World Real Interest Rates and Business Cycles in Open Economies: A Multiple Shock Approach*

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Abstract: While the world real interest rate is potentially an important mechanism for transmitting international shocks to small open economies, much of the recent quantitative and empirical research finds little evidence that it significantly affects output, investment, and the trade balance. We re-examine the importance of world real interest rate shocks using an approach that reverses the standard real business cycle methodology. Rather than specifying the stochastic processes for the shocks, and then solving and simulating a small open economy real business cycle model to evaluate how much and how well these shocks explain business cycle dynamics, we use the model to back out the shocks which ensure a perfect fit with the model’s (observable) endogenous variables. We then use variance decompositions to examine the importance of each shock. We apply this methodology to Canada and find that world real interest rate shocks can play an important role in explaining the cyclical variation in a small open economy. In particular, they explain up to 23 percent of fluctuations in output and more than half of the fluctuations in the trade balance.

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1. Introduction

The world real interest rate is a primary mechanism by which foreign shocks are transmitted to small open economies. Changes in the world real interest rate can affect behavior along many margins: they affect households by generating wealth effects, intertemporal substitution effects, and portfolio allocation effects, and they affect firms by altering incentives for domestic investment. It is surprising, then, that much of the existing empirical literature finds that world real interest rate movements are not important in explaining the dynamics of small open economies. This literature (see for example, Mendoza (1991), Correia, Neves, and Rebelo (1992, 1995), and Schmitt-Grohe (1998)) finds that world real interest rate shocks have small effects on output, consumption, and labor hours, and even more surprisingly, on investment, the trade balance, and net foreign asset holdings.

In obtaining these findings, the authors mentioned above follow the “standard” international real business cycle approach. They build a dynamic stochastic model of a small open economy. Then they parameterize the model, including the processes for the stochastic shocks – one of which is the world real interest rate. Finally, they simulate the model and/or conduct impulse responses to evaluate the role of interest rate shocks.

There are however, three difficulties with this approach. First, there is no consensus on a good proxy for world real interest rates. A wide variety of nominal interest rates, price indices, and inflation expectations have been used to construct measures of world real interest rates. The 3-month U.S. T-Bill rate, the rate of return on the S&P 500, the LIBOR rate, as well as weighted average of several countries’ T-Bill rates, have been employed as nominal interest rates, for example. Similarly, the consumer price index, GNP, and GDP deflators have been employed as measures of the price level.\footnote{Mendoza (1991), Schmitt-Grohe (1998), van Wincoop (1993), Beaudry and Guay (1996), and Barro and Sala-i-Martin (1990) use the U.S. and other countries’ 3-month T-bill rate. Schmitt-Grohe (1998) and Correia, Neves, and Rebelo (1992, 1993) use the S&P 500 index. Gagnon and Unferth (1996) use the Euro-market interest rates on certificates of deposit. Kose (1998) and Senhadji (1998) use the LIBOR rate. With respect to prices, van Wincoop (1993) and Barro and Sala-i-Martin (BSM) 1990) use the CPI, Beaudry and Guay (1996) use the GNP deflator, and Schmitt-Grohe (1998) uses the GDP deflator. For modeling inflation expectations, the Livingston Survey, as well as many ARMA and AR specifications have been employed. Obstfeld and Rogoff (1995), while discussing the issues.} These different measures are not necessarily correlated with each other, as Table 1 shows for four ex ante real interest rates. Half of the correlations are less than 0.25. Second, as discussed extensively in Ingram, Kocherlakota, and Savin (IKS) (1994a, 1994b, 1997) models in which the number of unobservable exogenous shocks is less than the number of observable endogenous variables are singular. This is because the model implies that some of the observable variables are related deterministically, which is fundamentally inconsistent with the data. Hence, there are an infinity of ways in which the importance of shocks – even orthogonal shocks – in driving business...
cycles can be calculated. Finally, in any model with model shocks, it is impossible to determine the impact of any single shock, because these shocks are in fact correlated in the data. For example, Baxter and Crucini (1993) find that the assumption that the world interest rate is uncorrelated with shocks in a small open economy is “empirically indefensible”. At best, then, only a range of estimates – depending on the ordering of the shocks – can be obtained on the importance of world interest rate shocks.

The purpose of this paper is to pursue an alternative methodology to assessing the empirical importance of world real interest rates on small economies. We begin with a standard dynamic stochastic small open economy model in which shocks to world interest rates and domestic productivity are augmented with shocks to depreciation and preferences. We then use the model’s Euler equations, data on the model’s endogenous variables, as well as estimated decision rules for the capital stock and net foreign assets, to recover the exogenous shocks implied by the model and the data. In other words, our methodology reverses the standard approach. Rather than specifying a shock process and using the model to solve for the endogenous variables, we let the model and the endogenous variables tell us the exogenous shocks – including the world real interest rate – that are consistent with the model. Finally, we perform variance decompositions on the shocks backed out from the model. By varying the ordering of the shocks, we generate a range of estimates on the importance of each of the shocks.

Our approach deals with all three difficulties highlighted above. First, because we back out the real interest rate shocks, we avoid the problems associated with calculating the appropriate world real interest rate. Second, because our model is nonsingular, we can evaluate the importance of the world real interest rate in businesses cycles without violating any relationships implied by the model. Third, by examining all possible orderings of shocks we do not take a particular stance on the relationship between or orthogonality of the shocks.

We apply our methodology to quarterly Canadian data from 1961 to 1996. Our backed-out real interest rate measure is quite different from the other measures; this finding is similar to findings in Beaudry-Guay (1996) and vanWincoop (1993). Our variance decompositions indicate the world

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2Our methodology draws from work by Ingram, Kocherlakota, and Savin (1994a, 1994b). This methodology, which finds its roots in the works of Hall (1986) and Parkin (1988), has recently been implemented in a variety of settings in the dynamic stochastic macroeconomic literature: Ingram, Kocherlakota, and Savin (IKS) (1994a) back out exogenous shocks of a nonsingular version of the standard closed economy real business cycle model to examine the importance of the productivity shocks. To study the cyclical behavior of home production IKS (1997) generate realizations of market and non-market hours. Using a similar methodology, Baxter and King (1998) back out the realizations of technology and preference shocks, and Ambler and Paguet (1994) back out the time series of

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real interest rate shocks can play a significant role in explaining Canadian business cycle fluctuations. If world interest rates shocks are ordered first, they explain 23 percent of Canada’s output variation. They also account for a significant fraction of variation in Canada’s external balances: up to 70 (52) percent of the variation in asset holdings (trade balance) is explained by these shocks. We also find that the shocks to preferences and depreciation are important in understanding the dynamics of open economies. These findings contrast with the results of Mendoza (1991) and Schmitt-Grohe (1998). Their results correspond to the lower bound of our range of estimates, which is the estimates obtained when real interest rates are not ordered first in the variance decompositions.

The rest of this paper is organized as follows: in section 2, we present our dynamic stochastic small open economy model. In section 3, we calibrate the model to Canada and present our methodology on recovering the exogenous shocks. Our results are presented in section 4, and section 5 provides some concluding remarks.

