Linear Feedback Rules in Non-Linear Models with Rational Expectations[1].

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Abstract

The increasing use of inflation targeting by central banks over the last 10 years has generated a new interest in monetary policy rules. Svensson (1997) has drawn a distinction between instrument rules, such as that of Taylor, which are backward looking, and targeting rules which are specifically forward looking. In this paper we describe some computationally simple methods for linearising a non-linear model with rational expectations using stochastic perturbation and show that a properly specified control rule derived via dynamic programming contains both backward and forward looking features.

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1. Introduction

The increasing use of inflation targeting by central banks over the last 10 years has generated a new interest in monetary policy rules. Svensson (1997) has drawn a distinction between instrument rules, such as that of Taylor, which are backward looking, and targeting rules which are specifically forward looking. In this paper we describe some computationally simple methods for linearising a non-linear model with rational expectations using stochastic perturbation and show that a properly specified control rule derived via dynamic programming contains both backward and forward looking features. The literature in this area is voluminous and continues to grow and there is still considerable interest in improving existing algorithms and in deriving methods that can be applied to non-linear, dynamic, stochastic models.

Given the clear advantages of the forward-looking targeting rule over the backward looking instrument rule, at first glance the superiority of the optimal feedback rule may appear odd. In fact, the terminology is misleading because in the standard linear, quadratic Gaussian case, the optimal rule contains both forward-looking and backward looking elements. There are certain conditions under which the feedback component of the optimal rule becomes a constant function of the lagged state, but even in this case there will still remain a forward-looking element that comes through the so-called tracking gain of the control rule (Holly and Corker, 1984). This of course quite separate from the forward-looking element that arises when private agents formulate expectations rationally.

In principle there are clear advantages to having monetary policy expressed as a rule - at least formally- since in a forward-looking world building credibility through the transparency of the decision-making process, may make the process of inflation targeting more effective. What we propose to do in this paper is to describe some methods for computing the 'optimal' control rule using dynamic programming while also allowing expectations to be forward-looking. For this to be entirely convincing as a feasible solution we must assume that there is a sufficient commitment technology in place to prevent the issue of time inconsistency raising its head. Given that Central Bank independence is an important element in the conduct of inflation control, it seems unlikely that the Central Bank would play hard and fast with the expectations of the public . The dynamic programming solution is also straightforward to calculate and provides a natural benchmark against which to compare other 'handcrafted' rules of the instrument or targeting variety.

In this paper we consider the design of feedback rules for inflation targeting. We adopt a stochastic linearisation approach in order to produce a linear reduced form version of a small model of the UK economy. We then use the method of Christodolakis (1987) in order to take into account the presence of forward-looking expectations. We then solve for the dynamic programming optimal control rule and use the method of Amman(1996) in order to ensure that the saddlepoint features of a forward looking uncovered interest parity condition are satisfied along with the dynamic programming solution. Because the model is now linear it is straightforward to compute the policy frontier that traces out the trade-off between inflation volatility and output volatility We then explore how these methods perform when there are shocks to the economy that drive the inflation rate away from its desired path.

2. The Method

In this section we describe the steps we go through in order to (1) linearise a non-linear rational expectations model, (2) estimate a reduced form, (3) convert it into state space form and (4) compute the optimal control solution while satisfying the saddlepoint requirements of the rational expectations solution.

2.1 The Linearization

Assume we can write our non-linear, stochastic model as:

$$F_i(y_t, y_{t-1}, ...y_{t-s}, y_{t+1}, ..., y_{t+v}, x_t, x_{t-1}, ..., x_{t-r}, \varepsilon_t) = 0 \tag{2.1}$$

$$i = 1, m \quad t = 1, T$$

There are m endogenous variables, y, with a maximum lag of s, a maximum lead of v and a maximum of r lagged values of n exogenous variables, x; e_t is m-dimensional vector of white noise processes. We can expand (2.1) about some initial path to give:

$$\sum_{j=0}^{s} \left[\frac{\partial F_i}{\partial y_{t-j}}\right]_o \widetilde{y}_{t-j} + \sum_{j=0}^{v} \left[\frac{\partial F_i}{\partial y_{t+1+j}}\right]_o \widetilde{y}_{t+1+j} + \sum_{j=0}^{r} \left[\frac{\partial F_i}{\partial x_{t-j}}\right]_o \widetilde{x}_{t-j} + \left[\frac{\partial F_i}{\partial \varepsilon_t}\right]_o \widetilde{\varepsilon}_t \tag{2.2}$$

$$i = 1, m \quad t = 1, T$$

The perturbations to the initial path are defined as:

$$\widetilde{y}_t = y_t - y_{ot}, \quad \widetilde{x}_t = x_t - x_{ot}, \quad \widetilde{\varepsilon}_t = \varepsilon_t - \varepsilon_{ot} \tag{2.3}$$

In general (2.2) is time-varying. However, we want to obtain an approximation to this time-varying representation. We will also only be interested in some subset of the endogenous variables, the targets and a subset of the exogenous variables, among which will be the policy instruments. Assume $v = 0$, then we can write this representation in vector polynomial form in the lag operator L, as:

$$A(L)\widetilde{y}_t + C(L)\widetilde{y}_{t+1} + B(L)\widetilde{x}_t = \widetilde{\varepsilon}_t \tag{2.4}$$

