach to economic growth, we have chosen to use employment $L^d = L_1^d + L_2^d$ in the place of labor supply L in equation (55), relative to which the impact of the research unit on productivity growth is measured. The neoclassical approach in general immediately inserts the full employment assumption into (55), the law of motion of labor productivity on the level of firms. Equation (41) then provides the aggregate production function of the economy, based on (besides the use of the capital stock K) the employment L_1^d of labor in the production sector (including Harrod neutral technical change at rate $\widehat{A}$). We assume in addition to this usual Cobb–Douglas type of production function an externality, based on Romer (1986), with respect to the level of labor productivity A generated by firms, which increases aggregate output Y and productivity beyond the effects generated by the firms themselves. This can be seen more clearly if one rewrites equation (41) as follows:

$$Y = K^\beta \left(A^{\frac{1-\beta+\xi}{1-\beta}} L_1^d\right)^{1-\beta} = K^\beta (xL_1^d)^{1-\beta}$$

now with $x = A^{\frac{1-\beta+\xi}{1-\beta}}$ as measure of aggregate labor productivity in contrast to A, its expression on the level of individual firms. The above approach synthesizes Uzawa's (1965) allocation of employed labor between productive activities and technological change or briefly 'research' activities L_1^d, L_2^d and Romer's (1986) externality approach as in Lucas (1988), see also Barro and Sala-i–Martin (1995, Ch.4) for a detailed investigation of this approach in the context of neoclassical growth theory.

In view of our methodology of using linear relationships as much as possible in the initial phase of model formulation we have to reformulate the equations (55), (41) in terms of the fixed proportions production technology used in the main part of the paper which gives rise to the following expressions:

$$
\begin{aligned}
\widehat{A} &= \eta\frac{L_2^d}{L^d}, \quad \eta > 0 && (42)\\
Y &= \min\{y^pK, AL_1^d \cdot A^\xi\} = \min\{y^pK, A^{1+\xi}L_1^d\} = \min\{y^pK, xL_1^d\},\ x = A^{1+\xi} && (43)
\end{aligned}
$$

The last expression in (58) shows that the aggregate production function is of the same type as the one used in the body of the paper, with the exception that productivity growth

$$n_l = \widehat{x} = (1+\xi)\widehat{A} = (1+\xi)\eta\frac{L_2^d}{L^d}$$

is now based on the allocation of labor within firms between production work L_1^d and research work L_2^d ($L^d = L_1^d + L_2^d$ total employment).

motion

$$\dot{h} = \beta_{h_1}((\gamma - \widehat{x}) - n) + \beta_{h_2}(V - \overline{V}) \tag{44}$$

where we interpret $\gamma - \widehat{x}$ as trend rate of the increase in labor demand, here contrasted with the rate of increase in labor supply n, and where $V - \overline{V}$ is as usual the actual tension measure on the market for labor. Firms therefore speed up their efforts to increase labor productivity A when they see a tighter labor market or a rate of increase in labor demand that exceeds the growth rate of labor supply, and they do this by employing more labor in the research sector (for a given L_1^d which in our model is determined by a Keynesian demand constraint on the market for goods).

The extension of the 20D dynamics of this paper to include the endogenous generation of technical change by firms is thus given by the addition of the following two laws of motion

$$\widehat{x} = n_l = (1+\xi)\frac{h}{1+h}, \quad h = L_2^d/L_1^d \tag{45}$$

$$\dot{h} = \beta_{h_1}(\gamma - (1+\xi)\frac{h}{1+h} - n) + \beta_{h_2}(V - \overline{V}) \tag{46}$$

Note that the first equation enters the model only as an algebraic equation so that the dimension of the dynamics is increased only by '1' to 21D. The model now exhibits fluctuating rates of labor productivity increases and corresponding fluctuations in the research activities of firms and thus now incorporates the effects of endogenous technological change into the business fluctuations it generates.

In order to see the consequences of such an extension at first in a way as straightforward as possible, let us make three further assumption which allows for the simplest possible treatment of the resulting 21D dynamics. We assume first that natural growth n is also endogenously determined and described by the following law of motion

$$\dot{n} = \beta_{n_1}((\gamma - \widehat{x}) - n) + \beta_{n_2}(V - \overline{V}), \tag{47}$$

i.e., by a law of the same type as the one used for $\dot{h}$. This law of motion is based on the assumption that there is a subsistence sector in the background of the economy, from which people joining the labor force can be attracted or to which they can be repelled. Labor market characteristics that signal employment V above the norm $\overline{V}$ and trend growth of labor demand $\gamma - \widehat{x}$ above the current growth rate of labor supply n both induce this latter growth rate to increase (through additions from the subsistence sector) and vice versa (plus, of course, also the occurrence of mixed situations). As an extreme case we, for the time being, assume next $\beta_{n_1} = \infty \quad (\beta_{n_2} < \infty)$ which implies the equality

$$n = \gamma - \widehat{x}$$

at each moment in time, with γ the given growth rate of the world economy.

This assumption ($\gamma = n + n_l(h)$), finally coupled with the assumption $\beta_{h_2} = 0$, implies that the dynamic equations (1) – (20) remain unchanged, since h is a given magnitude then and since the only changes in the laws of motion (1) – (20), due to the new equations describing productivity and labor supply growth, concern the equations (**??**), (**??**)

$$\dot{l}_f^{we} = \beta_l(l_f^{de} - l_f^{we}) + (n + n_l - (g_k - \delta))l_f^{we} \tag{48}$$

$$\widehat{l}^e = n + n_l - (g_k - \delta) \tag{49}$$

which now have to use $n + n_l$ in the place of the given term γ, due to the definition of l^e and l_f^e in terms of efficiency units. If n adjusts with infinite speed to $\gamma - n_l$ the term $n + n_l$ is of the dynamics. We add that we have assumed here $\alpha_g = 0$ for reasons of simplicity, i.e. we abstract from employment in the government sector in this appendix, which makes the above subscript 'f' redundant. Note furthermore that the term n_l is not involved in any of the algebraic equations supplementing the dynamics (1) – (20). This in sum gives that neither the steady values of the state variables in (1) – (20) nor the dynamics (1) – (20) themselves will change in this special situation under the above addition (60), (46), (47), $\beta_{n_1} = \infty, \beta_{h_1} = 0$, of endogenous productivity and labor force growth to the model, up to the use of the constant h in the determination of the allocation of labor between production and research activities.

What then are the new insights that can be gained when technical change of Uzawa–Romer–Lucas type is added to the disequilibrium growth dynamics of this paper? In the considered special situation there are none from the formal point of view (and only some with respect to economic interpretation), since the mathematical features of the model remain as they were before. There is thus a special case for the above extension of the 20D dynamics where the generation of technical change by the activities of firms does not matter with respect to what has been shown in the main part of the paper.

Concerning the dynamics, one can of course again add the law of motion

$$\dot{h} = \beta_{h_2}(V - \overline{V}) \tag{50}$$

and its (algebraic) consequences

$$n_l = (1+\xi)\frac{h}{1+h}, \quad n = \gamma - n_l$$

and thus have endogenously generated fluctuations in labor productivity (which at present are still completely offset by opposite fluctuations in labor supply growth as far as the trend growth terms in the investment and quantity adjustment equations of our model are concerned). With respect to steady state calculations we furthermore then easily get that h_0, n_{l_0} and n_0 are in the present situation subject to zero–root hysteresis (due to the zero eigenvalue that corresponds to equation (50)) and are therefore also determined through historical conditions (initial values and shocks), see Chiarella, Flaschel, Groh and Semmler (1999) and Flaschel (1999) for the details of a treatment of such situations.

In the general case where

$$\dot{h} = \beta_{h_1}(\gamma - \hat{x} - n) + \beta_{h_2}(V - \overline{V}), \quad \hat{x} = (1+\xi)\frac{h}{1+h} \tag{51}$$

$$\dot{n} = \beta_{n_1}(\gamma - \hat{x} - n) + \beta_{n_2}(V - \overline{V}), \quad \hat{x} = (1+\xi)\frac{h}{1+h} \tag{52}$$

holds, we finally have a fully interdependent dynamics, due to the reformulated equations

$$\dot{l}_f^{we} = \beta_l(l_f^{de} - l_f^{we}) + (n + n_l - (g_k - \delta))l_f^{we}, \quad l_f^{de} = (1+h)y \tag{53}$$

$$\widehat{l^e} = n + n_l - (g_k - \delta), \quad l^e = xL/K \tag{54}$$

of the 20D part of our dynamics. It is however beyond the scope of this appendix to offer theoretical or numerical investigations of the resulting 22D dynamics. We therefore conclude this appendix by providing a complete representation of the 4D model of section 5 augmented by the above general case of endogenously generated technical change, including thereby the dynamics of the ratio h, and endogenous, but sluggish adjustment of the growth rate of labor supply n, into the core 4D dynamics used for analyzing debt deflation.

following form:

$$\begin{aligned}
\hat{w}^e &= \kappa[\beta_{w_1}(\frac{(1+h)y}{l^e}-\bar{V})+\kappa_w\beta_p(y/y^p-\bar{U})] && (55)\\
\hat{p}_y &= \kappa[\kappa_p(\beta_{w_1}(\frac{(1+h)y}{l^e}-\bar{V})+\beta_p(y/y^p-\bar{U})] && (56)\\
\hat{l}^e &= n+(1+\xi)\frac{h}{1+h}-[\alpha_1^k(\rho-r_d)+\alpha_3^k(y/y^p-\bar{U})] && (57)\\
\dot{d}_f &= [\alpha_1^k(\rho-r_d)+\alpha_3^k(y/y^p-\bar{U})+\gamma](1-d_f)-\rho-\hat{p}_y d_f && (58)\\
\dot{h} &= \beta_{h_1}(\gamma-(1+\xi)\frac{h}{1+h}-n)+\beta_{h_2}(\frac{(1+h)y}{l^e}-\overline{V}) && (59)\\
\dot{n} &= \beta_{n_1}(\gamma-(1+\xi)\frac{h}{1+h}-n)+\beta_{n_2}(\frac{(1+h)y}{l^e}-\overline{V}) && (60)
\end{aligned}$$

with $l^e = xL/K$ now (by definition) and with the algebraic equations

$$\begin{aligned}
y &= \bar{U}y^p + d_1(\frac{w^e}{p_y}-(\frac{w^e}{p_y})_o)+d_2(d_f-d_f^o), \qquad d_1, d_2 \le 0\\
\rho &= y-\delta-\frac{w^e}{p_y}(1+h)y-r_d d_f, \quad r_d \text{ given.}
\end{aligned}$$

as in the 4D core dynamics of the paper.