2. The Model

Our model builds on Mendoza’s (1991) classic small open economy real business cycle model. The economy is populated by a large number of infinitely lived, identical households. The representative household maximizes expected lifetime utility given by:

$$\frac{\beta^t E_0}{1 - \gamma} \left[ c_t^{\eta \theta} l_t^{1 - \theta} \right]^{1 - \gamma} - 1$$

(1)

where $c_t$ is consumption in period $t$, $l_t$ is leisure, $\eta_t$ is a preference shock, $\theta$ is the consumption share parameter $\beta$ is the discount factor and $\gamma$ is the household’s coefficient of relative risk aversion. Note that we use a constant discount factor, rather than the endogenous discount factor in Mendoza (1991). Endogenous discount factors are used to ensure that models of small open economies with time separable preferences have a stationary steady state with accurate, well-defined dynamics around that steady-state. However, the problem of solving models with non-stationary steady states does not apply to our framework, because it does not require us to solve for the steady-state.

The economy produces an internationally tradable good, $y_t$, according to:
\[ y_t = z_t k_t^\alpha n_t^{1-\alpha}, \quad 0 < \alpha < 1 \]  

(2)

where \( k_t \) is the domestic capital stock at the beginning of the period \( t \), \( n_t \) is labor hours, \( \alpha \) governs the share of income accruing to capital, and \( z_t \) is the realization of the technology shock.

Following Baxter and Crucini (1993), we specify the following law of motion for capital:

\[ k_{t+1} = (1 - \delta_t) k_t + \phi \left( \frac{i_t}{k_t} \right) \]  

(3)

where \( i_t \) is investment and \( \delta_t \) is an exogenous depreciation shock. \( \phi(.) \) represents the standard adjustment cost function, with \( \phi(.) > 0 \), \( \phi(.)' > 0 \), and \( \phi(.)'' < 0 \).

The representative household has access to world capital markets to borrow and lend foreign financial assets \((A_t)\). Asset holdings evolve according to:

\[ A_{t+1} = nx_t + (1 + r_t) A_t \]  

(4)

where \( nx_t \) is net exports measured in units of the domestic consumption good, and \( r_t \) is the exogenously determined stochastic risk-free real interest rate from period \((t-1)\) to \( t \). To prevent the representative household from playing a Ponzi game, we impose the condition:

\[ \lim_{t \to \infty} E_0 (A_{t+1} \frac{1}{(1 + r_t)}) = 0 \]  

(5)

Finally, the aggregate resource constraint is:

\[ c_t + i_t + nx_t \leq y_t, \]  

(6)

In our model there are four exogenous shocks, the world real interest rate and a productivity shock – which are the shocks in Mendoza’s model – as well as a preference shock and a depreciation shock. Because our model has four endogenous variables, consumption, investment, labor hours, and net foreign assets, we need four exogenous shocks to insure that the model is non-singular. Singular models, that is, models with fewer exogenous unobservable variables than endogenous observable variables, imply deterministic relationships between the observable variables. These relationships are clearly violated in the data (any time the variance-covariance matrix is non-singular).\(^4\)

IKS refer to such models as singular and state that “using a singular model when the variance-covariance matrix of the data is nonsingular is equivalent to solving a set of inconsistent linear equations; there is no solution.” In the language of Ingram, Koehlerlakota, and Savin (1994a), Mendoza’s model is a singular one.

The representative household in this economy faces the following optimization problem:

\(^4\) Our study is the first one in its class using depreciation and preference shocks in a small open economy framework. In a two country open economy business cycle model, Stockman and Tesar (1995) find that introduction of preference shocks significantly improves the fit of the model to the data. Ambler and Paguet (1994) find that introduction of the depreciation shocks into an otherwise standard closed economy real business cycle model gets the predictions of the model closer to the data. Greenwood, Hercowitz, and Huffman (1988) study a model where
\[
\max E_0 \prod_{r=0}^{\infty} \beta \left[ c_t^{\eta, \theta} i_t^{1-\theta} \right]^{1-\gamma} - 1 \frac{1-\gamma}{1-\gamma}
\]

s.t.
\[
c_t + i_t + A_{t+1} - (1 + r_t) A_t \leq z_t k_t^a n_t^{1-\alpha} \quad (\lambda_{1t})
\]
\[
k_{t+1} = (1 - \delta) k_t + \phi \left( \frac{i_t}{k_t} \right) k_t \quad (\lambda_{2t})
\]

where \( \lambda_{1t} \) and \( \lambda_{2t} \) are the Lagrange multipliers. First order conditions are:

\[
c_t : \eta \theta c_t^{\eta, \theta (1-\gamma)^{-1}} (1 - n_t)^{(1-\theta)(1-\gamma)} - \lambda_{2t} = 0 \quad (7)
\]
\[
n_t : -c_t^{\eta, \theta (1-\gamma)} (1 - \theta)(1 - n_t)^{(1-\theta)(1-\gamma)^{-1}} + \lambda_{1t} z_t k_t^a (1 - \alpha) n_t^{-\alpha} = 0 \quad (8)
\]
\[
i_t : -\lambda_{1t} + \frac{f \phi}{f_i} \lambda_{2t} = 0 \quad (9)
\]
\[
k_{t+1} : -\lambda_{2t} + \beta E_t \left[ \lambda_{1t} + \alpha \frac{y_{t+1}}{k_{t+1}} + \lambda_{2t} \{ \phi \left( \frac{i_{t+1}}{k_{t+1}} \right) - \frac{i_{t+1}}{k_{t+1}} \frac{f \phi}{k_{t+1}} + (1 - \delta_{t+1}) \} \right] = 0 \quad (10)
\]
\[
A_{t+1} : -\lambda_{1t} + \beta E_t [ \lambda_{1t} (1 + r_{t+1}) ] = 0 \quad (11)
\]

Equations (7) and (8) yield

\[
\frac{(1 - \alpha) y_t}{n_t} = \frac{c_t}{(1 - n_t)} \frac{(1 - \theta)}{\eta \theta} \quad (12)
\]

while (9) and (10) yield

\[
\phi \lambda_{1t} = \beta E_t \left[ \lambda_{1t+1} + \phi \lambda_{1t+1} \left\{ \phi \left( \frac{i_{t+1}}{k_{t+1}} \right) - \frac{i_{t+1}}{k_{t+1}} \frac{f \phi}{k_{t+1}} + (1 - \delta_{t+1}) \right\} \right] \quad (13)
\]

where \( \phi_t = \left( \frac{-f \phi}{f_i} \right)^{-1} \).

the marginal efficiency of investment is a stochastic shock that is similar to the depreciation shocks we consider here.
Equation (11) governs the dynamics of foreign asset holdings. Equation (12) equates the marginal rate of substitution between consumption and leisure to the marginal product of labor. Equation (13) is the intertemporal efficiency condition pertaining to the domestic capital stock.

3. Recovering the Exogenous Shocks

3.1. Calibration

Since it is impossible to estimate both the structural parameters of the model (here $\alpha$, $\theta$, $\beta$, $\gamma$ and the parameters of $\phi(.)$) and the realizations of exogenous unobservables (here $z_t$, $\eta_t$, $\delta_t$ and $r_t$), we calibrate the structural parameters prior to estimating shock realizations.\(^5\) The consumption share parameter $\theta$, is chosen to be consistent with average labor hours allocation of 30 percent of the endowment of non-sleeping time to market activities and is equal to 0.34. The risk aversion parameter, $\gamma$, is set to 1.5, the share of capital income in the production, $\alpha$, is equal to 0.32 following Mendoza (1991) and Schmitt-Grohe (1998). The initial value of the depreciation rate is equal to 0.025, and the discount factor, $\beta$, is set to 0.988, both are widely used figures in the real business cycle literature (see Ingram, Kocherlakota, and Savin (1994a.)