In order to derive a constant coefficient, linear representation we perturb a subset of the instrument vector, the expectational variables and the error process. The perturbations are a sequence of orthogonal white noise processes. The use of a white noise perturbations is one way of meeting a basic identifiability condition (Hannan, 1971) that the perturbations have an absolutely continuous spectrum with spectral density non-zero on a set of positive measures in $(-\pi,\pi)$. The white noise sequence, since it contains all frequencies, will excite all of the dynamic modes of the non-linear model. Define $\widetilde{z}_t = (\widetilde{y}_{t+1}, \widetilde{x}_t, \widetilde{\varepsilon}_t)'$ as a stacked vector then:

$$E[\widetilde{z}_i(s)] = 0; \;\; E[\widetilde{z}_i(s)\widetilde{z}_j(t)] = \lambda_i \; \delta_{ij} \; \delta_{st} \tag{2.5}$$

$$i, j = 1, ..., p; \quad s, t = 1, ..., T$$

with $p = 2m + n$; δ_{ij} is the Kronecker delta and λ_i sets the size of the perturbation to the instruments, the expectational terms and to the equation error processes. We can then write the relationship between the target variables, the perturbed instruments, the expectational terms and the errors as an autoregressive distributed lag model:

$$A_0 y_t = A_1 y_{t-1} + ... + A_s y_{t-s} + D_1 y_{t+1} + B_1 x_t + ... + B_r x_{t-r} \tag{2.6}$$

Since the perturbations are orthogonal by construction, A_0 is an identity matrix and the matrices A_1 to A_s are diagonal, so each equation can be estimated separately.

2.2 State Space form

For the case of $\varepsilon_t = 0$, the state vector is simply defined as:

$$z_t' = (y_t, ..., y_{t-s-1}, x_{t-1}, ..., x_{t-r-1}) \tag{2.7}$$

and the state transition matrix:

$$z_t = A z_{t-1} + B x_t + D z_{t+1} \tag{2.8}$$

where:

$$A = \begin{bmatrix} \Pi_1 & \Pi_2 & ... & \Pi_s & \Omega_2 & ... & \Omega_r \\ I & 0 & ... & & & & 0 \\ 0 & I & & & & & : \\ : & 0 & I & & & & \\ & : & 0 & I & & & \\ & & : & 0 & I & & \\ 0 & 0 & 0 & 0 & ... & I & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \Omega_1 \\ 0 \\ : \\ \\ \\ \\ 0 \end{bmatrix}, D = \begin{bmatrix} \Gamma_1 \\ 0 \\ : \\ \\ \\ \\ 0 \end{bmatrix}$$

z is a $f = n \times (s + r - 1)$ dimensional vector, x is a m-dimensional vector. The transition matrix A is $f \times f$, B is $f \times m$, D is $f \times f$.

Since the state space form contains forward-looking expectations we could follow the approach of Blanchard and Kahn (1980), Sims (1996), Amman and Kendrick (1998), Anderson (1998), among others, and solve explicitly for the rational expectations solution. For the present let us hold the expectation fixed. We rewrite the state transition equation as:

$$z_t = A z_{t-1} + B x_t + C e_t \tag{2.9}$$

the vector e subsumes the expected values, and in general could also include any other *exogenous* variables as well as stochastic error terms.

2.3 The optimal Control Solution

To solve for the optimal control rule we define a loss function for the monetary authorities in terms of the state variables z and the control or instrument variables, x.

$$L_t = \frac{1}{2}\sum_{t=0}^{n}(z_t - z_t^d)'Q(z_t - z_t^d) + (x_t - x_t^d)'N(x_t - x_t^d) \tag{2.10}$$

where the superscript defines desired values for the state variables and the policy instruments, Q is a symmetric, semi-positive definite $f \times f$ matrix, and N is a symmetric $m \times m$ positive definite matrix.

To minimize (2.10) subject to the state transition equation (2.9) we can apply the well-known method of dynamic programming to compute an optimal control rule of the form:

$$x_t = K_t z_{t-1} + k_t \tag{2.11}$$

where K_t $(t = 1, , T)$ are a sequence of feedback control matrices and k_t $(t = 1, T)$ represents what is known as the tracking gain in the control literature. These are solved for recursively by first solving the period T problem to obtain a solution for x_T conditional on x_{T-1}. This is used to write a value function for period T which depends on x_{T-1} and which in turn forms part of the objective function for the period $T-1$ problem. Using this procedure, along with the terminal conditions $H_T = Q$ and $k_T = h_T = Qz_T^d$ we can solve for the sequence of feedback control matrices and tracking gains as:

$$K_T = -(N + B'H_TB)^{-1}(B'H_TA) \tag{2.12.a}$$
$$k_T = -(N + B'H_TB)^{-1}B'(H_TCe_T - h_T - Nx_T^d) \tag{2.12.b}$$
$$H_{T-1} = Q + (A + BK_T)'H_T(A + BK_T) \tag{2.12.c}$$
$$h_{T-1} = k_{T-1} + (A + BK_T)'(h_T - H_TCe_T + Nx_T^d) \tag{2.12.d}$$

These are solved recursively to obtain the control rule: Note that the feedback gains, K_i , for $i = 1, T$, depend only on the (constant) matrices of the transition equation and the loss function. The feedback part of the control rule then feeds only off the lagged state vector z_{t-1}. By contrast, the tracking gains vary over time depending upon the current and future values of the exogenous variables and expectations in the vector e. This is the feedforward part of the control rule.

2.4 Incorporating Rational Expectations

The solution in (2.12) is the well-known regulator problem for a non-rational expectational model. However, in the vector e, which appears in the recursion for the tracking gain, we have an expectation of the state in the next period.