Proposition

Assume that the steady state of 4D subdynamics (55) – (58) is locally asymptotically stable for given values of h, n_l, n (set equal to their steady state values). Then: The steady states of the dynamics (55) – (60), which now are given by a ray in the phase space (parametrized by h), are locally asymptotically stable for all parameter values $\beta_{h_1}, \beta_{h_2}, \beta_{n_1}, \beta_{n_2}$ that are sufficiently small.

Sketch of proof:

1. Since the law of motion for n can be expressed as a linear combination of the three laws of motion for w^e, p_y, h we know that the determinant of the Jacobian of the full dynamics at the steady state must always be zero. One eigenvalue of this Jacobian is therefore fixed at zero.

2. Freezing n at its steady state value, it is easy to show for the remaining system that the determinant of its Jacobian must be negative if the one of the first four laws of motion is positive (which it is when the assumption of asymptotic stability of the proposition is taken account of).

3. In the case where the parameter values $\beta_{h_1}, \beta_{h_2}, \beta_{n_1}, \beta_{n_2}$ are all zero, the assumed asymptotic stability of the 4D core dynamics implies that the four eigenvalues of their Jacobian at the steady state have all negative real parts. Since eigenvalues depend continuously of the parameters of the dynamics, this conclusion also applies for all $\beta_{h_1}, \beta_{h_2}, \beta_{n_1}, \beta_{n_2}$ sufficiently small. Due to the sign of the determinant calculated for the first five laws of motion in such a case we know that the fifth eigenvalue (which now is no longer zero) must be negative, since this determinant is the product of these five eigenvalues. Of course, the sixth eigenvalue stays at zero, as was shown under 1. Q.E.D.

We only claim here in addition that the 6D extension of the original 4D dynamics, with its further intrinsic nonlinearities, will also allow for the other propositions made on the 4d case if the parameters $\beta_{h_1}, \beta_{h_2}, \beta_{n_1}, \beta_{n_2}$ are sufficiently small. Despite considerably increased complexity due to the addition of endogenous technical change and growth we thus see that in the case of endogenous growth of the Uzawa-Romer-Lucas type. Of course, the question remains how large the parameters $\beta_{h_1}, \beta_{h_2}, \beta_{n_1}, \beta_{n_2}$ may become before this close relationship between the 4D case and the 6D case gets lost.

In sum, we therefore conclude that endogenous technical change (and endogenous labor force growth) does not represent too big an issue or introduce considerations that are quite new for the disequilibrium growth model of this paper, compared to its other feedback chains and their stabilizing or destabilizing features considered in the main part of the paper. This is primarily due to the fact that we have assumed given trend growth terms, based on γ, in investment g_k and g_h (also in sales expectations and intended inventory changes), which introduces a Keynesian demand constraint here on the intensive form level of the model. This constraint prevents that savings and endogenous technical change work in a way as suggested by neoclassical approaches to endogenous growth.

Instead, and here we take again for simplicity the case of a given h and $n = \gamma - \widehat{x} = \gamma - \eta\frac{h}{1+h}(1+\xi)$, we have in the most basic situation of produced technological change, that increases in the parameters h, η, ξ will increase labor productivity, yet, due to the accompanying fall in labor force growth n, will not change the growth path of the economy otherwise. Or, to put it differently, should natural growth n not fall in such a situation, it will lead us to $n > \gamma - \widehat{x}$ and thus to an (on average) continuously decreasing rate of employment V. Again, this is primarily based on the assumption

$$g_k = \alpha_1^k(\cdot) + \alpha_2^k(\cdot) + \alpha_3^k(\cdot) + \gamma$$

and raises thereby the question about the forces that shape and might change the trend term γ in this investment equation in the course of time.

6 Debt effects and debt deflation

In section 5.1 we consider the Keen (1999) 3D growth cycle dynamics from the analytical point of view. We then extend these dynamics in section 5.2 by a law of motion for the price level that is a special case of the one used in the 20D case and analyze the features of these 4D dynamics. In section 5.3 we then investigate these 3D, 4D and also the general 20D dynamics numerically, with particular stress on the occurrence of debt deflation.

6.1 Debt accumulation

Let us first consider the steady state of the 3D dynamics (37) – (39). This unique steady state is given by:

$$V_o = \bar{V} + n_l/\beta_w \tag{1}$$

$$u_o = \frac{y^p - \delta - \rho_o - r_d d_f^o}{y^p}, \quad \rho_o = \rho_{min} + (n + n_l)/\alpha^k \tag{2}$$

$$d_f^o = 1 - \frac{\rho_o}{n + n_l} = \frac{\alpha^k - 1}{\alpha^k} - \frac{\rho_{min}}{n + n_l} \tag{3}$$

This set of steady state values shows that steady employment is the higher the higher the rate of technical progress and the lower the speed of adjustment of nominal wages. Profitability depends positively on the minimum rate of profit (which separates positive from negative net investment) and on the natural rate of growth, and negatively on the speed of adjustment of investment with respect to changes in the rate of profit, while just the opposite holds true for the debt capital ratio in the place of the rate of profit. Note that the rate of profits need not coincide with the rate of interest on loans in the steady state as there is no mechanism

$\alpha^k > 1$ holds (a necessary condition for a positive debt capital ratio in the steady state which needs to be coupled with an assumption on the relative size of ρ_{min} in order to get a positive steady state value for d_f).[21] Furthermore, the size of output per capital y^p should be such that the steady share of wages u_o is positive (and less than one which is always the case under the assumption just made).

Referring again to overtime work (here assumed to come about when the labor market is exhausted),[22] we do not exclude the case $V_o > 1$ from consideration, and also allow for steady rates of profit $\rho > n + n_l$. Note that the Goodwin (1967) growth cycle is obtained if $\alpha^k = 1, \rho_{min} = 0$ is assumed which gives $\dot{d}_f = \rho(1 - d_f) - \rho$ which remains zero if $d_f(0) = 0$.

Proposition 1

> *Assume $\alpha^k > 1, 0 < r_d < n + n_l$.*[23] *Then: The steady state (1) – (3) of the dynamics (37) – (39) is locally asymptotically stable for all admissible parameter values.*

Proof: See Chiarella and Flaschel (1999x).

We thus have the strong result that a partial debt financing of investment demand turns the center type dynamics of the original Goodwin (1967) growth cycle (all orbits are closed) into ones that imply convergence to the steady state, at least in a certain neighborhood of this steady state.

Proposition 2

> *Consider again the situation $\alpha^k > 1, 0 < r_d < n + n_l$. Assume furthermore as special case that $\beta_w = 0, n_l = 0$ holds, i.e., there is no adjustment in the wage share occurring when the other two state variables of (37) – (39) are changing. Then: For each level of the wage share u satisfying $y^p(1-u) - \delta - \rho_{min} > 0$ there exists a threshold value $\bar{d}_f \geq 0$ of the debt capital ratio d_f above which this ratio will increase beyond any bound according to the dynamics (37) – (39).*

Proof: Under the stated assumptions law of motion (37) becomes:

$$\dot{d}_f = \alpha^k r_d d_f^2 + [(1 - \alpha^k) r_d - \alpha^k \tilde{\rho}] d_f + (\alpha^k - 1)\tilde{\rho} - \rho_{min},$$

with $\tilde{\rho} = y^p(1-u) - \delta - \rho_{min} > 0$. The right hand side of this equation represents a polynomial of degree 2 $p(d_f) = c_o d_f^2 + c_1 d_f + c_2$ (with $c_o > 0, c_1 < 0$) with minimum at $d_f = -c_1/(2c_o) > 0$ and it exhibits of course only positive values after the larger of its two roots has been passed (if it is real, otherwise all values of $p(d_f)$ are positive even for all $d_f > 0$). Initial values of the debt to capital ratio d_f which lie to the right of this root therefore imply a purely explosive behavior of this ratio as long as there is no sufficiently strong counteracting change in the wage share u. Q.E.D.

We have pointed out at the end of the preceding section that, at a minimum, the side condition $\rho > \rho_{min} - \delta/\alpha^k$ should always be fulfilled to guarantee economically meaningful trajectories (along which gross investment should always stay nonnegative). The threshold for an explosive evolution of the debt capital ratio found to exist in proposition 2 may however still be so large that explosiveness can only occur in a domain where the system is not economically viable. In this case the proposition simply states that the dynamics will not always be globally stable from the purely mathematical point of view, but does not yet prove that critical developments in the debt capital ratio may also come about at initial situations to which there corresponds an economically meaningful environment. To show that such situations will indeed exist is the aim of proposition 3.

[21] Note that the steady debt ratio must always be smaller than one.

[22] see the 20D model for a more plausible treatment of overtime work.

[23] In the case $r_d = 0$ we have the Goodwin growth cycle dynamics coupled with an isolated adjustment process in the debt capital ratio.

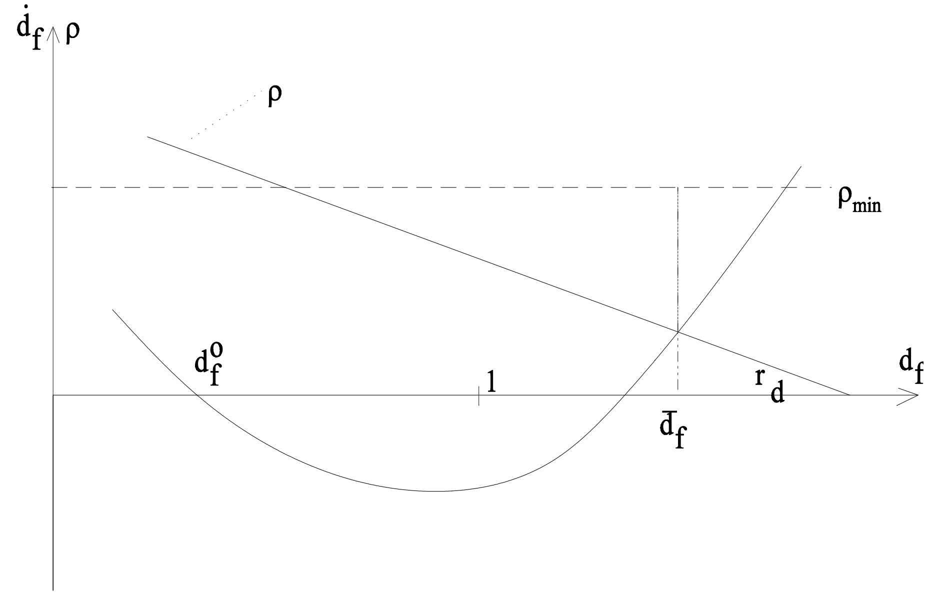


Figure 1: Debt dynamics at the steady share of wages.