We specify the following functional form for the adjustment cost function:

$$\phi\left(\frac{i}{k}\right) = \omega_1\left(\frac{i}{k} - \omega_2\right)^{\omega_3}$$

Adjustment cost parameters $\omega_1$, $\omega_2$, and $\omega_3$ are chosen so that the deterministic steady state of the model is the same as that without adjustment costs. This implies that $\phi(i/k) = i/k$, and $\phi(i/k)' = 1$. In addition, $\phi(i/k)^{\prime\prime}$ is set so that the elasticity of the marginal adjustment cost function, $\xi = -(\phi'/\phi)(i/k)^{-1}$ is equal to 15. This is the benchmark value used by Baxter and Crucini (1993). Together these three conditions determine the values of $\omega_1$, $\omega_2$, and $\omega_3$. We examine the sensitivity of our results to a wide range of changes in the calibrated parameters of the model in section 4.

3.2. Estimation of the Policy Functions

The typical approach for analyzing such a model is to calibrate its parameters, specify forcing processes of exogenous shocks and employ an approximation method to solve the model around its steady state. Then, the solution is used to generate artificial data of the endogenous variables to

evaluate the fit of the model to actual data. Following Ingram, Kocherlakota, and Savin (1994a, 1994b), our approach reverses this methodology: rather than produce simulated time series for endogenous variables, we use observable data of the endogenous variables as a solution to the model, and utilize orthogonality conditions implied by the Euler equations to uncover the exogenous shocks \( \{z_t, \theta_t, \delta_t, r_t\} \) consistent with the observable endogenous variables. Specifically, let \( s_t \) represent the current state of the economy. To implement our procedure, we approximate the policy functions for capital and asset holdings, \( k_{t+1} = k(s_t) \) and \( A_{t+1} = A(s_t) \), by first specifying the functional forms \( k_{t+1} = k^*(s_t, \psi_1) \) and \( A_{t+1} = A^*(s_t, \psi_2) \). Then, we estimate the parameter vectors \( \psi_1 \) and \( \psi_2 \).

We replace equations (11), (12), and (13) with their sample analogs to estimate \( \psi_1 \) and \( \psi_2 \):

\[
\frac{1}{T} \sum_{t=1}^{T} -\beta \lambda_{2t} \alpha y_{t+1} + \lambda_{2t} \{\phi(i_{t+1}) - \frac{i_{t+1}}{k_{t+1}} \frac{f \phi}{f k_{t+1}} + (1-\delta_{t+1})\} \psi_1 = 1 \tag{14}
\]

\[
\frac{1}{T} \sum_{t=1}^{T} \beta \frac{\lambda_{1t}}{\lambda_{1t-1}} (A_{t+1} - nx_t) Z_{2t} = 1 \tag{15}
\]

\[
\eta_t = \frac{(1-\theta)}{\theta} \frac{n_t}{1-n_t} \frac{c_t}{y_t (1-\alpha)} \tag{16}
\]

where \( \lambda_{1t} = c_t \theta (1-\gamma)(1-\theta)(1-n_t)^{1-\theta}y_t^{1-\gamma}n_t^{-\gamma} \) and \( \lambda_{2t} = \frac{\phi}{f} \). \( Z_{1t} \) and \( Z_{2t} \) are \((p_1 \times 1)\) and \((p_2 \times 1)\) vectors of instruments. Equation (16) identifies the series of preference shocks \( (\eta_t) \). To obtain the other series, we replace \( k_{t+1} \) and \( A_{t+1} \) with \( k^*(s_t, \psi_1) \) and \( A^*(s_t, \psi_2) \). For an arbitrary choice of \( \psi_1 \) and \( \psi_2 \) and given initial values of asset holdings and the stock of capital, we have sufficient structure to “back out” a series of shocks, \( \{z_t, \theta_t, \delta_t, r_t\} \). Given the calibrated parameters, observable series and the implied capital stock series, the \( z_t \) series comes from equation (2) and equation (3) yields the \( \delta_t \) series. Equation (4) in conjunction with the implied series of asset holdings yields the \( r_t \) series. Put differently, each choice of \( \psi_1 \) and \( \psi_2 \) implies a series.

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\( ^6 \) See Ingram (1995) for a brief explanation of this method and several other issues related to the solution and estimation of stochastic dynamic general equilibrium models.
of shocks. Our goal in estimating $\psi_1$ and $\psi_2$ is to choose values such that equations (14) and (15) hold as closely as possible.

We employ Hansen (1982) two step generalized method of moments estimator (GMM) to estimate the parameters $\psi_1$ from equation (14) and $\psi_2$ from equation (15). To implement the procedure, we specify the following approximations to the policy functions $k_{t+1} = k^*(s_t, \psi_1)$ and $A_{t+1} = A^*(s_t, \psi_2)$:

\[
\ln(k_{t+1}) = \psi_{11} + \psi_{12} \ln(c_t) + \psi_{13} \ln(\eta_t) + \psi_{14} \ln(z_t) + \psi_{15} \ln(k_t) + \psi_{16} (\delta_{t-1})
\]

\[
\ln(A_{t+1}) = \psi_{11} \ln(c_t) + \psi_{22} (r_t) + \psi_{23} \ln(z_t) + \psi_{24} \ln(\eta_t) + \psi_{25} \delta_t + \psi_{26} \ln(i_t)
\]

These approximate policy functions are computationally convenient yet include most relevant state variables. In each case, the effect of additional lagged variables is accounted for by the inclusion of $c_t$. The exclusion of the real world interest rate in the equation allows us to decrease the dimensionality of the problem; we are able to solve for $\psi_1$ and $\psi_2$ sequentially rather than simultaneously. The instrument vectors chosen are:

\[
Z_{it} = \left[ \varepsilon_{1,t-1}, \varepsilon_{1,t-2}, \hat{c}_t, \hat{c}_{t-1}, \hat{\epsilon}_{t-1}, \hat{\epsilon}_{t-2}, \hat{\epsilon}_{t-1}, \hat{\epsilon}_{t-2}, \delta_{t-1}, \hat{h}_{t}, \hat{h}_{t-1}, n\hat{x}_t \right]
\]

\[
Z_{2t} = \left[ \varepsilon_{2,t-1}, \varepsilon_{2,t-2}, \hat{c}_t, \hat{c}_{t-1}, \hat{c}_{t-2}, \hat{\epsilon}_{t-1}, \hat{\epsilon}_{t-2}, \hat{\epsilon}_{t-1}, \hat{\epsilon}_{t-2}, \delta_{t-1}, \hat{h}_{t}, \hat{h}_{t-1} \right]
\]

where $\hat{c}_t$ is the growth rate of $c$ in period $t$ and where $\varepsilon_{1,t-1}$ and $\varepsilon_{2,t-1}$ are the lagged error terms from the estimations.

3.3. Data

Quarterly values of $c_t$, $i_t$, and $nx_t$ for Canada from 1961 to 1996 are drawn from the IFS data. We seasonally adjust the data and convert it to real per capita values by adjusting for population growth, and using the GDP deflator implied by the real (1990 dollars) and nominal GDP series. Our consumption series, $c_t$, is household consumption expenditures, the investment series, $i_t$, is the sum of gross capital formation and inventory adjustments. Net exports, $nx_t$, is the difference between exports of goods and services and imports of goods and services. To be consistent with the model, we

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7 We choose initial values to be consistent with the data of the Canadian economy in 1961. We truncate the first 8 data points from recovered shock series so that the remaining subset of the series is not sensitive to our choice of starting values.