There is a large class of methods for solving this problem. However, there is in principle a difficulty with the computation of an optimal control solution in this case because of the time inconsistency problem first identified by Kydland and Prescott

(1977). In essence there are two components to this problem. The first concerns the use of Bellman's principle of optimality. Consider the problem of minimizing:

$$J(y_t, y_{t+1}, x_t, x_{t+1}) \tag{2.13}$$

where y are targets of policy, and the xs are instruments, subject to:

$$y_t = f_t(x_t, x_{t+1}) \tag{2.14.a}$$

$$y_{t+1} = f_{t+1}(y_t, x_t, x_{t+1}) \tag{2.14.b}$$

The first equation can be thought of as representing a situation in which expectations of what the policymaker will do tomorrow affects what happens today. Assuming, as is normal differentiability and an interior solution, the first order conditions for (2.13) are:

$$\frac{\partial J}{\partial x_t} + \frac{\partial f_t}{\partial x_t}\left(\frac{\partial J}{\partial y_t} + \frac{\partial J}{\partial y_{t+1}}\frac{\partial f_{t+1}}{\partial y_t}\right) + \frac{\partial f_{t+1}}{\partial x_t}\frac{\partial J}{\partial y_{t+1}} = 0 \tag{2.15}$$

$$\frac{\partial J}{\partial x_{t+1}} + \frac{\partial f_t}{\partial x_{t+1}}\left(\frac{\partial J}{\partial y_t} + \frac{\partial J}{\partial y_{t+1}}\frac{\partial f_{t+1}}{\partial y_t}\right) + \frac{\partial f_{t+1}}{\partial x_{t+1}}\frac{\partial J}{\partial y_{t+1}} = 0 \tag{2.16}$$

But the solution provided by the method of dynamic programming for period $t+1$ is:

$$\frac{\partial J}{\partial x_{t+1}} + \frac{\partial J}{\partial y_{t+1}}\frac{\partial f_{t+1}}{\partial x_{t+1}} = 0 \tag{2.16}$$

and for period t:

$$\frac{\partial J}{\partial x_t} + \frac{\partial f_t}{\partial x_t}\left(\frac{\partial J}{\partial y_t} + \frac{\partial J}{\partial y_{t+1}}\frac{\partial f_{t+1}}{\partial y_t}\right) + \frac{\partial f_{t+1}}{\partial x_t}\frac{\partial J}{\partial y_{t+1}} = 0 \tag{2.17}$$

Now while the FOC for period t corresponds to the first period FOCs in (2.15), the FOC for $t+1$ does not. This is because the model is not causal (i.e. $\frac{\partial f_t}{\partial x_{t+1}}$ is not equal to zero). The solution derived by dynamic programming is referred to as time-consistent. It ignores the term $\frac{\partial f_t}{\partial x_{t+1}}$. A way of deriving the time-inconsistent solution was originally proposed by Buiter (1981). Note that the feedback part of the control rule in (2.12.a) does not depend on the matrix C. So the contingent part of the rule that depends on the lagged state is independent of the expectational term. However, the current state and the actual value of the instrument depend on the expectation through the tracking gain. So the problem is to compute the jump in the expectational variables. To do this we follow the procedure of Amman and Kendrick (1992) and treat this part of the problem iteratively.

- Step 1: For initial assumptions about the expected path for the expectational state, z_{t+j}, $j = 1, T-1$, and a terminal condition for T, compute the feedback and tracking gains. Store the feedback gains.

- Step 2: Update the vectors $E^{j+1}z_{t+1}$, for $i = 1, T$, where j is an iteration counter, using $E^{j+1}z_{t+i} = \lambda E^{j}z_{t+i} + (1-\lambda)^{j+1}z_{t+i}$, for $i = 1, T$, where λ is the relaxation factor (Fisher et al, 1986).

- Step 3: Recompute the tracking gain matrices.

- Step 4: Test whether: $\mid E^{j+1}z_{t+i} - \ ^{j+1}z_{t+i} \mid < \varepsilon$, for $i = 1, T$, where ε is an arbitrarily small convergence criteria. If not true go to step 2.

- Step 5: Stop.

This provides what we can refer to as the expectations-consistent optimal rule that satisfies (2.16). However, as is well known that is very far from being the end of the matter since, *ex post*, there is an incentive for policymakers to renege on previous commitments and to act in a time-inconsistent way. Because economic agents are assumed to be aware of this possibility the first best, expectations consistent policy is not actually implementable. Since Kydland and Prescott's paper the literature on economic policy has been dominated by various proposals for resolving this difficulty. The standard model uses a surprise supply function. A positive inflation bias results because the level of output the policymaker aims for is higher than the natural rate. However, as Bean(1998) persuasively argues it is more convincing to see the inflation bias coming from political considerations. Democratic politicians may seek an electoral advantage by inflating the economy prior to an election. Thus the act of delegation to an independent central bank will be enough to rid economic policy of its inflationary bias[2].For the remainder of this paper we assume that central bank independence is sufficient to eliminate any inflation bias in policy.

3. An Application

In this section we provide an application to the UK. We use a small non-linear model of the UK economy in order to generate a linearization and then use the linearization to examine a policy question in which the Bank of England uses the short term interest rate in order to pursue a target path for the rate of inflation. Expectations are forward looking in the foreign exchange rate market so the effective exchange rate is determined by an uncovered interest parity condition. The expected change in the exchange rate is equal to the risk adjusted interest rate differential.