Proposition 3

Assume as before $\alpha^k > 1, 0 < r_d < n + n_l$ and as special case again that $\beta_w = 0, n_l = 0$ holds. Then: For the steady state value of the wage share, u_o,[24] the threshold value $\bar{d}_f \geq 0$ of the debt capital ratio d_f of proposition 2 implies a rate of profit $\bar{\rho} \in (0, \rho_{min})$. The considered dynamics (37) – (39) therefore becomes divergent for values of d_f that lie in an economically meaningful part of the state space.

Proof: We show that the situation depicted in figure 1 obtains under the given assumptions: First, we show that the threshold value $\bar{d}_f$ must be larger than one in the considered situation. To see this it suffices to show that the polynomial considered in proposition 2 is still negative at $d_f = 1$. At $d_f = 1$ we have $\dot{d}_f = -\rho = y^p(1-u_o)-\delta-r_d$. This gives $\dot{d}_f = -[\rho_o+(r_d d_f^o - r_d)]$. Inserting the steady state values (2), (3) into this expression then implies $\dot{d}_f = -[\rho_{min}(1 - \frac{r_d}{n}) + \frac{1}{\alpha^k}(n - r_d)] < 0$ due to the assumption $n > r_d$. From this result it follows that the second root of the considered polynomial $\bar{d}_f$ must be larger than one (while the first coincides with the steady state due to our assumption $u = u_o$[25]).

Let us now calculate the rate of profit at this threshold value $\bar{d}_f$. Since we have $\dot{d}_f = 0$ at this value (yet not a steady rate of employment, but instead a falling one), we get for $\bar{\rho} = y^p(1-u_o) - \delta - r_d \bar{d}_f$ the expression:

$$0 = \alpha^k(\bar{\rho} - \rho_{min})(1 - \bar{d}_f) - \bar{\rho}$$

which in turn gives

$$\bar{\rho} = \frac{\alpha^k \rho_{min}(1 - \bar{d}_f)}{\alpha^k(1 - \bar{d}_f) - 1} = \frac{\rho_{min}}{1 - 1/(\alpha^k(1 - \bar{d}_f))}$$

Due to the above considerations we know however that the denominator of this expression is larger than one which implies that $\bar{\rho}$ must lie in the open interval $(0, \rho_{min})$. Q.E.D

[24]Note here that the steady state value of V, the rate of employment, is no longer uniquely determined in the considered case.

[25]Note that the smaller root can be negative, meaning that firms are creditors not debtors in the steady state, if $\rho_{min} > 0$ and if the parameter α^k is sufficiently close to one.

the threshold value $\bar{d}_f$ it will be caught in a situation where d_f is monotonically increasing accompanied by a falling rate of employment V until the domain of economically meaningful values for these two state variables is left. We stress that this result is obtained on the basis of a wage share that remains fixed at its steady state value and which therefore neither improves nor worsens the considered situation through its movements in time. This result will also hold true for all adjustments in the wage share that are sufficiently slow. At present it is however not clear whether a strongly falling wage share (based on a high value of the parameter β_w), which significantly improves the profitability of indebted firms, can lead us back to the steady state. This may depend on the size of the implied change in gross investment and its consequences for the change of the debt of firms.

For sufficiently small parameter values β_w we however know that the dynamics will produce explosiveness of the debt to capital ratio d_f and implosiveness for the rate of employment V beyond threshold values $\bar{d}_f, \bar{\rho}$. For sufficiently high debt, measured relative to the level of the capital stock, we thus get that debt accumulation feeds itself and will lead to larger and larger debt capital ratios at least if there is no sufficient support for the rate of profit from downward changes in the wage share. Yet, as there is no price deflation, there cannot be a 'perverse' adjustment (a rise) of the wage share in such a situation of depressed profitability and high debt accumulation. Such a problematic situation comes about when there is sluggish or no downward adjustment in the level of nominal wages, but – due to insufficient goods demand, which is not yet a possibility in the considered model of Keen (1999) – downward adjustment in the price level causing increases in the real wage and the wage share. This scenario will be investigated by a suitable 4D simplification of the general 20D model in the next subsection.

6.2 Debt deflation

Let us thus now extend the model (37) – (39) to include into it in a minimal way the possibility for price level deflation and thus the possibility for the occurrence of debt deflation (high levels of debt combined with declining profitability due to falling output prices). To do so we set to zero we set all parameters of the general 20D model that characterize the fiscal and monetary authority, the foreign sector, the housing sector and the asset markets equal to zero and thus get in particular given rates of interest (no Keynes-effect, no cumulative asset market behavior), with all interest rates equal to the then given rate of interest on loans. We will furthermore ignore the delayed Metzlerian quantity adjustment on the market for goods and assume that firms adjust their labor force with infinite speed which identifies employment l^{de} with the employed workforce l^{we}. We assume finally that inflationary expectations remain fixed at their steady state level (no Mundell effect) by setting adjustment coefficients equal to zero there too. This gives rise to the following dynamics for w^e, p_y and d_f coupled with an investment driven growth path, via the dynamics l^e:[26]

$$\begin{aligned}
\hat{w}^e &= \kappa[\beta_{w_1}(l^{de}/l^e - \bar{V}) + \kappa_w \beta_p (y/y^p - \bar{U})] && (4)\\
\hat{p}_y &= \kappa[\kappa_p(\beta_{w_1}(l^{de}/l^e - \bar{V}) + \beta_p (y/y^p - \bar{U})] && (5)\\
\hat{l}^e &= -[\alpha_1^k(\rho - r_d) + \alpha_3^k (y/y^p - \bar{U})] && (6)\\
\dot{d}_f &= [\alpha_1^k(\rho - r_d) + \alpha_3^k (y/y^p - \bar{U}) + \gamma](1 - d_f) - \rho - \hat{p}_y d_f && (7)
\end{aligned}$$

where the Metzlerian feedback mechanism simplifies to the following static (and again linearized) relationship:

$$y^d = y^e = y = y(\frac{w^e}{p_y}, d_f) = \bar{U}y^p + d_1(\frac{w^e}{p_y} - (\frac{w^e}{p_y})_o) + d_2(d_f - d_f^o), \qquad d_1, d_2 \leq 0$$

[26] $\bar{V}, \bar{U}$ the NAIRU utilization rates of the labor force and the capital stock.

case (and its richer concept of aggregate demand) in order to integrate the effects of price inflation and deflation in the simplest way possible.

We assume that the propensity to invest dominates the propensity to consume with respect to the impact of real wages $\frac{w^e}{p_y}$ on consumption and investment (the orthodox point of view) and also assume that output depends negatively on the debt to capital ratio d_f. The partial derivatives of the function $y(\frac{w^e}{p_y}, d_f)$ are therefore both assumed as negative in the following $(d_1, d_2 < 0)$. Since l^{de} is strictly proportional to output y, due to the fixed proportions technology assumed, we have that this employment magnitude exhibits the same type of dependence on the real wage and the debt to capital ratio as output y. Finally we of course again have $\rho = y - \delta - \frac{w^e}{p_y} l^{de} - r_d d_f$ for the rate of profit ρ.

Note that this shortcut of the originally delayed quantity adjustment process of Metzlerian type demands that the steady state value of this function y must be equal to $y^p \bar{U}$ in order to get a steady state solution for this 4D simplification of the 20D dynamics. Note also that the budget equations of the credit-giving institution (here the pure asset holders) are no longer subject to the problem we observed for the banks of the 3D Keen model. Note furthermore that Goodwin type dynamics are obtained when $r_d, d_f(0), d_1, d_2$ are all zero,[27] while the more general Rose (1967) type of real wage dynamics demands $r_d, d_2 = 0$ (with wage flexibility as a stabilizing factor and price flexibility destabilizing if (as is assumed) $d_1 < 0$ holds). Finally, the Fisher debt mechanism is obtained (due to $d_2 < 0$) by setting $\beta_{w1}, \kappa_w, d_1 = 0$.

The above represents the simplest way to integrate from the perspective of the 20D model the dynamics of the price level into our representation of the Keen (1999) model by abstracting from Metzlerian delayed output adjustment, from the distinction between the inside and the outside employment rate, from inflationary expectations, the housing sector, a fiscal and monetary authority, a foreign sector and from endogenous interest rate determinations. The reformulated goods market representation does allow for Rose-effects of traditional type (where price flexibility is destabilizing) and for Fisher debt effects (where price flexibility should also be destabilizing), but it excludes Mundell-effects for example (that would also demand the inclusion of inflationary expectations into the above model).

We first calculate the interior steady state of the dynamics (4) – (7) which is uniquely determined up to the steady level of prices p_y and is given by:[28]

$$
\begin{aligned}
d_f^o &= 1 - r_d/\gamma && (8)\\
y_o &= y^p \bar{U}, \quad l_o^{de} = y_o l_y^e, \quad l_o^e = l_o^{de}/\bar{V} && (9)\\
\rho_o &= [y_o - (\frac{w^e}{p_y})_o l_o^{de} - \delta - r_d d_f^o =] \quad r_d && (10)\\
(\frac{w^e}{p_y})_o &= \frac{y_o - \rho_o - \delta - r_d d_f^o}{l_o^{de}} && (11)\\
p_y^o &= \text{determined by initial conditions} && (12)\\
w_o^e &= p_y^o (\frac{w^e}{p_y})_o && (13)
\end{aligned}
$$

Due to the new form of the investment function[29] $I/K = \alpha_1^k(\rho - r_d) + \alpha_3^k(y/y^p - \bar{U}) + \gamma + \delta$ we now have a different steady debt capital ratio which is solely determined by trend growth γ in its deviation from the given rate of interest r_d on loans. We again assume that $\gamma - r_d > 0$ holds in order to get a positive steady state ratio d_f. The two NAIRU's on the labor and the goods market, $\bar{V}, \bar{U}$, and our consistency assumption that y is equal to $y^p \bar{U}$ in the steady

[27] also in the further special case where $\alpha_1^k = 1, \alpha_3^k = 0, \gamma = r_d$ holds.

[28] We use l_y^e to express employment per unit of output measured in efficiency units (a given magnitude).

[29] which must be nonnegative along the relevant trajectories of the dynamics.

employment labor intensity, l^{de}, l^e in the usual way. Having determined the rate of profit through the rate of interest on loans implies on this basis a well defined level of real wages, $(\frac{w^e}{p_y})_o$, which is positive if y^p is chosen sufficiently high relative to γ, δ, r_d and $\bar{U}$. This real wage level then determines the nominal wage level on the basis of a given price level which is determined through historical (initial) conditions.

Proposition 4

Assume $0 < r_d < \gamma, d_2 = 0$ and $\beta_p, \kappa_p = 0$,[30] *i.e., the price level is a given magnitude in this special case. Assume furthermore that the investment parameter α_1^k is chosen such that $\alpha_1^k r_d - \gamma > 0$ holds true. Then: The steady state (8) – (13) of the dynamics (4) – (7) is locally asymptotically stable for all other admissible parameter values.*

Proof: See Chiarella and Flaschel (1999x).