8 Smith and Zin (1997) use the GMM to estimate the policy functions, and generate realizations of endogenous variables in their real business cycle model.

9 Series other than the hours are seasonally adjusted by the data source. To seasonally adjust the quarterly hours series we subtract from each observation the mean for observations in the relevant quarter and add back in the mean of all observations.
exclude government expenditures from our output measure. So, output, $y_t$, is the sum of $c_t$, $i_t$, and $nx_t$. Quarterly labor hours, population, and employment data are taken from the OECD Intersectoral Database (ISDB). Following King, Plosser and Rebelo (1988) total hours worked, $n_t$, is defined as the product of hours worked per week in the manufacturing sector and the employment rate normalized by the weekly time endowment.

### 4. Results

In this section, we first examine the results of our estimation exercise by focusing on the moments of the model produced shocks. We also provide an intuitive account of the mechanics of our model economy. Next, we study the moments of several world real interest rate measures in the data, and compare these with the moment implications of the model. Then, we examine the importance of exogenous shocks in inducing business cycles in our model economy.

#### 4.1. Moments of the Shocks in the Model

The coefficients from this estimation and the associated standard errors are provided in table 1. Since the number of parameters estimated in each case is less than the number of instruments, we are able to conduct a test of the over-identifying restrictions. The null hypothesis is that the over-identifying restrictions are satisfied. In each case, there is not sufficient evidence to reject this hypothesis at conventional statistical significance levels.

Figure 1 shows the series of shocks implied by the model using the benchmark parameterization and several data series. Figure 1.1 shows that the series of technology shocks mimics loosely the output series though the latter grows more rapidly. From figure 1.2, we see that the depreciation series is highly variable and occasionally negative. The series of preference shocks mirrors the hours series over much of the series and they share a similar upward trend (figure 1.3). Figure 1.4 shows that the implied interest rate is positive only about 60 percent of the time, and shows little persistence.

Table 2 presents volatility and comovement properties of the estimated shocks. The most volatile shock is the world real interest rate. The depreciation shock is half as volatile while the preference and technology shocks are much less volatile. The technology shock is positively

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10 See Watson (1991), King, Plosser, Stock, and Watson (1991), and Beaudry and Guay (1996) for the same treatment of excluding government expenditures from aggregate output.

11 See Ambler and Paquet (1994) and Ingram, Kocherlakota, and Savin (1994a) for a discussion of occasionally negative depreciation and highly variable depreciation rates. They argue that a composite capital series represents many highly substitutable capital goods whose marginal productivities need not move together. Thus, there is substitution across capital types with fixed but differing depreciation rates and the composite depreciation rate can be highly variable.
correlated with the world real interest rate shock, and negatively correlated with the depreciation and preference shocks.\textsuperscript{13} There is a negative correlation between the technology shock and the depreciation shock suggesting that depreciation tends to be low when productivity is high. These results lend support to the findings by Baxter and Crucini (1993.) They find that fluctuations in the world real interest rate are correlated with domestic shocks in a two country model calibrated to represent a large and a small open economy. They claim that because of this, assuming the small open economy faces a constant world real interest rate or one uncorrelated with domestic shocks might induce misleading results. We reach the same conclusion from a different perspective: we estimate the exogenous shocks that exactly replicate the macroeconomic time series of a small open economy, and conclude that these shocks are correlated.

Contemporaneous correlations between the model produced shocks and macroeconomic aggregates are presented in table 3. There is a low positive correlation between the output and the world real interest rate. Depending on the foreign asset position of the economy, an increase in the world real interest rate can increase the capital income received abroad increasing both consumption and leisure. This is captured by the model as the positive (negative) correlation between the model produced interest rate and consumption (labor hours). In response to an increase in the world real interest rate, investment and labor hours go down. To reconcile this with a positive correlation (0.14) between aggregate output and the world real interest rate, notice that there is a relatively high positive correlation between the world real interest rate and the technology shocks produced by the model. If the economy receives a positive technology shock, and faces an increase in the world real interest rate at the same time, it is not surprising to see a positive correlation between the world real interest rate shock and output.

As one would expect, there is a positive correlation (0.38) between the world real interest rate shock and the net exports. There is a low positive correlation (0.15) between investment and the model produced depreciation shock. An increase in the depreciation of capital implies higher marginal product of capital, and investment increases to rebuild the capital stock. There is relatively high negative correlation (-0.67) between technology shock and depreciation shock. As depreciation goes down, and the economy receives positive productivity shocks, these two have a positive impact on the investment. Since the representative agent increases her labor supply to rebuild the capital stock in response to a sudden increase in the depreciation rate, the model predicts that there is positive correlation (0.50) between the depreciation shock and labor hours.

\textsuperscript{12} Following the standard practice in the real business cycle literature, we detrend the series using Hodrick and Prescott (HP) (1997) filter.
We examine the sensitivity of our results with respect to changes in the deep parameters of the model. In particular, we study whether the results in tables 2 and 3 change for reasonable changes in the risk aversion coefficient, the discount factor, the elasticity of marginal adjustment cost, and the share of capital income. In general, we find the results to be very robust. For example, changes in the parameters do not affect the signs of the correlations between the shocks and output fluctuations: the interest rate is weakly procyclical. While changes in the parameters affect the volatility of the shocks, their effects on the comovement properties of the shocks are quite small.

4.2. Moments of the Shocks in the Data

An important message in the previous section is that the exogenous shocks produced by the model are correlated. This implies that researchers constructing small open economy models to replicate the time series properties of macroeconomic aggregates should pay attention to the comovement of the external shocks (such as the world real interest rate shock) with the domestic shocks (such as productivity and preference shocks.) In this section, we demonstrate another concern with such experiments: when researchers choose a proxy for the world real interest rate shock from available data, there is no clear “best” choice, and each of the proxies used by various researchers is inconsistent with the world real interest rate implied by the model along some important dimensions.

We compare the real interest rate implied by the model with four different interest rate measures. First, the return on three-month U.S. T-bills is the measure used by Hercowitz (1986), Mendoza (1991), Gaudin and Yi (1996), and Schmitt-Grohe (1998) in their small open economy real business cycle models, and by Beaudry and Guay (1996) in their closed economy real business cycle model. A recent paper by Gagnon and Unferth (1995) suggests that the U.S. real interest rate shows large and persistent deviations from the world real interest estimate they construct. Considering this, we construct a second world interest rate measure using a weighted average of the returns on three-month T-bills of seven industrialized countries. In computing the weights, we used the PPP-adjusted numbers for real GDP reported by the Penn World Tables (PWT 5.6, 1994.) Barro and Sala-i Martin (1990) and van Wincoop (1993) employ a similar interest rate measure constructed as a weighted average of interest rates in developed economies.

13 Obstfeld and Rogoff (1995) argue that that the world real interest rate shocks partially capture the common supply shocks affecting all countries in the global economy.