3.1 The Linearized Model

The linearization was obtained by passing white noise through both the short term interest rate and the (exogenised) exchange rate for 92 periods and storing the effect of these stochastic perturbations on inflation $(RPIX) = \pi$, and output growth $=g$. In order to smooth the use of the interest rate, r, as an instrument we also included as an endogenous variable, the first difference of the interest rate, Δr. The vector y in (2.4) is now $y' = (\pi, g, eer, \Delta r)$. We then estimated a distributed lag model of

[2]It is interesting to note that on the day that the UK government announced that the Bank of England was to have operational independence, interest rates on UK 10 year bonds fell relative to German 10 year bonds by almost half of a percentage point.

inflation and output growth on the interest rate and the exchange rate, eer, with five lags in inflation and output growth, and eight in the interest rate and the exchange rate. We use an uncovered interest parity condition that the expected change in the exchange rate is equal to the interest rate differential between the domestic interest rate and the overseas interest rate, rw. We also want to allow for the possibility of independent shocks to inflation and output growth. So the vector of exogenous variables, is now defined as $e' = (eer_{t+1}, rw_t, \xi_{\pi t}, \xi_{gt})$, where the last two elements are designed to allow for shocks to inflation and output growth, and eer_{t+1} is the expected exchange rate. This means our linearization takes the structural form:

$$A_0 y_t = A_1 y_{t-1} + ... + A_s y_{t-5} + D_1 e_t + B_1 x_t + ... + B_8 x_{t-8}$$

where:

$$A_0 = \begin{bmatrix} 1 & 0 & \alpha_{13} & 0 \\ 0 & 1 & \alpha_{23} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} \beta_1 \\ \beta_2 \\ 1 \\ 1 \end{bmatrix}$$

$$\sum_{i=1}^{5} A_i = \begin{bmatrix} \alpha_{i11} & 0 & \alpha_{i13} & 0 \\ 0 & \alpha_{i22} & \alpha_{i23} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \sum_{i=6}^{8} A_i = \begin{bmatrix} 0 & 0 & \alpha_{i13} & 0 \\ 0 & 0 & \alpha_{i23} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \sum_{j=2}^{8} B_j = \begin{bmatrix} \beta_{j1} \\ \beta_{j2} \\ 0 \\ 0 \end{bmatrix}$$

and $B_2(4) = -1$.

This produced a 39 dimensioned state vector.

The linearisation method also provides some diagnostic information. For example, a simple regression of inflation on output resulting from the perturbations to the interest rate gives:

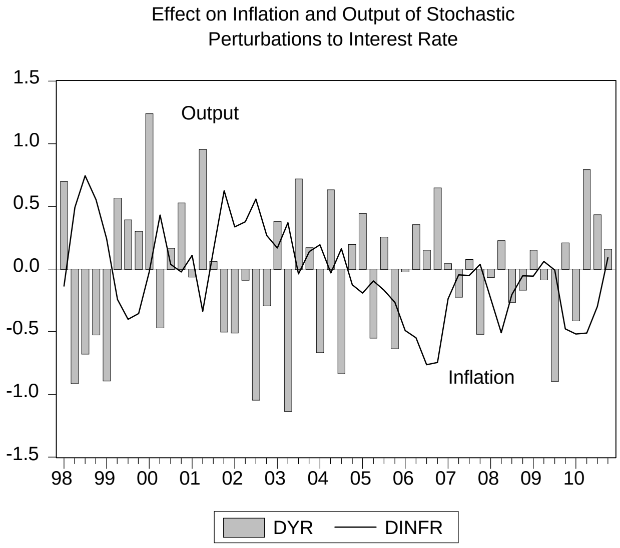

Figure 1: Stochastic Perturbations to Interest Rate

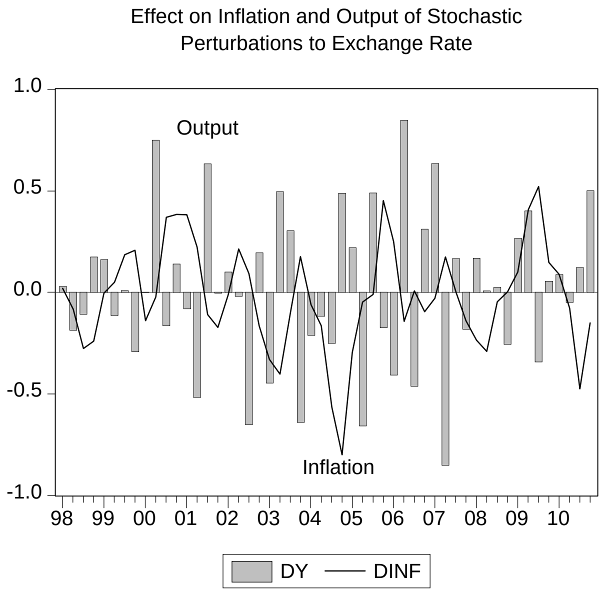

Figure 2: Stochastic Perturbations to Exchange Rate

Dependent Variable: $\Delta\pi$		
Method: *Least Squares*		
Variable	Coefficient	t-Statistic
C	-0.008102	-0.559238
$\Delta\pi_{t-1}$	0.428850	4.693387
$\Delta\pi_{t-2}$	-0.544334	-4.835404
$\Delta\pi_{t-3}$	0.259059	2.250824
$\Delta\pi_{t-4}$	-0.704680	-6.439743
$\Delta\pi_{t-5}$	0.004484	0.073881
δy_{t-1}	0.226832	6.375036
δy_{t-2}	-0.095983	-2.328444
δy_{t-3}	0.098812	2.170162
δy_{t-4}	-0.033774	-0.728274
δy_{t-5}	0.106045	2.534038
δy_{t-6}	0.067147	2.078174
δy_{t-7}	0.032141	1.098118
δy_{t-8}	0.074717	2.538922
R-squared	0.766780	
Durbin-Watson	2.305172	