We thus have that the steady state of the reduced dynamics (4),(6),(7) (where there is no adjustment of prices due to the demand pressure on the market for goods) is locally asymptotically if the influence of the debt to capital ratio d_f on the level of output and employment, is sufficiently weak. Furthermore, since the determinant of the full 4D dynamics is always zero these dynamics will be convergent with respect the three state variables w^e, l^e, d_f also for all speeds of adjustments β_p (and parameters κ_p) chosen sufficiently small, since the eigenvalues of the full dynamics are continuous functions of the parameters of the model.

Proposition 5

Assume now (as was originally the case) that $d_2 < 0$ holds. Then: The steady state (8) – (13) of the dynamics (4) – (7) is not locally asymptotically stable for all price adjustment speeds β_p chosen sufficiently large.

Proof: The interdependent part of the dynamics (4) – (7) can be reduced to the dynamics of the state variables $\omega^e = \frac{w^e}{p_y}$, the real wage, and again l^e, d_f, as follows:

$$\begin{aligned}
\hat{\omega}^e &= \kappa[(1-\kappa_p)\beta_{w_1}(l^{de}/l^e - \bar{V}) - (1-\kappa_w)\beta_p(y/y^p - \bar{U})] && (14)\\
\hat{l}^e &= -[\alpha_1^k(\rho - r_d) + \alpha_3^k(y/y^p - \bar{U})] && (15)\\
\dot{d}_f &= [\alpha_1^k(\rho - r_d) + \alpha_3^k(y/y^p - \bar{U}) + \gamma](1-d_f) - \rho \\
&- \kappa[\kappa_p(\beta_{w_1}(l^{de}/l^e - \bar{V}) + \beta_p(y/y^p - \bar{U})]d_f && (16)
\end{aligned}$$

The coefficient of β_p in the trace of the Jacobian of these dynamics at the steady state is:

$$\omega_o^e \kappa(1-\kappa_w)\beta_p(-d_1)/y^p + \kappa\beta_p(-d_2)/y^p > 0$$

up to the possibility that either κ_w or κ_p can be equal to one.[31] Therefore the trace of J can always be made positive by choosing the parameter β_p sufficiently large. Q.E.D.

The local stability result for the 3D Keen model is therefore overthrown in the case where relative goods demand is negatively dependent on the debt capital ratio and where the price level adjusts with respect to demand pressure on the market for goods with sufficient speed. In such a case, we conjecture and will tests this assertion numerically, that a process of deflation will continue without end accompanied by higher and higher debt ratios of firms which eventually will lead to zero profitability and bankruptcy.

Proposition 6

[30] This implies $\kappa = 1$.

[31] The first expression shows the strength of the destabilizing Rose or price level flexibility effect and the second is the Fisher debt effect.

nominal wages are completely fixed ($\beta_w, \kappa_w = 0$). Then: The dynamics (4) – (7) is monotonically explosive, implying higher and higher real wages and debt to capital ratios, for initial debt capital ratios chosen sufficiently high (in particular larger than 1) and all real wage levels above their steady state value.

Proof: The dynamics in the proof of the preceding proposition then reduces to

$$\hat{\omega}^e = -\beta_p(y/y^p - \bar{U}) \tag{17}$$

$$\dot{d}_f = [\alpha_1^k(\rho - r_d) + \alpha_3^k(y/y^p - \bar{U}) + \gamma](1 - d_f) - \rho - \beta_p(y/y^p - \bar{U})d_f \tag{18}$$

since l^e no longer feeds back onto ω^e and d_f. Since both ω^e and d_f are larger than their steady state values, we get from the first law of motion that ω^e must be rising further (due to falling price levels caused by $y < y^p\bar{U}$). Furthermore, since also $\rho - r_d, 1 - d_f < 0$ holds $\dot{d}_f$ must be larger than

$$\gamma(1 - d_f) - r_d - \beta_p(y/y^p - \bar{U})d_f > -\gamma d_f - \beta_p(y/y^p - \bar{U})d_f.$$

If therefore $-\beta_p(y/y^p - \bar{U}) > \gamma$ has come about by choosing d_f sufficiently high, $\dot{d}_f > 0$ must be true so that both ω^e and d_f will be rising which further strengthens the conditions for their monotonic increase. Q.E.D.

We thus get as in proposition 3, but more much easily and more pronounced (through the occurrence of price deflation), that there will indeed occur situations of now debt *deflation* where profitability falls monotonically and where the debt of firms is increasing beyond any limit, therefore leading to economic collapse sooner of later.

Proposition 7

Assume as always $0 < r_d < \gamma$ and $\alpha_1^k > 1$. Assume furthermore that $\beta_p = 0, \kappa_p = 1$, i.e., the price level is determined by cost push considerations solely and hence by a conventional markup equation of the type

$$p_y = (1+m)\frac{wL^d}{Y} = (1+m)wl_y = (1+m)w^e l_y^e.$$

Assume that the given markup m is such that the implied real wage ω^e (in efficiency units) is equal to its steady state level. Next, assume a given level of nominal wages (measured in efficiency units), i.e., $\beta_{w1} = 0, \kappa_w = 0$.[32] Assume finally that the investment parameter α_3^k is chosen such that $\alpha_3^k > y^p(1 - \omega^e l_y^e)\frac{\gamma - r_d}{r_d}$ holds true.[33] Then: The steady state (8) – (13) of the dynamics (4) – (7), which can then be reduced to adjustments of the debt to capital ratio basically, is locally asymptotically stable for all values of the parameter $d_2 < 0$.

Proof: In the assumed situation we have $\hat{p}_y = 0$ due to the given level of nominal wages and thus get as single independent law of motion for the debt capital ratio d_f:

$$\dot{d}_f = [\alpha_1^k(\rho(d_f) - r_d) + \alpha_3^k(y(d_f)/y^p - \bar{U}) + \gamma](1 - d_f) - \rho(d_f).$$

We have to show that the derivative of the right hand side of this equation is negative at d_f^o. Note first that $\rho'(d_f) = y'(d_f)(1 - \omega^e l_y^e) - r_d = d_2(1 - \omega^e l_y^e) - r_d$ holds with a real wage

[32] The nominal wage is therefore growing in line with labor productivity.

[33] This inequality is equivalent to the inequality

$$\alpha_3^k > [(\gamma - r_d)^2 + 2(\gamma - r_d) + \delta(\gamma/r_d - 1)]/\bar{U}.$$

evaluated at the steady state turns out to be

$$-(\gamma - r_d d_f) + (\alpha_1^k - 1)\rho'(d_f)(1 - d_f) - d_2(-\alpha_3^k / y^p (1 - d_f) + (1 - \omega^e l_y^e) d_f)$$

with $d_f = 1 - r_d/\gamma, 1 - d_f = r_d/\gamma$. This expression must be negative since $r_d < \gamma, d_f < 1, \alpha_1^k > 1, \rho' < 0$ and due to $\alpha_3^k > y^p(1 - \omega^e l_y^e) d_f/(1 - d_f) = y^p(1 - \omega^e l_y^e)\frac{\gamma - r_d}{r_d}$. Q.E.D.

In a similar way it can also be shown that the above derivative is negative for all $d_f \in (0, d_f^o)$, i.e., there is convergence to the steady state for all positive debt capital ratios below the steady state ratio. It is however not possible to provide an easy expression for the upper limit of the basin of attraction of the steady state (which may be less than 1).

We have formulated proposition 7 with a view to an intended policy application which we only sketch here. Consider the case where the debt capital ratio d_f is so large that there are cumulative forces at work (as in proposition 6) which would lead to higher and higher debt and lower and lower profitability in the considered economy. In the case considered in proposition 7 there are three possible ways to break this catastrophic tendency in the evolution of the economy. 1. An increase in nominal wages w^e which under the assumptions of proposition 7 causes an immediate increase in the price level p_y and thus an immediate decrease in the ratio d_f, which (if strong enough) may lead the economy back to the basin of attraction of its steady state. 2. A decrease in the rate of interest r_d on loans which moves the steady state of the economy to a higher sustainable debt capital ratio. 3. A decrease in the sensitivity of output y (through appropriate fiscal policies) with respect to d_f, i.e., a parameter d_2 that is smaller in amount (which may enlarge the basin of attraction of the steady state).

There is therefore scope for economic policy to move the economy out of regions of developing debt deflation into regions where it converges back to the steady state. The details of such possibilities must however here be left for future research,

6.3 Some numerical investigations

In this subsection we provide some numerical examples for the propositions on the 3D and 4D dynamics presented in the preceding subsections and will also present a few simulation runs of the general 20D dynamical system which will be further investigated in a companion paper to the present one, see Chiarella, Flaschel and Zhu (1998b), with respect to the various feedback mechanisms it contains and with respect to shape and size of its basins of attraction.

6.3.1 The 3D dynamics

We start the numerical analysis of the 3D dynamics by stressing again that they are of the Goodwin (1967) growth cycle type (where all orbits are closed curves around the steady state) when one assumes with respect to their parameters: $r_d = 0, \rho_{min} = 0, \alpha^k = 1$. There are also further cases where the closed orbit structure is obtained as we shall see in the following.

As a first example we now consider the case where there holds: $\alpha^k = 1.5; \beta_w = 0.5; n = 0.03; n_l = 0.03; \rho_{min} = 0; \bar{V} = 0.9; r_d = 0.05; y^p = 0.45; \delta = 0.1$; and where we exercise a very large shock on the debt capital ratio which gives it three times the size of its steady state value (from which the dynamics starts). The first to notice is that the debt capital ratio converges back to its steady state value in a time span of approximately fifty years and this in a strictly monotonic way while the real cycle keeps its shape basically. The result is that the size of this cycle is shock dependent since the disappearance of motion in the debt ratio makes the wage-share employment-rate dynamics again self-contained and thus of closed orbit type (the size of which depends as in the Goodwin growth cycle model on the history of the economy). If there is strong convergence of d_f back to its steady state value

Goodwin cycle mechanism comes to a rest once the debt ratio has become sufficiently small. The figure 2 provides an example of this type.