14 These countries (interest rate series) are the U.S.A (3-month T-bill rate), the U.K. (3-month T-bill rate), Germany (deposit rate), Italy (discount rate), Japan (deposit rate), Canada (3-month T-bill rate), and Belgium (3-month T-bill rate.) To get the real interest rate, we deflate each country’s interest rate with the inflation rate in its own CPI. Then, we apply the weights to get the world real interest rate measure. Since France does not have the interest rate series dating back to 1961, we are unable to include France data. Gagnon and Unferth (1996) estimate the common component of the ex post real interest rate of nine countries. They use the 3-month and 12-month Euro-market interest rates on certificates of deposit.
Third, we consider the quarterly S&P500 total return index. This is the measure used by Correia, Neves, and Rebelo (1992, 1995), and Schmitt-Grohe (1998) in their small open economy models. This interest rate represents the return to a portfolio of common stocks in the 500 largest American companies. Fourth, we use the six-month LIBOR (the London Interbank Offer Rate) considering that this rate is adopted as a benchmark interest rate measure by international organizations and commercial banks when they provide loans.\footnote{See World Economic Outlook (1993, p. 83) for the use of this measure as a proxy for real cost of borrowing for developing economies. This rate is used by Senhadji (1998) and Kose (1998) in their small open economy models calibrated to represent a typical developing country.}

We consider ex-post as well as ex-ante real interest rates following Beaudry and Guay (1996).\footnote{Van Wincoop (1993), Hercowitz (1986) consider ex-post real interest rate. Gaudin and Yi (1996) consider ex-ante interest rate. Beaudry and Guay (1996) consider both ex-post and ex-ante series, and we think that it might helpful to see the behavior of both of these series as well.} Our quarterly inflation rate is computed using the U.S. CPI. Ex-post real interest rates are calculated by subtracting the quarterly inflation rate from the nominal interest rates. Ex-ante real interest rates are calculated by subtracting the previous year’s inflation from the current nominal interest rates.

Table 4 presents volatility of the world real interest rate measures in the data. The volatility does not differ much across different interest rate measures except that the S&P 500 return exhibits much higher volatility. The model produced real interest rate is almost 70 percent more volatile than the three interest rate measures in the data. However, the S&P real return is two times more volatile than the model produced interest rate. Van Wincoop (1993), using a multi-country real business cycle model, finds that the model is unable to generate real interest rate volatility close to that in the data. His real interest rate measure is similar to our weighted measure. In particular, he finds that the model produced interest rates are one-third to one-sixth as volatile as the actual interest rates. Unlike his results, our findings suggest that the volatility of the weighted interest rate measure is roughly half as volatile as the model produced real interest rate.\footnote{We also study the mean of the interest rate measures. The mean of the model produced world real interest rate is quite close to the mean values of the some of the interest rates in the data. In particular, the averages of the real U.S. rate and the weighted world interest rate are 1.01 and 1.35 percent respectively while the mean of the model produced world rate is 1.27 percent.}

In table 5 we examine the comovement between the main macro aggregates and the world real interest rate measures in the data. There is a low negative correlation between the most of the interest rate measures and the aggregate output. One exception to this regularity is the S&P 500 return that has a small positive correlation with the output. Our model predicts a positive correlation between the world real rate and the aggregate output. Similar to aggregate output, while the model predicts a positive correlation between consumption and the world real interest rate, there is a low
negative correlation between most of the interest rate measures and consumption in the data. All the interest rate measures are negatively correlated with investment, and with hours, and positively correlated with trade balance as predicted by the model.

To further document comovements between the data and interest rates, we present the lead, lag, and contemporaneous cross-correlations in Table 6. The model predicts low negative correlation between the lead interest rate and current output. Most of the interest rates in the data demonstrate the same low negative correlation. However, the model does not capture the negative correlation between the lead output and the current interest rate seen in the data.

Using a closed economy real business cycle model, Beaudry and Guay (1996) find that the model produced real interest rate is procyclical since technology shocks induce strong and sudden changes in both consumption and investment. In contrast, they observe a low negative correlation between the interest rate and aggregate output in the data. Our findings are in line with theirs, as our model (data) also predicts procyclical (countercyclical) world real interest rate.

While the model produced interest rate captures some important features of the interest rate measures previously used by researchers, there are notable differences. First, the model produced interest rate is more volatile than the observed interest rates, and some of the correlations with aggregate macroeconomic variables are not consistent with those of the interest rates in the data. Second, we point out that different types of world real interest rate proxies exhibit different time series behavior. In the previous section, we conclude that the model produced shocks are correlated. The findings presented in this section coupled with the result of correlated exogenous shocks suggest that the standard small open economy model requires multiple correlated shocks in addition to the world real interest rate and domestic productivity shocks to closely replicate the macroeconomic time series.

4.3. Variance Decompositions

Understanding the role of world real interest shocks in explaining macroeconomic fluctuations in small open economies is an important research topic in international macroeconomics literature. Some researchers have attempted to answer this question using singular small open economy models. The results of preceding sections suggest that these singular models are not suited to this task. Moreover, the conventional interest rate measures in the data are not fully compatible with the interest rate series produced by the model. A nonsingular model with multiple correlated shocks is required to examine the role of the interest rate shocks in inducing macroeconomic fluctuations in small open economies. Can our model economy shed some light on this? In this section, we use our model to study the importance of the world real interest rate shocks and domestic shocks in explaining business cycles in Canada.
Measuring the contribution of a particular shock to business cycle fluctuations in our multi-shock model is a non-trivial exercise since shocks are correlated with each other. When shocks are correlated with each other, the standard approach taken in the literature to determine the contribution of a particular shock to business cycle fluctuations can produce misleading results. We apply a variance decomposition method borrowed from the vector autoregression (VAR) literature to determine the relative importance of shocks in explaining business cycle fluctuations in our model.\footnote{For examples of the standard approach see Prescott (1986), and Aiyagari (1992). Ingram, Kocharlakota, and Savin (1994a, 1994b), Cochrane (1994), and King (1995) provide extensive discussions of the standard approach and its} We decompose the variances of macroeconomic variables into fractions explained by exogenous shocks. Let \((\hat{r}_t, \hat{z}_t, \hat{\delta}_t, \hat{\eta}_t)\) denote the vector of time series of these four shocks. The source of the variation in endogenous variables of the small open economy is fluctuations in current and lagged values of these shocks. Since the shocks are correlated with each other, the order of precedence of these shocks is crucial in determining their relative importance in explaining the variance of a particular variable.

To illustrate, the ordering \((\hat{r}_t, \hat{z}_t, \hat{\delta}_t, \hat{\eta}_t)\) implies that \(\hat{r}_t\) will affect the comovements between \(\hat{r}_t\) and any other shock, and \(\hat{z}_t\) is responsible for any comovements between \(\hat{z}_t\) and the remaining three shocks and so on. To capture the impact of a particular shock on other shocks, we first run the regression of \(\hat{z}_{t-L}\), where \(l\) is the number of lags, on the vector of \((\hat{r}_t, \hat{r}_{t-1}, ..., \hat{r}_{t-L})\), where \(L\) is the maximum number of lags. Let \(e'_{t-L}\) denote the residuals from this regression. The interpretation of \(e'_{t-L}\) is that it captures the fluctuations in \(\hat{r}_{t-L}\) that are not correlated with current, future or past fluctuations in the \(\hat{z}_t\). In the same fashion, we can run the regression of

\[
\hat{\delta}_{t-L} \quad \text{on} \quad (\hat{r}_t, \hat{r}_{t-1}, ..., \hat{r}_{t-L}, e'_{t-1}, e'_{t-L}) \\
\hat{\eta}_{t-L} \quad \text{on} \quad (\hat{r}_t, \hat{r}_{t-1}, ..., \hat{r}_{t-L}, e'_{t-1}, e'_{t-L}, e^\delta_{t}, e^\delta_{t-1}, ..., e^\delta_{t-L})
\]