Note that the relationship is specified in terms of the change in the perturbation (this restriction is easily accepted: a Wald test gave a p-value of 0.65) This confirms the unit root in the process for inflation. In Figures 1 and 2 we have plotted the paths for inflation and output as an outcome of one particular stochastic realisation. In the first figure we show the effect on inflation and output of just the realisation for the interest rate. In Figure 2 we show the effect of just the realsiation for the exchange rate. THe figures confirm some of the sylised facts of inflation-output relationships. Output tends to lead, responding to interest rates first. The effect on inflation comes through later. There isalso considerable persistence in inflation. In figure 2 we show the effect of the exchange rat on output and inflation. Now the effect on inflation and output is more or less immediate. Negative shocks to the exchange rate raise both output and inflation.

3.2 Some Illustrative Simulations

In this section we turn to some illustrative simulations designed to highlight some of the features of the methodology. In particular we want to demonstrate how the optimal control rule can be forward-looking and respond to anticipated shocks and how this forward-lloking component interacts for the forward-looking feature of the exchange rate market. One of the perceived drawbacks of a feedback rule is that it appears to be invariant to changes in expected shocks to the economy emanating from exogenous variables. For example, if a downturn in world economic activity is expected or energy prices are expected to rise sharply, a feedback rule will not produce a change in monetary policy until the effects of the exogenous events show up in the lagged state vector. While it is true that the feedback rule only works off the lagged state, it is not true that the optimal control rule is not forward-looking and capable of responding in anticipation of future shocks. This forward-looking role is provided by the tracking gain, the second part of the control rule.

We are particularly interested in the relative roles of the feedback and tracking gains when shocks to inflation are anticipated and when they are not. However, there is a complication because the expected exchange rate appears in the tracking gain when expectations are rational. In order to disentangle the forward looking part of the control rule from the forward-looking exchange rate, we first examine a version of the model in which the exchange rate does not appear.

We consider shocks of two types. First there is an unanticipated shock to the initial state. Inflation turns out to be 5 percentage points higher than expected. In the absence of any monetary response the effect on the path for inflation relative to the base inflation rate, is shown in Chart 1. As we have noted above there is considerable persistence in the inflation rate. When the optimal control rule is used (with no exchange rate pass-through) the path for inflation in Chart 1 does return to base more quickly, but much of the inflationary spurt is unavoidable, even though interest rates are raised by 5 percentage points. Note, that there is no forward-looking element to the interest rate jump (Chart 2). Once the shock occurs there is nothing else to anticipate.

The second shock is an anticipated shock to inflation. The inflation rate is expected to receive a 2 percentage point shock in each of periods 3 and four, so the shock is expected (with certainty) to occur in 6 months time. The forward looking nature of the optimal control rule is now clear. In Table 1 we show the response of inflation and interest rates to the expected shock. The tracking gain component of the optimal control rule generates an immediate jump in the interest rate in response to the expected rise in inflation. Once the shock has passed, the tracking gain term drops back to zero.

Table 1

	Inflation	*Interest Rate*	*Tracking gain*
Initial State	0	0	0
1	0.00	3.33	3.33
2	0.00	6.20	3.07
3	2.00	5.79	2.76
4	3.83	5.34	1.27
5	3.88	4.56	0.00
6	4.16	3.97	0.00
7	3.59	3.44	0.00
8	2.84	3.02	0.00

In Chart 7 onwards we show the effect of allowing for the exchange rate channel. The forward looking response of the exchange rate now delivers a considerable increase in the potency of monetary policy. The monetary contraction triggers an immediate jump in the exchange rate which bears down on inflation. However, this is also associated with larger, and more volatile, output losses. In Chart 8 we show the interest rate outcome. What is particularly striking is that the tracking gain contribution is negative. This is because the tracking gain includes a term in the expected exchange rate. The tracking gain *leans* against the jump in the exchange rate and is a measure of the extent to which monetary policy would have to be tighter in order to achieve the same inflation path without the help of the exchange rate appreciation.

In Charts 11 to 14 we show the outcome when the inflation shock is anticipated. The inclusion of the exchange rate channel enhances the effectiveness of monetary policy considerably. As before, the forward-looking part of the control rule triggers a monetary tightening in anticipation of the future shock. This actually reduces inflation prior to the shock as the exchange rate appreciates.