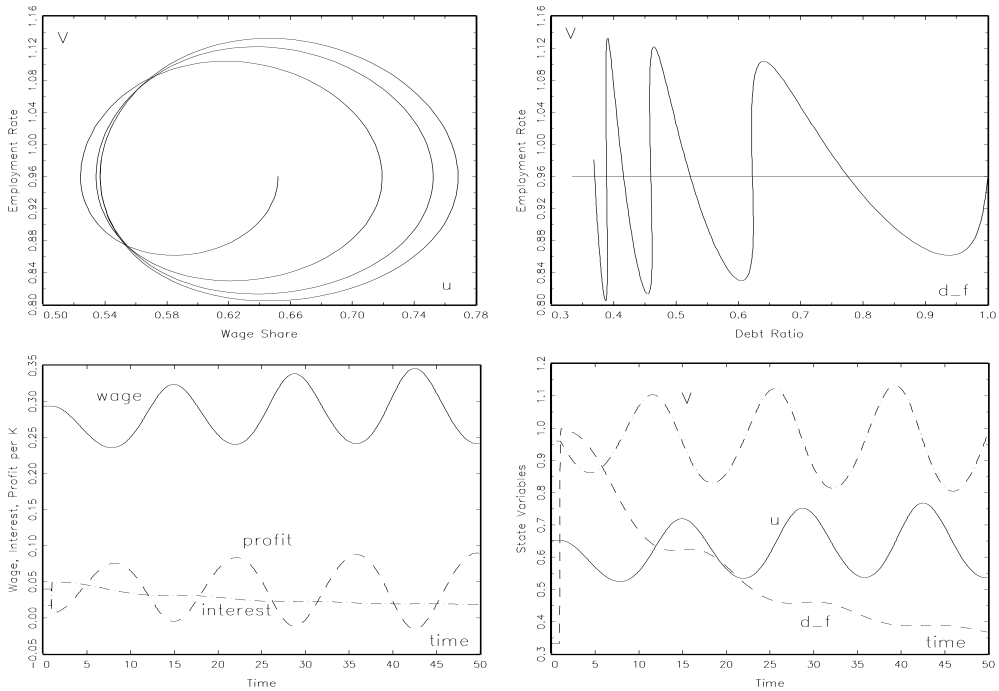


Figure 2:
Debt convergence and shock-dependent persistent cyclical growth.

In the next 3D example in figure 3 we make use of less sensitive investment behavior now based on a minimum rate of profit that is larger than 0. In this case we get sluggishly convergent Goodwin-type growth cycle behavior which we exhibit in the following figure for the time interval [200.260]. As in many other convergent cases we have here only a weak reduction in amplitude over time, in particular since debt is relative small for many reasonable choices of the parameters α^k, ρ_{min}. We now also observe a basically positive correlation of the employment rate and the debt to capital ratio.

The parameters underlying figure 3 are the following ones: $\alpha^k = 1.3; \beta_w = 0.5; n = 0.03; n_l = 0.03; \rho_{min} = 0.01; \bar{V} = 0.9; r_d = 0.05; y^p = 0.45; \delta = 0.1$, i.e., the change only concerns the assumed investment behavior in comparison to the preceding situation.

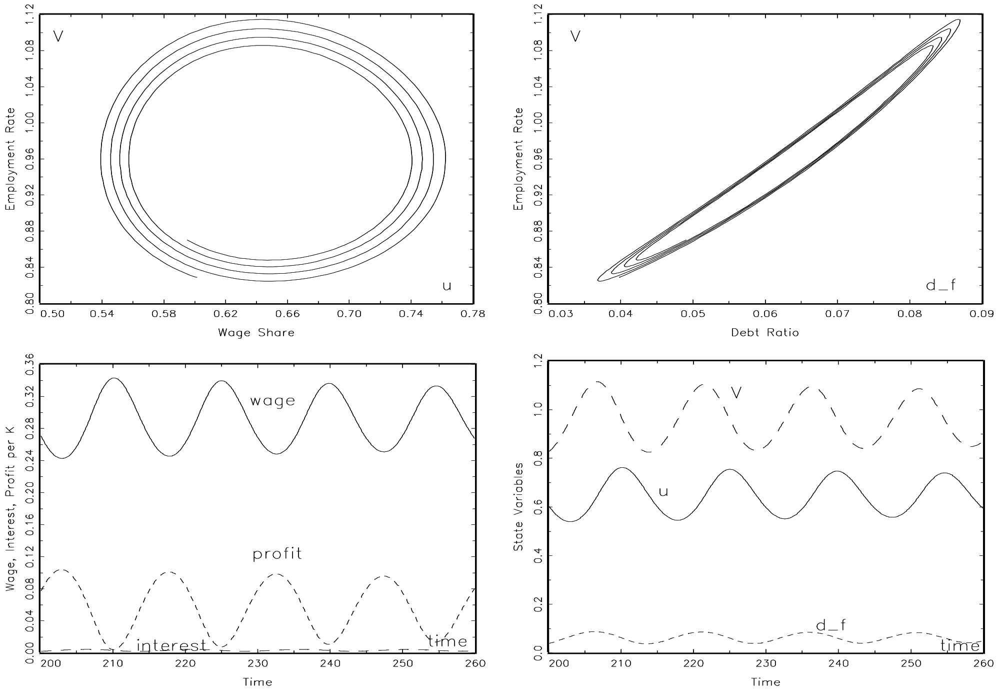


Figure 3:
Slow Convergence via debt-financed investment.

We conclude from these and various other simulations that the Goodwin real growth cycle often plays the dominant role in the shaping of the dynamics, while the debt dynamics either dies out or leads to Goodwin cycles with slowly declining amplitude, or leads to strongly explosive behavior (not shown) if the shocks in the debt ratio are made very large. It is therefore time to add the nominal price dynamics to the real growth model with firms that finance part of their investment via debt.

6.3.2 The 4D dynamics

We consider now the 4D dynamics with both wage and price level adjustment and assume as starting point the following parameter set: $\alpha_1^k = 1.3; \alpha_3^k = 1.3; \beta_{w1} = 0.3; \beta_p = 0.5; \kappa_w = 1; \kappa_p = 0.5; \gamma = 0.06; \bar{V} = 0.9; \bar{U} = 0.9; r_d = 0.04; y^p = 0.45; \delta = 0.1; l_y^e = 2; d_1 = -0.5; d2 = -0.1$. We thus assume now that price adjustments are based on demand pressure as well as a wage cost-push term, and that wage adjustments (expressed in efficiency units) are fully incorporating price inflation ($\kappa_w = 1$), a situation in which the real wage dynamics depend only on demand pressure in the market for labor and not on that in the market for goods. There is thus only a stabilizing Rose (1967) effect present with respect to real wage adjustment (since $d_1 < 0$ holds and since goods market equilibrium presently irrelevant for real wage dynamics). This effect is of course the stronger the larger the parameter β_{w1} becomes. Furthermore the debt effect on output is comparably weak here: $d_2 = -0.1$, and the steady debt ratio as well as the moving one (and thus also interest payments) are small in the present situation, which in sum gives rise here to a fast cyclical adjustment of the employment rate, of the wage share

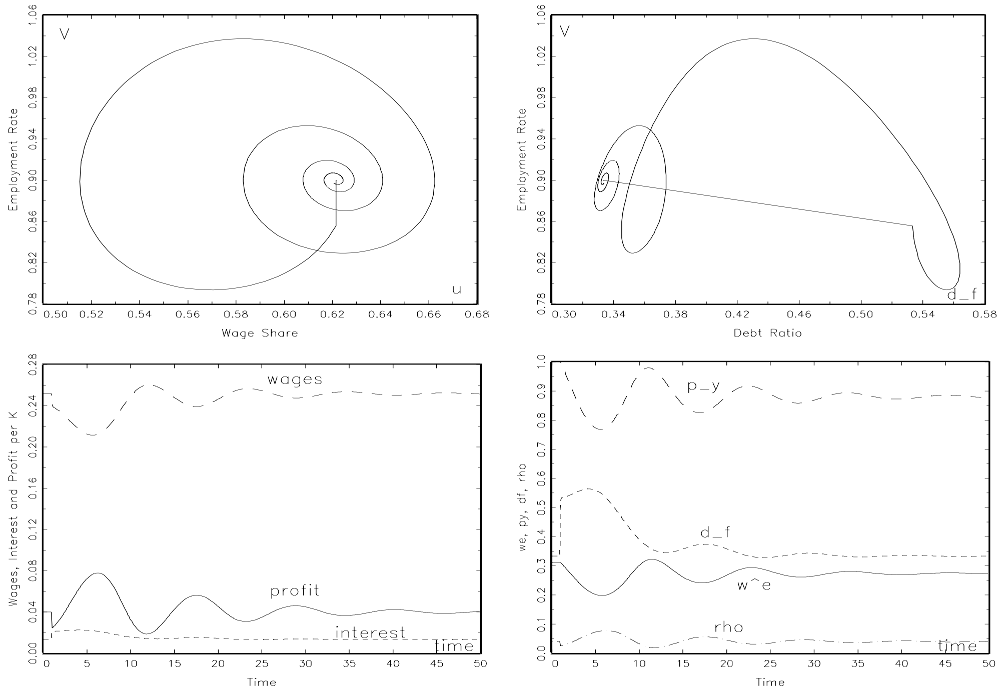
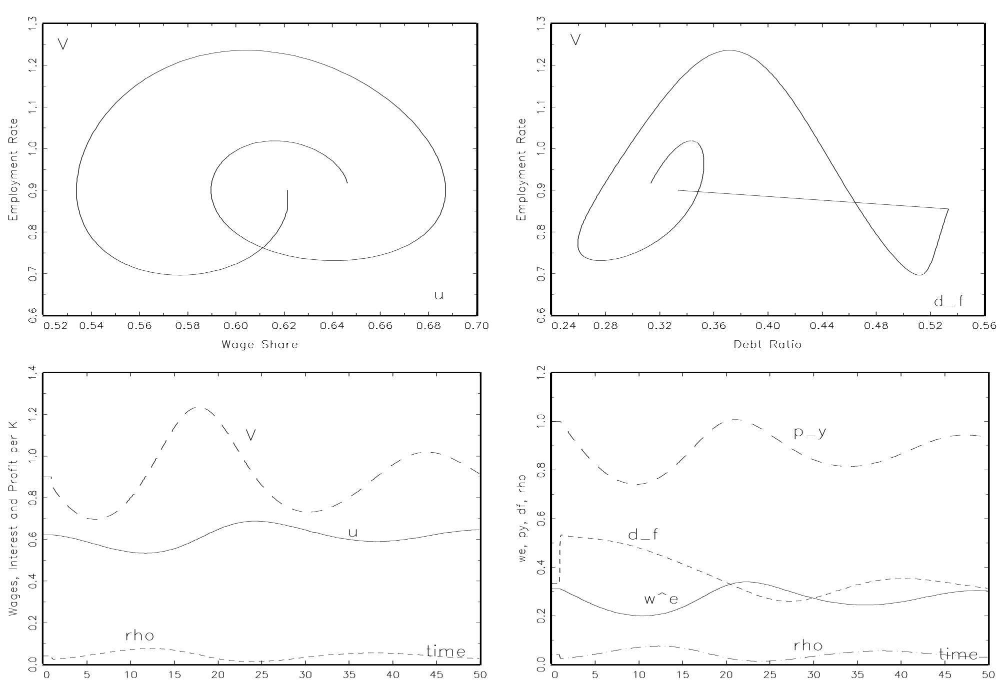


Figure 4:
Faster convergence through a stabilizing Rose effect.