Suppose that we would like to determine the contribution of different disturbances to output using this particular shock ordering. Then, we can get the variance decomposition of output into the four exogenous shocks by running the following regression,

\[
y_t = \sum_{l=0}^{L} b_{\epsilon_t l} \hat{z}_{t-l} + \sum_{l=0}^{L} b_{\epsilon\delta l} e'_{t-l} + \sum_{l=0}^{L} b_{\epsilon\delta l} e^\delta_{t-l} + \sum_{l=0}^{L} b_{\epsilon\eta l} e^\eta_{t-l} + \epsilon_t
\]

\footnote{For examples of the standard approach see Prescott (1986), and Aiyagari (1992). Ingram, Kocharlakota, and Savin (1994a, 1994b), Cochrane (1994), and King (1995) provide extensive discussions of the standard approach and its}
\[ y_t \cdots y_t^n + y_t^r + y_t^\delta + y_t^n + \epsilon_t, \]

As \( S \) takes very large values, the variance of \( \epsilon_t \) goes to zero since current and lagged values of the four shocks account for all of the variation in output fluctuations. The fraction of the variance of output explained by each shock is given by

\[ q^r = \frac{\text{var}(y_t^r)}{\text{var}(y_t)}, \quad q^\delta = \frac{\text{var}(y_t^\delta)}{\text{var}(y_t)}, \quad q^n = \frac{\text{var}(y_t^n)}{\text{var}(y_t)} \]

We repeat this procedure for all possible twenty-four orderings. The results of this exercise are presented in table 7. We present the maximum and minimum fraction of variance explained by each shock depending on the ordering. The small open economy assumption implies that the domestic shocks should not have any impact on the external shocks. Considering this, we then focus on the ordering when the interest rate shock precedes the other shocks. The results are highly sensitive to the ordering imposed on the shock structure. If we put the interest rate shock first, it explains almost 23 percent of output volatility. When we consider all possible orderings, depending on the ordering, the total contribution of the world real interest shock to output fluctuations ranges from a quarter of a percent to 23 percent. The productivity shock explains at most (least) 77 (25) percent of output fluctuations. Both depreciation shocks and preference shocks account for sizeable fractions of output volatility depending on the ordering we employ.

We consider the sources of fluctuations in other macroeconomic aggregates as well. The world real interest rate shocks play a smaller role in accounting for the consumption variation. In the previous section, we reported that there is a high correlation between the preference shocks and consumption fluctuations. The variance decompositions further support the relation between the two: the fraction of consumption volatility explained by preference shocks is between 11 percent and 59 percent. The world real interest shocks, depending on the information ordering adopted, can explain up to 35 (38) percent of investment (labor hours) fluctuations. Examining only the maximum contribution of the shocks reveals that while productivity shocks explain the bulk of investment variation (57 percent), preference disturbances account for more than half of the fluctuations in labor hours (52 percent.) In the preceding section, we find that there is a high correlation between productivity (preference) shocks and investment (labor hours) dynamics.

The variables that are potentially affected the most by the world real interest rate disturbances are the trade balance and asset holdings in small open economies: the interest rate shocks account for shortcomings. See Ingram, Kocharlakota, and Savin (1994a), Kouparitsas (1997) and Kose (1998) for similar variance decomposition methods.
up to more than 52 percent of the trade balance variation when it is ordered as the first shock. If we consider all orderings, we observe that the contribution of the interest rate shock to trade balance fluctuations is always greater than 14 percent. The volatility of the trade balance movements explained by of the technology shock ranges from 6 percent to 32 percent. While the world real interest rate shock explains up to 70 percent of the variation in asset holdings, the productivity shock accounts for up to 31 percent of the volatility.

We study the sensitivity of our results to our selection of benchmark parameterization. As the risk aversion coefficient increases from 1.5 to 5, the volatility of the world real interest rate shock goes down. This reduces the fraction of output variance explained by the world real interest rate shock from 22.63 to 15.66 when the interest rate shock ordered first. Changes in the elasticity of the adjustment cost does not have any major impact in the results. An increase in the share of capital income from 0.32 to 0.40 induces a relative increase in the volatility of productivity shocks, and, in turn, the productivity shocks become more important in explaining output fluctuations. In general, while the results are sensitive to some of the parameters, this does not affect our main conclusions.

Depending on the order of orthogonalization, we find that the world real interest rate shock can account for 23 (52) percent of output (trade balance) variation in a small open economy. This is contrary to the findings of some recent studies examining business cycles in Canada: Mendoza (1991), using a singular small open economy real-business cycle model calibrated for Canada, examines the impact of world real interest rate fluctuations, represented by the real return on the U.S. T-bills, on the second order moments of the simulated data. His results indicate that the interest rate shocks have only “minimal” effects on model variables. Schmitt-Grohe (1998) investigates the importance of the interest rate shocks in transmitting the business cycle fluctuations from the U.S. economy to Canada using a small open economy real business cycle model. She uses two different series to estimate the interest rate processes: the U.S. three month Treasury bill rate and the S&P 500 quarterly index. Her results indicate that the role of the interest rate shock is minor, since this shock does not induce large responses in output and employment.

19 He predicts that the world real interest rate fluctuations might potentially play a more important role in highly indebted developing countries. Kose (1998) constructs a small open economy model to study the contribution of the world real interest rate shocks to the macroeconomic fluctuations in small open developing economies and shows that the world interest rate shocks explain only a minor fraction of aggregate economic activity. Kose and Riezman (1998) extend this analysis using the data of several highly indebted African countries, and find that the world real interest rate disturbances do not generate significant macroeconomic fluctuations in these economies.

20 Correia, Neves, and Rebelo (1992, 1995), using a similar small open economy model to Mendoza’s one, analyze the role of the interest rate shocks in a small open economy model calibrated to represent Portugal economy. Their findings suggest that the world real interest rate shocks play a minor role in the business cycle dynamics of Portugal economy. Gaudin and Yi (1996) examine the role of the world real interest rate shocks in explaining the trade balance movements. They employ a small open economy model, and derive the reduced form equation describing
Using the exogenous shocks estimated by the actual data, Mendoza (1991) and Correia, Neves, and Rebelo (1992, 1995) try to match certain moments of the macroeconomic aggregates, Schmitt-Grohe (1998) focus on replicating certain impulse responses of the macroeconomic aggregates. In the preceding section we emphasize the differences between the world real interest rate shocks produced by the model and the interest rate series, including those used by Mendoza (1991), Correia, Neves, and Rebelo (1992, 1995), and Schmitt-Grohe (1998), in the data. We employ a different methodology to evaluate the importance of the world real interest rate shocks and find that these shocks, that are fully consistent with the data, might play an important role in explaining aggregate economic activity.

While the importance of the depreciation and preference shocks has already been discussed in the real business cycle literature, our study is the first one raising this issue in the context of small open economy models. Ambler and Paguet (1994) find that introduction of the depreciation shocks into an otherwise standard closed economy real business cycle model gets the predictions of the model closer to the data. In a two country business cycle model, Stockman and Tesar (1995) find that introduction of preference shocks significantly improves the fit of the model to the data. Our results indicate that both depreciation and preference shocks can be important in understanding the dynamics of macroeconomic aggregates in open small economies.