4. Calculating the Policy Frontier

In this section we show how the linearisation can be used in a straightforward way to compute the policy frontier. Since an inflation target regime involves the use of one instrument, the interest rate, while the objective function includes both inflation and output, we do not have strict controllability. The trade-off is in terms of the relative volatility of inflation and output. We can trace out the *policy frontier* by varying the relative weights on inflation and output in the objective function and calculate the effect on the volatility of inflation and output. Since we have a linearized model of the form:

$$z_t = Az_{t-1} + Bx_t + Ce_t + \xi_t \tag{4.1}$$

where $\xi_t \sim N(0, \Omega)$ and:

$$x_t = K_t z_{t-1} + k_t$$

The state covariance $\Sigma_t = E(z_t z_t')$ evolves under control as:

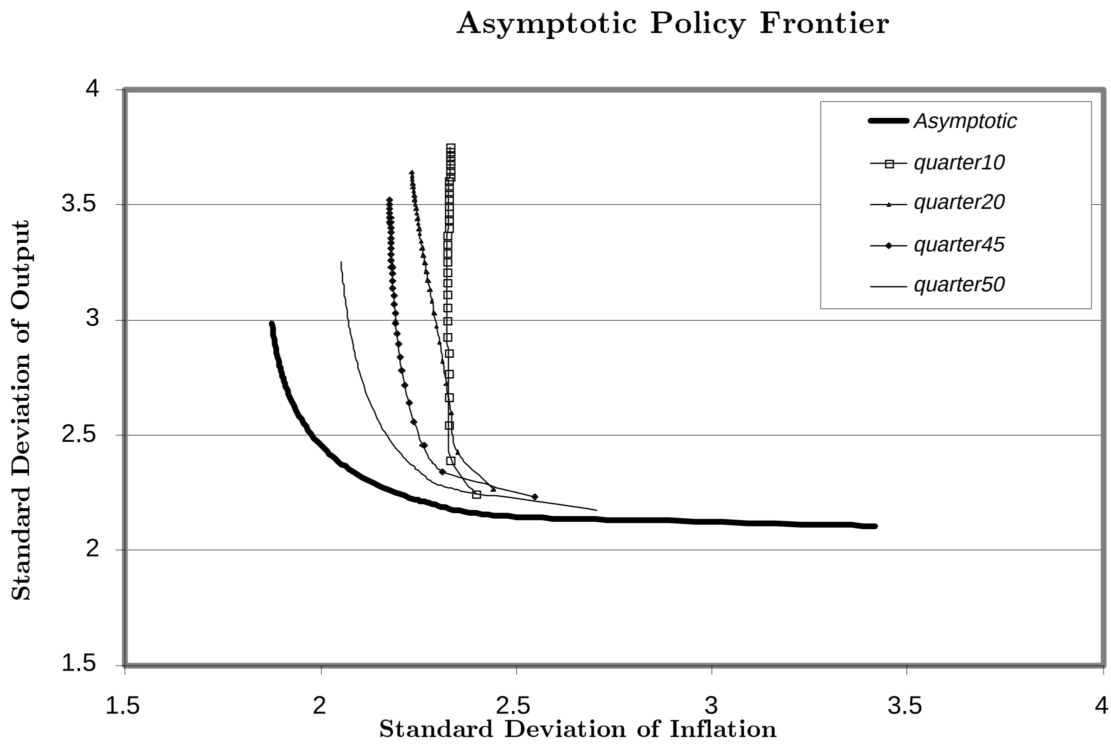

Figure 3: Asymptotic State Variance. The weight on output has been fixed at 5 and the weight on inflation varies from 0 to 30.

$$\Sigma_{t+1} = (A + BK_t)\Sigma_t(A + BK_t)' + Bk_tk_t'B' + \Omega \tag{4.2}$$

where the covariance of the tracking gain is:

$$k_tk_t' = (N + B'H_tB)^{-1}B'(H_tCe_t - h_t)(H_tCe_t - h_t)'B[(N + B'H_tB)^{-1}]' \tag{4.3}$$

We set the initial variance of inflation and output to 1.51 and 2.2 respectively. Each point on the frontier corresponds to a weight on inflation which varies between 0 and 30, while the weight on output is fixed at 5. When the weight on inflation is zero stabilising output leads to a large increase in the variabiilty of inflation, while stabilising inflation suggests that in the medium term relevant to the conduct of monetary policy, the the policy frontier is almost rectangular. This confirms the finding of Svensson (1997) that in order to stabilize output, inflation targeting may be difficult to implement in the short-run, for the simple reason that Central Bank have imperfect control over current inflation.

4. Discussion and Conclusions

We shown in this paper that a properly specified control rule derived by the method of dynamic programming has both a forward and a backward looking dimension. The feedback part of the rule responds to the lagged state, but the forward-looking tracking gain allows monetary policy to respond in anticipation of future shocks. We have also described a linearisation method for a nonlienar model that allows the use of a control rule with rational expectations.

Our approach also addresses in a straightforward way the concerns of Svensson and others that monetary policy needs a forward-looking dimension. Information about the future path for inflation not already reflected in the lagged state can be incorporated into the current setting for monetary policy through the tracking gain part of the optimal control rule

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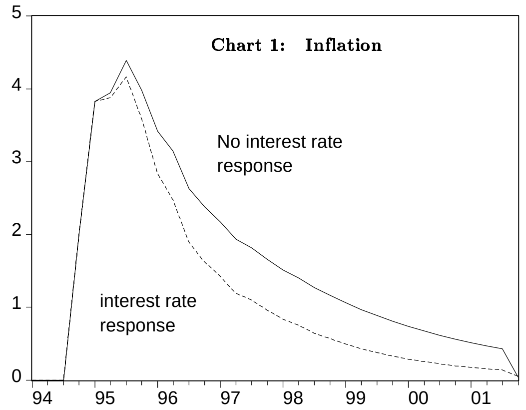

Figure 4: The effect on inflation of an unanticipated shock to inflation. No exchange rate channel

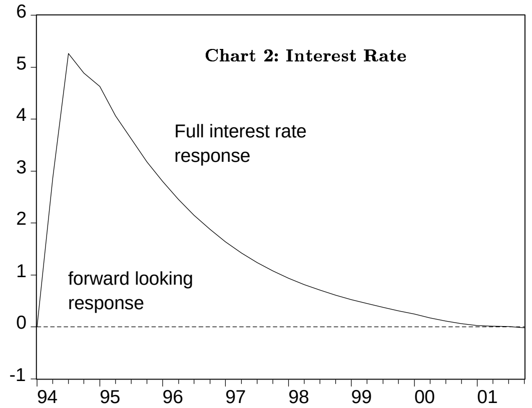

Figure 5: The interest rate response to an anticipated shock to inflation. No exchange rate channel.