Note however that the initial phase of the shown dynamics exhibits high (and even rising) debt and falling price levels which however creates no long-lasting problem for the economy here. We expect that this situation will change when the wage adjustment speed is decreased or the price adjustment speed increased and the parameter d_2 made more negative, because of the normal Rose-effect with respect to real wage adjustments and a destabilizing Fisher debt effect. A partial example for this is shown in the next figure 5. Yet, also in this figure we have still a rising rate of profit despite high debt and falling prices and thus still a situation where the conflict about income distribution helps to prevent debt deflation to become a real threat for the rate of profits of firms.



Less convergence through more sluggish wages.

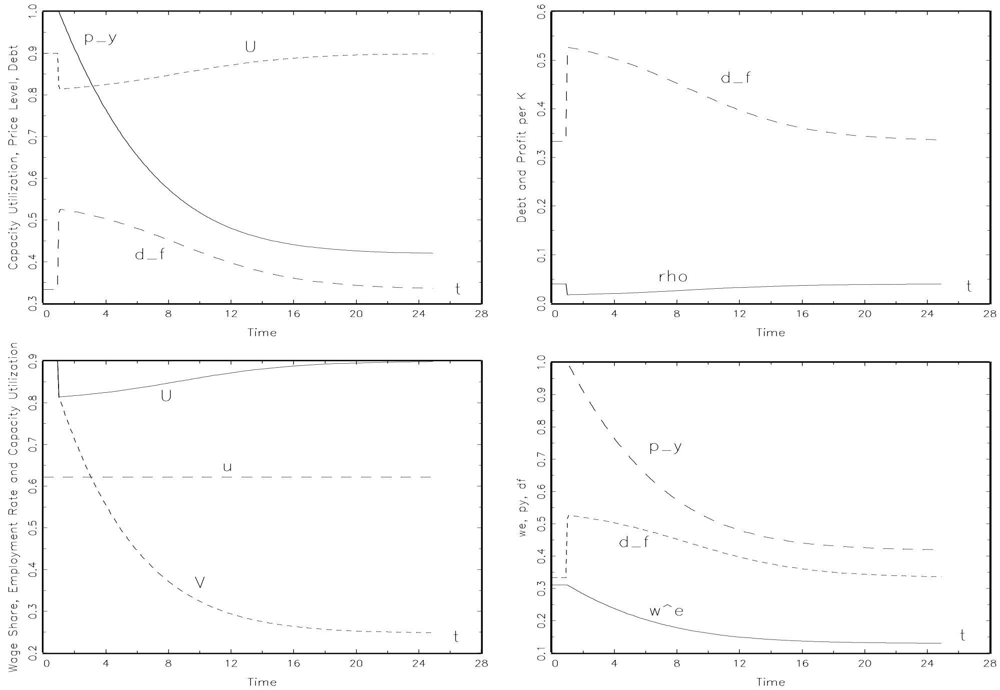


Figure 6:
Deflation and converging debt.

This situation is assumed away in figure 6 where we have $\beta_{w1} = 0$ coupled with $\kappa_w = 1$ implies that wages are following prices passively such that the wage share stays constant (furthermore we now also assume $d_2 = -0.2, \beta_p = 0.552$ with respect to the preceding set of parameter values).

As figure 6 shows we have a marked dip in the rate of profit when the sudden increase in the debt ratio occurs (at $t = 1$), which nevertheless becomes slowly reversed thereafter since the debt ratio declines back to its steady value and since deflation does again not make the dynamics a collapsing one. Note however that, though the rate of capacity utilization converges back to its normal rate, the rate of employment shows no similar tendency as there is no demand pressure effect from the rate of employment on the share of wages.[34] Increasing the size of the shock in the debt to capital ratio further will, however, eventually lead to divergence and thus to economic breakdown.

[34] See Fair (1997) for an empirical study of wage and price Phillips curves where only demand pressure in the goods market is important.

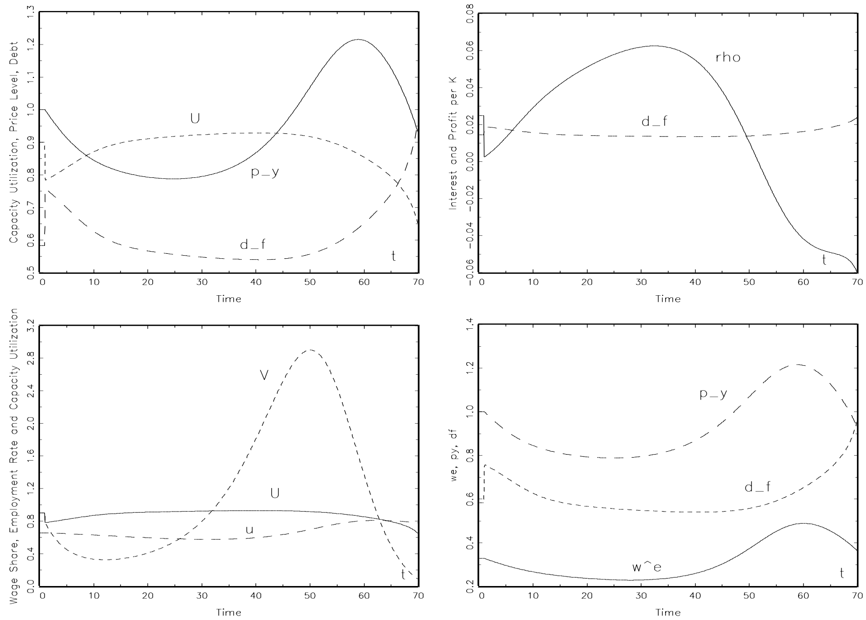


Figure 7:
Debt deflation in the case of a sluggishly adjusting wage share.

The figure 7 is based on the following parameter set: $\alpha_1^k = 1.3; \alpha_3^k = 1.3; \beta_{w1} = 0.1; \beta_p = 0.1; \kappa_w = 1; \kappa_p = 0.5; \gamma = 0.06; \bar{V} = 0.9; \bar{U} = 0.9; r_d = 0.025; y^p = 0.45; \delta = 0.1; l_y^e = 2; d_1 = 0; d_2 = -0.03$, i.e., we have a sluggishly reacting price and wage levels, now coupled with a low rate of interest on loans and thus a higher steady state ratio for d_f. There is little movement in the wage share at first and there is no real wage effect on output (no Rose-effect), but only the small negative effect of increasing debt on y. As we can see the dynamics is explosive in the present case, with at first rapidly rising profitability, due to the decline in debt and in the wage share occurring after the initial increase of debt at $t = 1$. Later on, however, the wage share starts rising, lowering the rate of profit significantly which then leads to increasing debt to capital ratios, falling capacity utilization and falling prices, and to economic breakdown soon thereafter (though the wage share seems to start declining again).

Clearly, there is debt deflation in the final phase of the time series shown, and the question may therefore be posed whether positive price shocks, placed appropriately in such periods of deflation, can prevent economic collapse, extending its life beyond the 70 years that if here runs before (numerical) breakdown occurs. In the next figure we have added such positive price shocks (at $t = 58, 70$) to the dynamics just shown and indeed get that these shocks counteract debt deflation for some time, by stopping the occurrence of falling price levels, restoring profitability and lowering the debt to capital ratio, which also leads to higher capacity utilization due to its negative dependence on debt d_f. Note however that employment reacts in an extreme fashion and with long swings (basically due to the sluggish adjustment of nominal wages in the face of a large disequilibrium in the market for labor).

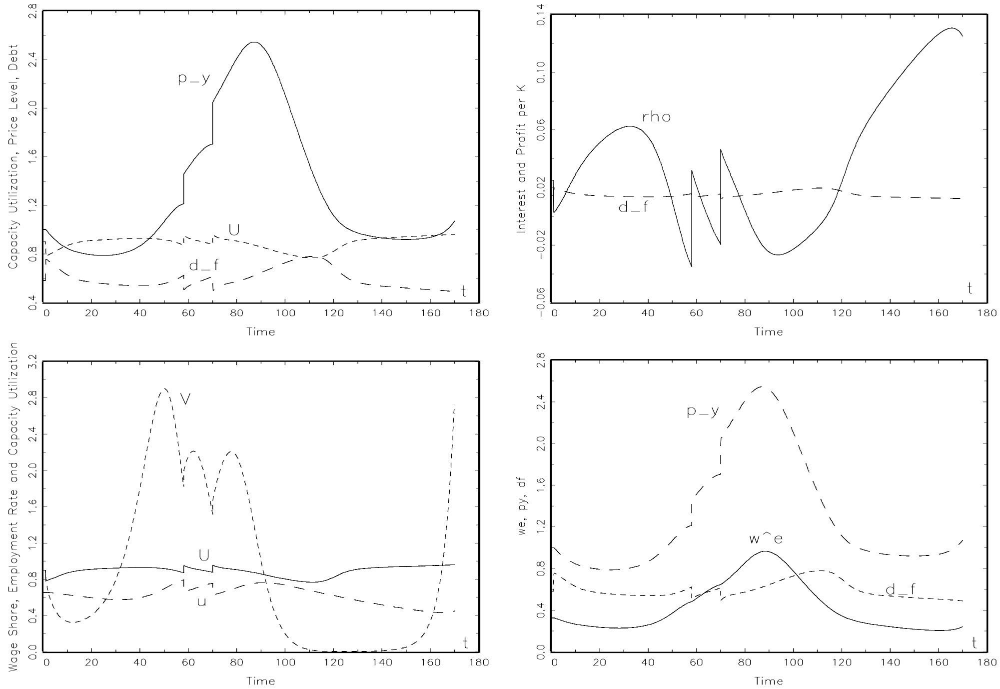


Figure 8:
Positive price shocks (temporarily) stop debt deflation.

This closes our investigation of basic growth cycle models with debt-financing, the possible occurrence of deflation and the role of the wage share in such a situation. Further numerical investigations will be provided in a companion paper to the present one where also basins of attraction, eigenvalue analysis and more will be calculated in order to study the Fisher debt effect and the Rose feedback mechanism in more detail than was possible here. We have seen that (with and without profitability increasing adjustments in the wage share) debt will often converge back to its steady state value after debt shocks of considerable size. Undamped fluctuations are however possible and may lead to periods of strong debt deflation where positive price shocks may help to avoid economic collapse. Increasing price flexibility further will, however, lead to even stronger explosiveness (not shown), in the present model due to the joint working of the Rose real wage and the Fisher debt effect if both parameters d_1, d_2 are significantly below zero.