5. Conclusion

Our paper provides an alternative method to evaluate the role of the world real interest shocks in explaining business cycles in small open economies. We argue that a method aimed at examining this question should rigorously deal with the three problems: First, a convincing proxy to the world real interest rate is not available. Second, a non-singular small open economy model is needed to provide a thorough evaluation of the importance of the various shocks. Third, since world interest rate fluctuations may be correlated with domestic disturbances, assuming no correlation can generate misleading results. Most importantly, with correlation across shocks, the share of business cycle fluctuations attributable to a single shock depends upon the ordering of the shocks.

Our methodology builds on that introduced by Ingram, Kocherlakota and Savin (1994a, 1997) and provides practical solutions to each of these problems. By including as many shocks as endogenous observable variables, we obtain a non singular model in which the data does not violate any deterministic relationships implied by the model. In using orthogonality conditions implied by the Euler conditions to estimate decision rules, we obtain sufficient structure to identify the shock the behavior of the trade balance. Their results indicate that the real interest rate has a minor role in driving the
realizations implied jointly by the model and the data. Thus, we are not required to make any arbitrary assumptions regarding the processes generating these shocks and run no risk of missing the impact of relevant correlations across shocks. Finally, since the world real interest rate is one of the recovered shocks, we need not choose a possibly misleading proxy.

This fundamentally different approach yields results which differ in important ways from earlier work. An implication of prior research is that the world real interest rate plays at best a small role in explaining business cycles in small open economies. For example work by Mendoza (1991), Correia, Neves, and Rebelo (1992, 1995), and Schmitt-Grohe (1998) finds the world real interest rate to be unimportant in explaining fluctuations in output, consumption, labor hours and investment. In much of this work, even the trade balance and net foreign asset holdings are relatively unaffected by the world real interest rate shocks. Collectively, these findings suggest that the prominence given the world real interest rate in international macroeconomic theory may be misguided.

In contrast, we find that the world real interest rate shocks can be quite important. The world real interest shock can account for as much as 23 percent of the output fluctuations. This shock plays a smaller role in accounting for the consumption variation but can explain up to 35 percent of investment fluctuations and 38 percent of labor hours fluctuations. The variables that are potentially affected the most by the world real interest rate disturbances are the trade balance and asset holdings in small open economies. Indeed we find that as much as 52 percent of the trade balance variation and 70 percent of the variation in asset holdings can be explained by movements in the world real interest rate. Our research, then, reasserts that the world real interest rate may be an important part of the transmission mechanism of business cycles to small open economies.

There are interesting future research avenues to be explored. We observe that the model is unable to generate the correlation between the world real interest rate and output fluctuations. It may be interesting to consider the deficiencies of the standard model and improve on those dimensions, so the model does a better job in terms of replicating the data. Our study abstracts from monetary and fiscal policy shocks that are both important in understanding business cycle dynamics in open economies. We plan to examine the role of these shocks in a more complex small open economy model using the methods developed here.


References:


Ingram, B., Kocherlakota, N., and N. E. Savin, 1994b, Rational expectations shock estimation, mimeo, University of Iowa.


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_{11}$</td>
<td>2.405 (0.563)</td>
<td>$\psi_{21}$</td>
<td>3.883 (5.255)</td>
</tr>
<tr>
<td>$\psi_{12}$</td>
<td>3.094 (0.539)</td>
<td>$\psi_{22}$</td>
<td>-0.258 (0.134)</td>
</tr>
<tr>
<td>$\psi_{13}$</td>
<td>-2.64 (0.429)</td>
<td>$\psi_{23}$</td>
<td>4.469 (5.930)</td>
</tr>
<tr>
<td>$\psi_{14}$</td>
<td>-1.414 (0.541)</td>
<td>$\psi_{24}$</td>
<td>-3.854 (4.528)</td>
</tr>
<tr>
<td>$\psi_{15}$</td>
<td>-0.107 (0.144)</td>
<td>$\psi_{25}$</td>
<td>2.862 (1.611)</td>
</tr>
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<td>$\psi_{16}$</td>
<td>0.143 (0.050)</td>
<td>$\psi_{26}$</td>
<td>-1.090 (0.530)</td>
</tr>
</tbody>
</table>


Numbers in parentheses are standard errors associated with the parameters. The J-Statistic is the chi-square test value for the number of over-identifying restrictions (8 in each case). Numbers in brackets are the critical values to reject the null hypothesis that the over-identifying restrictions are met at the 10% level of significance.
Table 2  
Selected Statistics of the Estimated Shocks  
(Volatility and Comovement)  

<table>
<thead>
<tr>
<th>Variable</th>
<th>Volatility</th>
<th>Correlation with</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r</td>
<td>δ</td>
</tr>
<tr>
<td>r</td>
<td>4.73</td>
<td>1.00</td>
</tr>
<tr>
<td>δ</td>
<td>2.27</td>
<td>-0.61</td>
</tr>
<tr>
<td>η</td>
<td>0.49</td>
<td>-0.29</td>
</tr>
<tr>
<td>z</td>
<td>0.48</td>
<td>0.52</td>
</tr>
</tbody>
</table>

The technology (z) and preference (η) shocks are detrended by the Hodrick and Prescott (HP) (100) filter. The world interest rate (r) and depreciation (δ) shocks are in levels. Volatility is measured as the standard deviation of detrended series.
Table 3

Selected Statistics of the Estimated Shocks
(Comovement with Main Macroeconomic Aggregates)

<table>
<thead>
<tr>
<th></th>
<th>Correlation w/ Mean</th>
<th>$r$</th>
<th>$\delta$</th>
<th>$\eta$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td></td>
<td>0.14</td>
<td>-0.05</td>
<td>0.05</td>
<td>0.71</td>
</tr>
<tr>
<td>Consumption</td>
<td></td>
<td>0.11</td>
<td>-0.14</td>
<td>0.53</td>
<td>0.41</td>
</tr>
<tr>
<td>Investment</td>
<td></td>
<td>-0.16</td>
<td>0.15</td>
<td>0.05</td>
<td>0.46</td>
</tr>
<tr>
<td>Net Exports</td>
<td></td>
<td>0.38</td>
<td>-0.21</td>
<td>-0.36</td>
<td>0.07</td>
</tr>
<tr>
<td>Labor Hours</td>
<td></td>
<td>-0.22</td>
<td>0.50</td>
<td>0.63</td>
<td>0.09</td>
</tr>
</tbody>
</table>

The technology and preference shocks are detrended by the HP (100) filter. The other shocks are in levels. All macro aggregates, except net exports, are logged and then HP(100) filtered. Data are in 1990 dollars from the first quarter of 1963 to the last quarter of 1996. Net exports is normalized by output, then filtered.