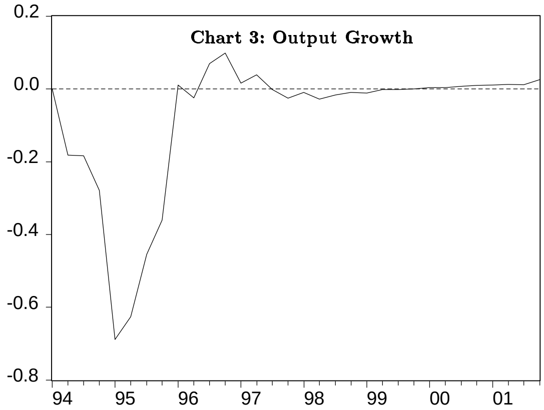

Figure 6: Effect on output growth of monetary response to unanticipated shock to inflation. No exchange rate channel.

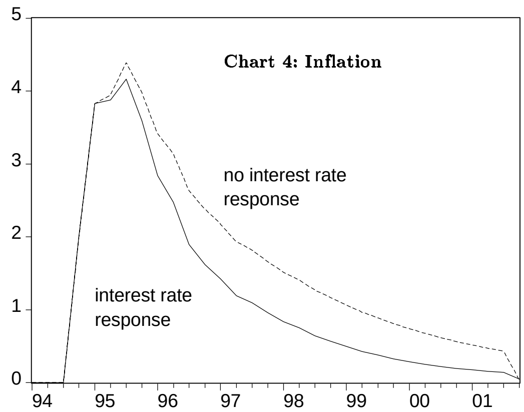

Figure 7: Response of inflation to an anticipated shock to inflation rate 6 months ahead. With and without monetary response. No exchange rate channel.

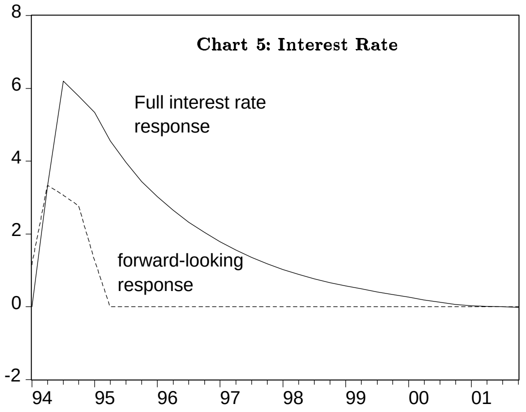

Figure 8: The optimal response of the interest rate to an anticipated shock to inflation rate 6 months ahead. No exchange rate channel.

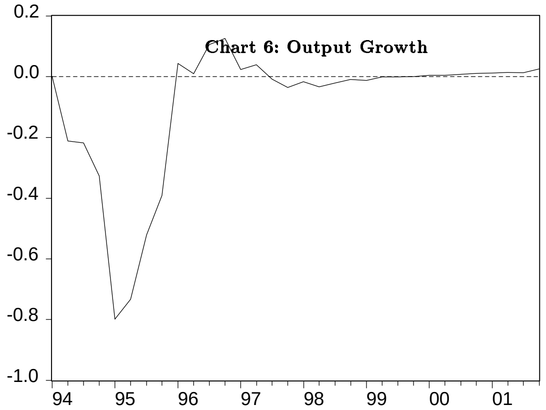

Figure 9: The optimal response of the output growth to an anticipated shock to inflation rate 6 months ahead. No exchange rate channel.

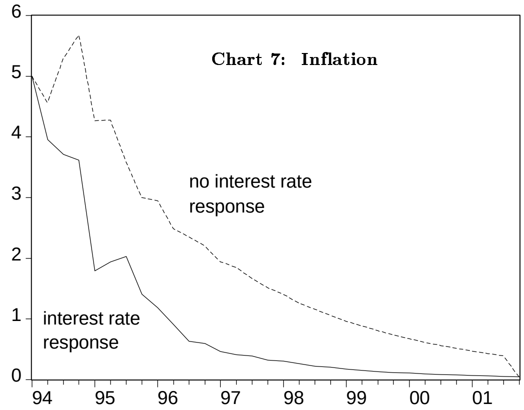

Figure 10: Inflation response to an unanticipated inflation shock. With and without a monetary response. Exchange rate channel.