6.3.3 The 20D dynamics

Let us first of all stress that debt financing is the least involved in the 20D dynamics in the case where $\alpha_1^k = 1, \alpha_2^k, \alpha_3^k = 0, \beta_{r_d} = 0, r_d(0) = \gamma$ holds. We then have $g^k = \rho^e + \delta$ which implies that only unexpected inventory changes have to be financed by loans (which should not matter very much for the dynamics of the model and thus should not allow debt deflation to play a significant role in this case). Furthermore, the qualitative properties of the original 18D dynamics should not be changed radically as far as the role of adjustment speeds is concerned if all expected profits are retained and not paid out as dividends as assumed in the 18D model, where fixed business investment was financed (in the background) via issuing new equities. We have used the above simplified situation to find cases where the steady state is asymptotically stable (not shown) and from where we could then start parameter modification and the investigation of destabilizing debt deflation in the 20D case.

We first thus show in figure 9 a case of asymptotic stability of the steady state of the 20D dynamics, see below and appendix 2 for the underlying parameter set. Note that this case already departs from the above reference situation to a considerable degree and that we instabilities here already present in the private sector of the economy. These destabilizing forces again basically derive from the Rose and the Fisher debt effect which in this extended framework can be schematically presented as follows:

$$\boxed{p_y \downarrow \rightarrow \omega^e \uparrow \rightarrow y^d \downarrow \rightarrow y^e \downarrow \rightarrow y \downarrow \rightarrow \hat{p}_y \downarrow \quad \text{(the Rose-effect)}}$$

$$\boxed{p_y \downarrow \rightarrow d_f \uparrow \rightarrow \rho^e \downarrow \rightarrow y^d \downarrow \rightarrow y^e \downarrow \rightarrow y \downarrow \rightarrow \hat{p}_y \downarrow \quad \text{(the Fisher-effect)}}$$

Note that the partial Rose -effect only works this way if investment reacts more sensitively to real wage changes than consumption in which case the cost effect of increasing real wages dominates the purchasing power effect they have in this model (as is the case in the following numerical simulations of the 20D dynamics). Note furthermore that asset markets are reacting very sluggishly in the situations considered in this subsection and that the inventory adjustment mechanism exhibits slow inventory adjustments coupled with fast sales expectations which give it (from a partial perspective) the features of a stable dynamic multiplier process. Finally, also the Mundell-effect of inflationary expectations is absent, due to the parameter choices made in the following. We consequently concentrate in this subsection on the two effects shown in the above boxes and on the role of the interest rate policy rule as a stabilizing instrument in such an environment (because of its close relationship to the Keynes-effect in the alternative case of a money-supply policy rule). Note finally that rates of return are equalized in the 20D case, in contrast to the 3D and 4D situations considered in the preceding subsections.

The parameter values underlying figure 9 are those specified in appendix 2 of this paper, up to $\beta_{r_2} = 0.5, \kappa_w = 0$ and the adjustment speed of the price level β_p which set equal to 0.2 in this case. We stress that the steady state is indeed asymptotically stable since also the price level will converge to a given level (and thus not fall forever) in the considered situation.

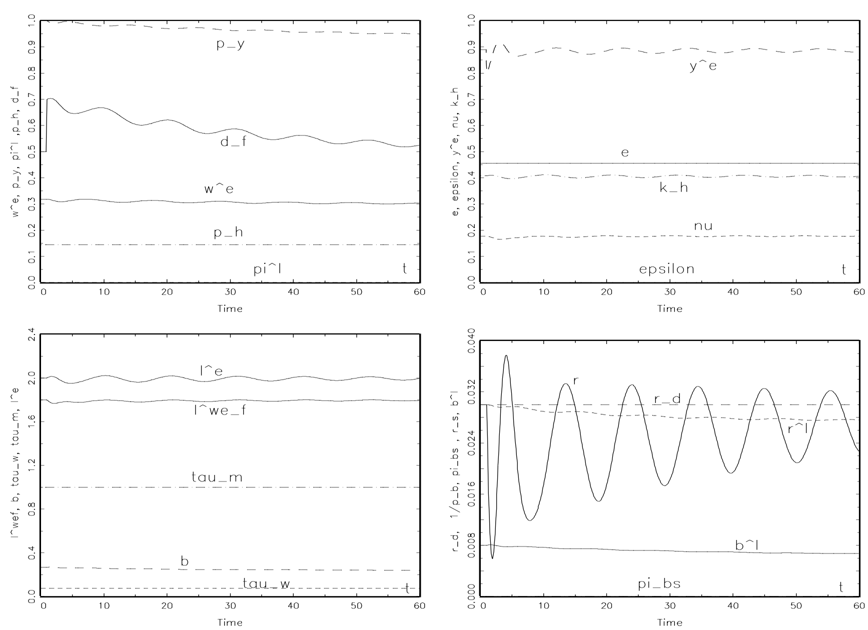


Figure 9:
Asymptotic stability in the 20D case.

Next, we increase the parameter reflecting price flexibility ($\beta_p = 0.35$) and indeed get a situation where the steady state is no longer attracting. We stress that monetary policy (the stabilizing Keynes effect) is needed in order to get this only slightly explosive situation. However, the type of monetary policy that is assumed seems to be too weak here to enforce again convergence to the steady state.

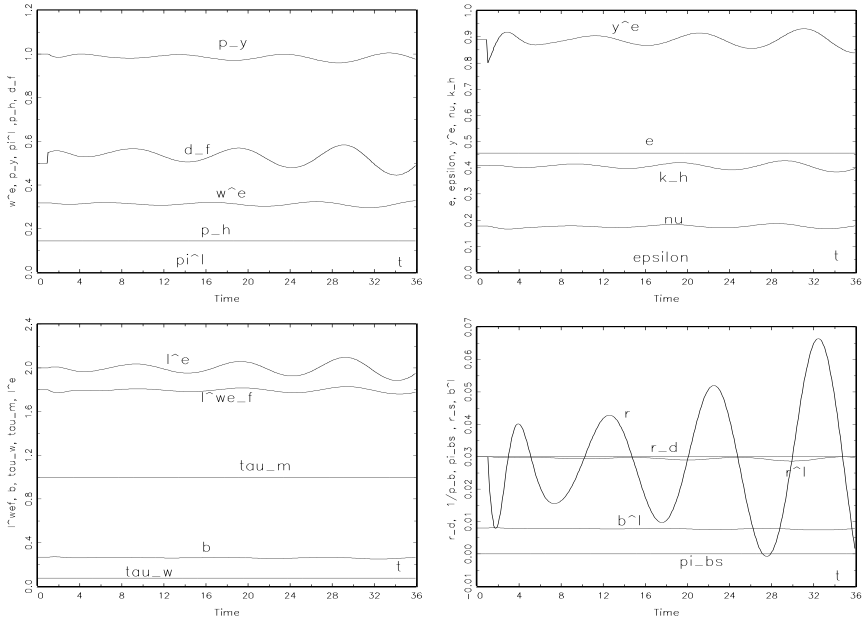


Figure 10:
Destabilizing price flexibility.

Next, we consider a case where there is some sort of isolated debt deflation, over the horizon shown, coupled with declining government debt and corresponding rates of interest. The parameters specific to this situation are:

$$\beta_p = 1, \beta_{r_2} = 1, \kappa_w = 1.$$

There are however no real effects visible over the horizon shown which only follow later on when the situation becomes more and more extreme.

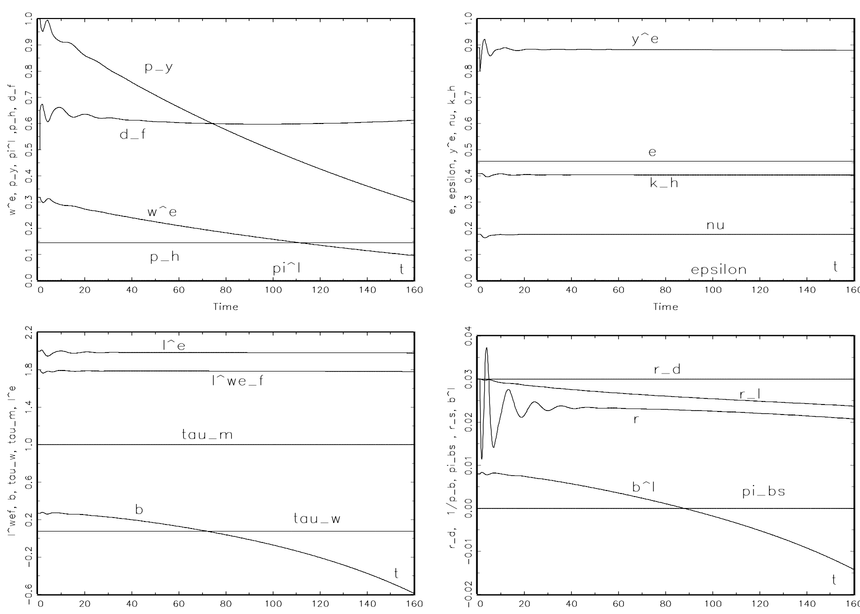


Figure 11:
Pure debt deflation.

The final situation presented in this subsection is given by figure 12 where the deflationary process just considered is interrupted from time to time by positive price shocks which stop the monotonic development shown in figure 11, decrease the real debt of firms and add fluctuations to the real magnitudes of figure 11. The parameters (and the full program) of this numerical example are provided in appendix 2 of this paper.

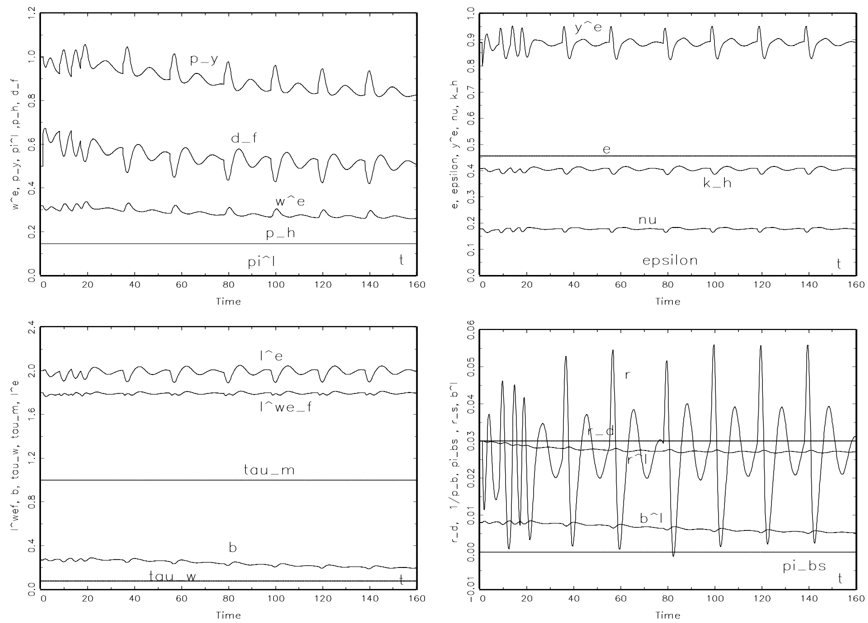


Figure 12:
Positive price shocks in order to stop debt deflation.