Table 4

Selected Statistics of the Real Interest Rate Measures
(Volatility)

<table>
<thead>
<tr>
<th></th>
<th>$r^a_{USA}$</th>
<th>$r^p_{USA}$</th>
<th>$r^a_W$</th>
<th>$r^p_W$</th>
<th>$r^a_{SP}$</th>
<th>$r^p_{SP}$</th>
<th>$r^a_L$</th>
<th>$r^p_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility</td>
<td>2.70</td>
<td>2.66</td>
<td>2.74</td>
<td>2.78</td>
<td>11.04</td>
<td>11.63</td>
<td>2.68</td>
<td>2.65</td>
</tr>
</tbody>
</table>

$r^x_y$: $x=a$, ex-ante; $x=p$, ex-post; $y=USA$, U.S.A. T Bill rate; $y=W$, weighted rate of developed economies; $y=SP$, S&P 500 return; $y=L$, Libor rate.
Table 5

Selected Statistics of the Real Interest Rate Measures
(Volatility and Comovement with Main Macroeconomic Aggregates)

<table>
<thead>
<tr>
<th></th>
<th>$r^a_{USA}$</th>
<th>$r^p_{USA}$</th>
<th>$r^a_{W}$</th>
<th>$r^p_{W}$</th>
<th>$r^a_{SP}$</th>
<th>$r^p_{SP}$</th>
<th>$r^a_{L}$</th>
<th>$r^p_{L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>-0.10</td>
<td>-0.11</td>
<td>-0.04</td>
<td>-0.03</td>
<td>0.05</td>
<td>0.04</td>
<td>-0.10</td>
<td>-0.11</td>
</tr>
<tr>
<td>Consumption</td>
<td>-0.04</td>
<td>-0.05</td>
<td>0.01</td>
<td>-0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>-0.06</td>
<td>-0.07</td>
</tr>
<tr>
<td>Investment</td>
<td>-0.11</td>
<td>-0.14</td>
<td>-0.07</td>
<td>-0.07</td>
<td>-0.06</td>
<td>-0.06</td>
<td>-0.10</td>
<td>-0.13</td>
</tr>
<tr>
<td>Net Exports</td>
<td>0.04</td>
<td>0.08</td>
<td>0.03</td>
<td>0.08</td>
<td>0.13</td>
<td>0.13</td>
<td>0.03</td>
<td>0.08</td>
</tr>
<tr>
<td>Labor Hours</td>
<td>-0.17</td>
<td>-0.18</td>
<td>-0.08</td>
<td>-0.09</td>
<td>0.01</td>
<td>0.01</td>
<td>-0.16</td>
<td>-0.17</td>
</tr>
</tbody>
</table>

$r^x_y$: $x=a$, ex-ante; $x=p$, ex-post; $y=USA$, U.S.A. T Bill rate; $y=W$, weighted rate of developed economies; $y=SP$, S&P 500 return; $y=L$, Libor rate. All macro aggregates, except net exports, are logged and then HP(100) filtered. Net exports is normalized by output, then filtered.

Table 6

Selected Statistics of the Real Interest Rate Measures
(Comovement with Output)

<table>
<thead>
<tr>
<th>$k$</th>
<th>$r_{Model}$</th>
<th>$r^a_{USA}$</th>
<th>$r^p_{USA}$</th>
<th>$r^a_{W}$</th>
<th>$r^p_{W}$</th>
<th>$r^a_{SP}$</th>
<th>$r^p_{SP}$</th>
<th>$r^a_{L}$</th>
<th>$r^p_{L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-0.07</td>
<td>-0.10</td>
<td>-0.01</td>
<td>-0.04</td>
<td>-0.07</td>
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<td>0.04</td>
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</tbody>
</table>

$r^x_y$: $x=a$, ex-ante; $x=p$, ex-post; $y=USA$, U.S.A. T Bill rate; $y=W$, weighted rate of developed economies; $y=SP$, S&P 500 return; $y=L$, Libor rate, $y=Model$, the model produced rate. In each cell $corr(y(t), r(t-k))$ is reported. The output is logged and detrended with the HP(100) filter.
<table>
<thead>
<tr>
<th></th>
<th>Ordering</th>
<th>$r$</th>
<th>$z$</th>
<th>$\delta$</th>
<th>$\eta$</th>
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<tbody>
<tr>
<td><strong>Output</strong></td>
<td>$r$ first</td>
<td>22.63 , 22.63</td>
<td>64.33 , 24.63</td>
<td>19.1 , 5.87</td>
<td>22.38 , 0.77</td>
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<td></td>
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<td>22.63 , 0.24</td>
<td>76.42 , 24.63</td>
<td>36.39 , 5.87</td>
<td>40.18 , 0.77</td>
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<td><strong>Consumption</strong></td>
<td>$r$ first</td>
<td>13.05 , 13.05</td>
<td>49.09 , 11.02</td>
<td>27.21 , 6.42</td>
<td>56.52 , 10.41</td>
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<td>20.96 , 4.45</td>
<td>49.09 , 11.02</td>
<td>28.46 , 6.42</td>
<td>58.70 , 10.41</td>
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<td><strong>Investment</strong></td>
<td>$r$ first</td>
<td>31.20 , 31.20</td>
<td>51.98 , 15.02</td>
<td>26.95 , 6.06</td>
<td>26.53 , 6.03</td>
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<td>37.60 , 6.08</td>
<td>57.05 , 15.02</td>
<td>42.47 , 6.06</td>
<td>43.78 , 6.03</td>
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<td><strong>Labor Hours</strong></td>
<td>$r$ first</td>
<td>34.75 , 34.75</td>
<td>33.93 , 10.38</td>
<td>26.54 , 10.55</td>
<td>34.86 , 8.54</td>
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<td></td>
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<td>34.75 , 2.49</td>
<td>38.73 , 10.38</td>
<td>49.26 , 10.55</td>
<td>51.86 , 8.54</td>
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<td><strong>Net Exports</strong></td>
<td>$r$ first</td>
<td>52.10 , 52.10</td>
<td>18.15 , 6.17</td>
<td>16.83 , 8.09</td>
<td>25.39 , 13.42</td>
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<td></td>
<td>all</td>
<td>52.10 , 14.42</td>
<td>31.21 , 6.17</td>
<td>35.80 , 8.09</td>
<td>59.09 , 13.42</td>
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<tr>
<td><strong>Asset Holdings</strong></td>
<td>$r$ first</td>
<td>70.22 , 70.22</td>
<td>8.92 , 6.70</td>
<td>9.30 , 6.18</td>
<td>15.52 , 12.40</td>
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<tr>
<td></td>
<td>all</td>
<td>70.22 , 14.14</td>
<td>30.52 , 6.70</td>
<td>42.02 , 6.18</td>
<td>43.84 , 12.40</td>
</tr>
</tbody>
</table>

$r$ first: All orderings in which the world real interest rate shock ordered first.
all: All shock orderings.
In each cell, the share the variable’s variance explained by a particular shock is reported. The first (second) number is the upper (lower) bound of variance decompositions. The technology and preference shocks are detrended by the HP (100) filter. The other shocks are in levels. All macro aggregates, except net exports and asset holdings, are logged and then HP(100) filtered. Net exports and asset series are normalized by output, then filtered.
Figure 1.1. Technology Shocks and Output.
Quarterly technology shocks implied by the model (dashed line) and measured output (solid line) for each period from 1963:1 to 1996:4. Each series is normalized by its 1963:1 value.

Figure 1.2. Depreciation Shocks.
Quarterly depreciation shocks implied by the model from 1963:1 to 1996:4.

Figure 1.3. Preference Shock and Hours.
Quarterly preference shocks implied by the model (dashed line) and measured hours (solid line) for each period from 1963:1 to 1996:4. Each series is normalized by its 1963:1 value.
Figure 1.4. Real Interest Rates.
Quarterly real interest rates implied by the model from 1963:1 to 1996:4.