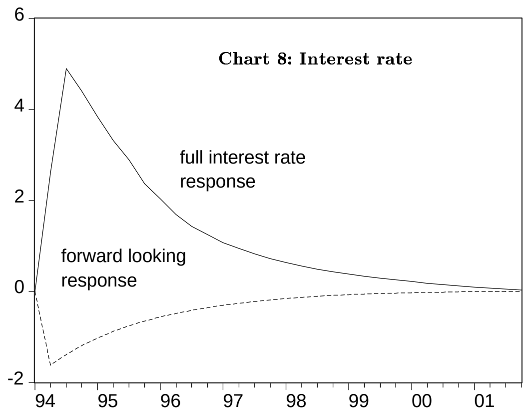

Figure 11: Optimal interest rate response to an unanticipated inflation shock. Exchange rate channel.

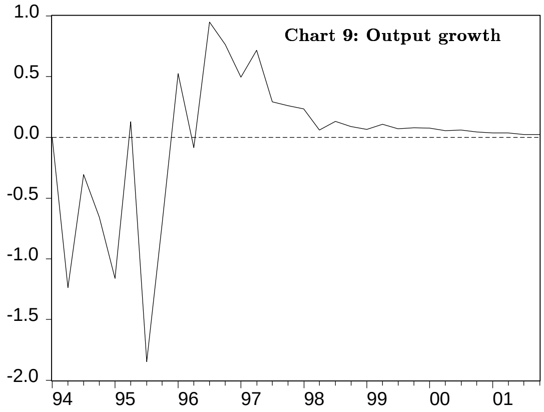

Figure 12: Response of output growth to monetary response to an unanticipated inflation shock. Exchange rate channel

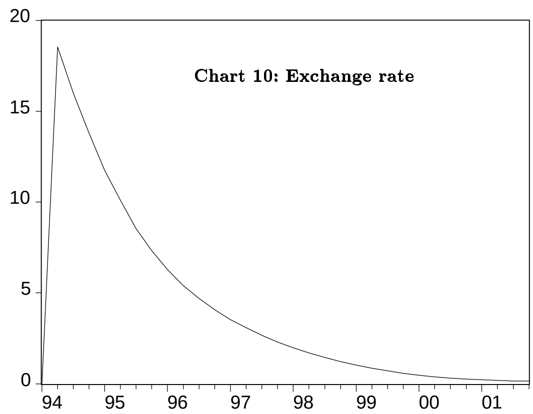

Figure 13: Response of exchange rate to monetary response to an unanticipated inflation shock..

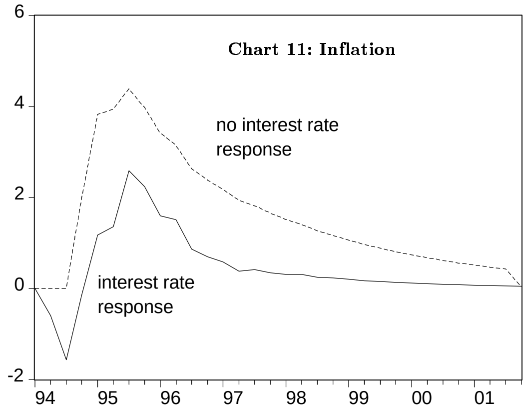

Figure 14: Inflation response to an anticipated inflation shock. With and without a monetary response. Exchange rate channel

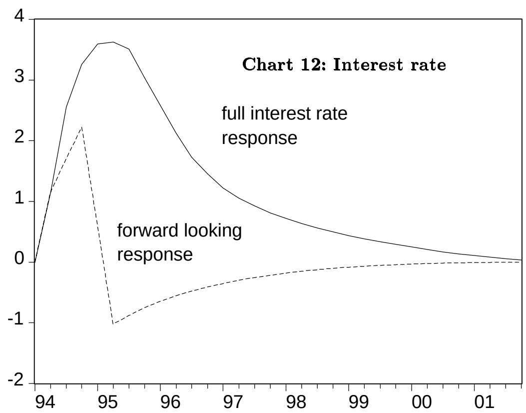

Figure 15: Optimal interest rate response to an unanticipated inflation shock. Exchange rate channel

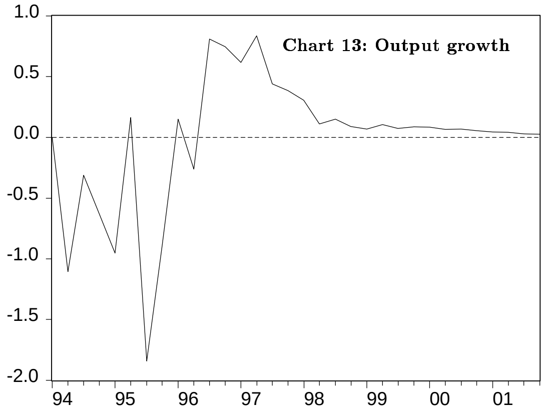

Figure 16: Response of output growth to monetary response to an unanticipated inflation shock. Exchange rate channel

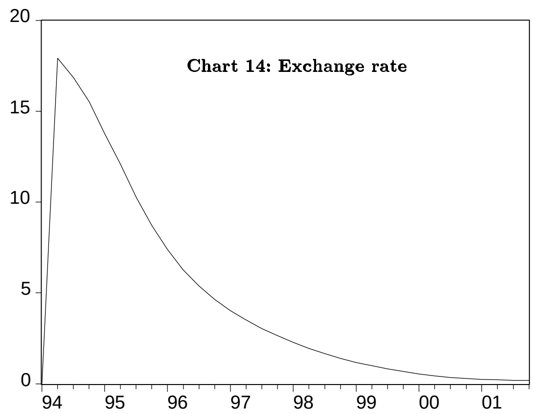

Figure 17: Response of exchange rate to monetary response to an anticipated inflation shock.