These few numerical examples of the working of the 20D dynamics (still with a simplifying choice of parameter values) show that much remains to be done for a proper demonstration of the consequences of debt deflation in a fully specified Keynesian model of monetary growth. Such investigations, which demand more numerical tools and more thorough parameter specifications (also with respect to empirically observed parameter sizes) must however here be left for future research.

7 Summary

We have applied in this paper the integrated Keynesian 18D dynamics of Chiarella, Flaschel and Zhu (1998a), with their prices and quantities adjustment processes, their growth laws, asset market descriptions and fiscal and monetary policy rules, to the problem of describing and investigating situations where high debt of firms becomes combined with deflationary processes on the goods market, leading to falling profitability when there is no accompanying sufficiently large fall in real wages.[35] To achieve this we have assumed as polar case to this earlier paper that firms use debt in the place of equities to finance their investment expenditures (fixed business investment and inventories) and have derived the growth law of the debt to capital ratio from the budget equation of firms. In contrast to the very

[35]real wages may even rise in such situations if prices fall faster than nominal wages.

where firms basically had no retained earnings, we now have pure profits of firms (over and above their debt service and factor costs) which in their relation to financial rates of return determine their investment plans. Though wealth effects on consumption and asset holdings are lacking in both of the considered dynamics we have in the present paper that the level of debt influences economic activity via investment behavior and thus may significantly influence the fluctuating growth patterns this model type generally gives rise to. Using loans in the place of equities implies that the rate of interest on loans has to be added to the endogenous variables of the model which has been done in this paper in the simplest way possible, by assuming that it adjusts to the long-term rate of interest on government bonds with a given time delay. The 18d dynamics thereby became a 20D dynamics which served as point of reference for various types of simpler dynamics we considered in this paper.

The most basic type of debt accumulation in a growing economy was obtained in the paper by making use of Keen's (1999) extension of the Goodwin (1967) growth cycle model which allows in addition to the reinvestment of profits by firms also for debt financed investment in this supply driven growth model thereby extending the dynamic interaction of the share of wages with real capital accumulation by the law of motion for the debt to capital ratio which feeds back into the real part of the dynamics via the pure rate of profit that it defines. This basic situation was investigated both analytically and numerically and gave rise to local stability assertions as well as global instabilities, depending on the size of the shock applied to the debt capital ratio. Integrating debt financing into the Goodwin growth cycle therefore gives rise to a new phenomenon, the occurrence of corridor stability, in this classical model of fluctuating growth.

Yet, in this extended approach, debt accumulation occurs without the possibility that firms have to face falling output prices simultaneously, a possibility that is not easily incorporated into a model where there is full capacity growth. In view of the established general 20D model we have therefore integrated as a next step into the 3D dynamics a demand constraint for firms on the market for goods, reflecting two basic goods market characteristics of the general case given by a negative impact effect of both real wages and debt on this demand constraint. Using this shortcut to a full description of goods market adjustment processes of the 20D case, we could then make use of the price Phillips curve of the 20D case in order to add as fourth law of motion to the 3D case a theory of price inflation based on demand pressure and cost push elements. In this extended 4D model, we could again show asymptotic stability of the steady state for sluggishly adjusting price levels and, providing a particular type of scenario for debt deflation, instability for price flexibility chosen sufficiently large. Furthermore, if wages do not fall to a sufficient degree, the possibility of debt deflation could be demonstrated and policies that possibly could stop such an outcome were sketched (again analytically as well as numerically).

This decisive step away from supply side driven capital accumulation to demand side determined growth patterns was however insofar preliminary as the shortcut of the feedback chain leading from expected demand to actual output to aggregate demand and back to expected demand is not an exact representation of this chain, even if all adjustment processes in this chain occur with infinite speed. The full feedback chain must therefore be approached if Keynesian growth is formulated as it should be formulated, with sluggish prices as well as quantities adjustments. In this respect this paper has offered however only a few preliminary numerical illustrations which also down played important, but for the current question not central, aspects of the general model, namely state activities (up to the use of a Taylor type monetary policy rule), asset market behavior, international aspects and the housing sector. As in the 4D dynamics we therefore concentrated in these examples on Rose type real wage dynamics and Fisher type debt deflations, which both stress the destabilizing potential of price flexibility in depressed situations due to the adverse effects on real wages and real debt. With the fully integrated 20D dynamics as a perspective we thus could show how the question of debt deflation may be approached with respect to monetary growth models of applicable nature, but must also admit here that much remains to be done in order to get a deeper understanding of processes of debt deflation, which, as shown, are currently an

We will return to these issues in future investigations of the general 20D model where more advanced mathematical tools will be used to determine with respect to speed of adjustment parameters regions of stability and boundaries where stability gets lost, basins of attraction and more. In this way we in particular hope to contribute to the understanding of the adjustment features of macrodynamical models, for the USA, Germany, Australia and other countries, that are actually applied.

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Appendix 1: Notation

The following list of symbols contains only domestic variables and parameters. Foreign magnitudes are defined analogously and are indicated by an asterisk ($*$).

A. Statically or dynamically endogenous variables:

Y	Output of the domestic good
Y^d	Aggregate demand for the domestic good
Y^p	Potential output of the domestic good
Y^e	Expected sales for the domestic good
Y_w^{Dn}	Nominal disposable income of workers
Y_c^{Dn}	Nominal disposable income of asset holders
L_1	Population aged 16 – 65
L_2	Population aged 66 – ...
L_0	Population aged 0 – 14
L^d	Total employment of the employed
L_f^d	Total employment of the work force of firms
$L_g^d = L_g^w$	Total government employment (= public work force)
L_f^w	Work force of firms
L^w	Total active work force
V_f^w	Employment rate of those employed in the private sector
$V = L^d/L$	Rate of employment ($\bar{V}$ the employment–complement of the NAIRU)
C_w	Real goods consumption of workers
C_c	Real goods consumption of asset owners
C_h^s	Supply of dwelling services
C_h^d	Demand for dwelling services
S_c^n	Nominal savings of asset holders
I	Gross business fixed investment
I_h	Gross fixed housing investment
$\mathcal{I}$	Planned inventory investment
N	Actual inventories
N^d	Desired inventories
r	Nominal short-term rate of interest (price of bonds $p_b = 1$)
r_l	Nominal long-term rate of interest (price of bonds $p_b = 1/r_l$)
$\pi_b = \hat{p}_b^e$	expected appreciation in the price of long-term domestic bonds
T^n	Nominal (real) taxes
G	Real government expenditure
ρ^e	Expected rate of profit of firms
ρ^a	Actual rate of profit of firms
ρ_h	Actual rate of return for housing services
K	Capital stock
K_h	Capital stock in the housing sector
w^b	Nominal wages including payroll tax
w	Nominal wages before taxes
w^u	Unemployment benefit per unemployed
w^r	Pension rate
p_y	Price level of domestic goods
p_x	Price level of export goods in domestic currency
p_m	Price level of import goods in domestic currency including taxation
p_h	Rent per unit of dwelling
$\pi^l = \hat{p}_y^e$	Medium-run expected rate of inflation
$p_b = 1/r_l$	Price of long-term bonds
$\pi_{bs} = \hat{p}_b^e$	Expected rate of bond price inflation
e	Exchange rate (units of domestic currency per unit of foreign currency: A$/$)
$\epsilon = \hat{e}^e$	Expected rate of change of the exchange rate
L	Labor supply
B	Stock of domestic short-term bonds

B_c	Short-term debt held by asset owners
B^l	Stock of domestic long-term bonds, of which B_1^l are held by domestic asset-holders and B_1^{l*} by foreigners
B_2^l	Foreign bonds held by domestic asset-holders
τ_m	Tax rates on imported commodities
X	Exports
J^d	Imports
τ_w	tax rate on wages, pensions and unemployment benefits
g_k	Actual rate of growth of the capital stock K
g_h	Actual rate of growth of the housing capital stock K_h
c_h^g	Actual consumption of goods per unit of capital
c_h^w	Actual consumption of housing services per unit of capital
d_g	Actual public debt / output ratio
D_f	Actual nominal debt (loans) of firms
r_d	Interest rate on the ratio of firms
d_f	Actual debt / output ratio of firms
l_y	Actual labor intensity (subject to Harrod neutral technical change)
l_f^{de}	Actual employment of firms per unit of capital
l_f^{we}	Actual labor force of firms per unit of capital
l^{de}	Actual employment (including the government sector) per unit of capital
l_g^{we}	Actual labor force of the government sector per unit of capital

B. Parameters[36]

δ	Depreciation rate of the capital stock of firms
δ_h	Depreciation rate in the housing sector
n	Natural growth rate of the labor force
n_l	Rate of Harrod neutral technical change
α_i^j	All α-expressions: behavioral or other parameters
α_s	Proportion of adaptively formed expectations
β_x	All β-expressions: adjustment speeds
γ	Steady growth rate in the rest of the world (here $= n + n_l$)
$\bar{V}$	NAIRU employment rate on the external labor market
$\bar{V}_f^w$	Normal employment rate of the employed
$\bar{U}$	Normal rate of capacity utilization of firms
$\bar{U}_h$	Normal rate of capacity utilzation of the stock of houses
κ_w, κ_p	Weights of short– and long–run inflation ($\kappa_w \kappa_p \neq 1$)
κ	$= (1 - \kappa_w \kappa_p)^{-1}$
y^p	Output–capital ratio
x_y	Export-output ratio
l_y^e	Actual labor intensity (in efficiency units)
j_y	Import-output ratio
$\bar{d}_g$	Desired public debt / output ratio
τ_c	Tax rates on profit, rent and interest ($\tau_c^* =$ ßtau_c)
τ_p	Payroll tax
c_1	Propensity to consume goods (out of wages)
c_2	Propensity to consume housing services (out of wages)

C. Further notation

$\dot{x}$	Time derivative of a variable x
$\widehat{x}$	Growth rate of x
$r_o, etc.$	Steady state values
$y = Y/K, etc.$	Real variables in intensive form
$D_f = D_f/(p_y K), etc.$	Nominal variables in intensive form
$\nu = N/K$	Inventory-capital ratio

[36] All parameters that follow represent positive magnitudes.