The application of the Kalman Filter to the Fisher Equation. Italian and German Term Structure of the Interest Rates.

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Abstract. Generalising the Fisher Equation into the Term Structure of Interest Rates we analyse the influence of the premium risk on the long run interest rate. The existence of the risk premium causes the inequality between the forward interest rates and the expected interest rates. We try to give an alternative view of the Fisher Theory by explaining the interest rate using the expected inflation rate and the variable component of the real interest rate. We estimate the regression using the Kalman Filter, under the hypothesis that the agents' expectations are adaptive rather than rational. We analyse the Italian interest rates and the German interest rates over the period 1980:1-1998:1.
We find that in the financial markets the monetary policy influences the agents’ expectations. Credible monetary policies create rational expectations and low volatility of the interest rates.
Non-credible monetary policies make the agents’ expectations adaptive and volatility of the interest rates high: the agents’ expectations change with the risk premium and with the uncertainty.
The comparative analysis between German results and Italian results explains clearly the different impact of the monetary policy on the expectations and on the volatility of the interest rates.
Low volatility is associated with a low risk premium (calculated as a component of the interest rate): under these conditions it’s not possible to hypothesise adaptive expectations and the Kalman Filter is not useful. High volatility is associated to higher risk premium, and the agents could have adaptive expectations rather than rational expectations.

Keywords: Term Structure of the Interest Rates, Fisher Eqaution, Adaptive Expectations, Rational Expectations, Kalman Filter, Garch model
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PRELIMINARY VERSION. COMMENTS WELCOME.

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1. **Introduction**

There is a large body of literature on the Fisher equation and of particular interest are the results obtained in the framework of rational expectations and efficient markets hypotheses.
In this paper we consider some of the previous results and evaluate empirically the differences between what the theory suggests and the evidence from empirical works. With respect to the latter, we study the behaviour of the interest rates in two financial systems: the Italian market of the Treasury bonds and the German "bunds" market. These exercises show the poor performance of the Fisher framework as guideline to model the two markets and we claim that is due to the fact that the Fisher theory disregards the existence of a particular time-varying unobservable variable, i.e. the premium risk. Hence we replace the traditional Fisher equation with a more complex state space form in order to capture the influence that the premium risk has on the volatility of the interest rates. We find that the risk premium seems to affect more the Italian term structure than the German one; we suggest that this may be ascribed to the different credibility of the monetary policy and hence to the different value that agents attach to their expectations, or to monetary shocks that, in the Italian case are not captured by the process characterising the interest rates.

In Section 2 we provide a short theoretical presentation of the Fisher equation and of the term structure rejecting the possibility to apply the "Expectations Theory" in the Italian zero coupon bond market. In Section 3 we report the estimates of the generalised Fisher equation using the Kalman filter and modelling the conditional variance throughout a GARCH (1,1) process. Section 4 concludes.

2. **Theoretical framework**

2.1 Generalisation of the Fisher equation into the term structure of the interest rates

The Fisher Equation $i_t = r_t + b\,\pi_t^e$ explains the relation between the nominal interest rate (i_t) and the expected inflation rate (π_t^e), under the assumption that agents have rational expectations and markets are efficient.
In the framework of the Fisher theory at any change in the inflation rate corresponds an equal change in the nominal interest rate meaning that the real interest rate (r_t) remains unchanged.
Empirically this may be proved by estimating the parameter b on the expected inflation rate equals to one.
The empirical evidence shows that nominal interest rates and the expected inflation rates don't change in the same proportion, probably because the economic models are misspecified or because the agents expectations are wrong *(Mundell R.A, 1963)*.

In this paper we first generalise the Fisher Equation into the term structure of the interest rates and secondly assume a real interest rate variable across the time according to a stochastic process.

We define the Fisher equation in a multivariate model in which we assume that:

- agents are risk adverse;
- in financial markets there are assets (zero coupon bonds) with different maturity that allow one to build the yield curve;
- the inflation follows a stochastic process that is able to affect the agents' behaviour.

The generalisation of the Fisher equation allows us to consider not only one interest rate but interest rates characterised by different maturity. Hence we do not estimate the equation looking at the coefficient b and testing that it's equal to one, rather we test whether the real interest rate is variable or not thereby determining whether the Expectation Theory works under these new assumptions.

The Expectations Theory

According to the Expectation Theory the return on a long term bond should be given by the yield curve and therefore the forward rate should be implied by the spot rates at the different maturities: hence the forward rate should be an unbiased estimator of the expected short interest rates.
In formal notation we can write:

(1) $\quad {}_t r_{n-m} = E_t[{}_{t+m}R_{n-m}]$

where ${}_t r_{n-m}$ is the forward interest rate, and $E_t[{}_{t+m}R_{n-m}]$ is the expected spot rate at the time t+m.
The Expectations Theory does not consider the existence of a risk premium, defined as that part of the return that exceeds the expected returns on short-term bonds.
The implied forward rate can be calculated using the spot yield curve and the spot rates of two or more bonds with different maturities

$$(1+{}_{t+1}\, r_n)^n = \frac{(1+R_{k+1})^{k+1}}{(1+R_1)}$$

A linear approach of the term structure defines the long-term interest rate as weighed average of the expected short interest rates.

Considering zero coupon bonds

$${}_tR_n = (m/n)\ ({}_tR_m + E_t\ {}_{t+m}R_m + \ldots\ldots + E_t\ {}_{t+n-m}R_m) + c$$
$${}_tR_n = (m/n)\ {}_tR_m + [(n-m)/n]\ E_t\ {}_{t+m}R_{n-m} + c$$

where c is a constant, m<n, m/n is a integer number and the sum of the short interest rates equals one.
Working on the above formulations we get:

$$E_t\ {}_{t+m}R_{n-m} - {}_tR_n = (m/n\text{-}m)\ ({}_tR_n - {}_tR_m) - c/n\text{-}m$$

in which the different return between a long term bond and a short term bond is an unbiased predictor of the changes occured in the long term interest rates, during the life of the short term bond.
If the difference between the two returns is positive then the long term interest rates should increase in the next m periods.
Considering the term structure of three zero coupon bonds (zcb) with different maturities, three-month, six-month, and one-year we can write:

(2) $\quad {}_{t+3}R_3 - {}_tR_3 = a + b\,[2\,({}_tR_6 - {}_tR_3)] + u_t$

(3) $\quad {}_{t+6}R_6 - {}_tR_6 = a + b\,[2\,({}_tR_{12} - {}_tR_6)] + u_t$

$$(4) \qquad [3\,({}_{t+3}R_3 - {}_{t}R_3) + 2({}_{t+6}R_3 - {}_{t+3}R_3) + ({}_{t+9}R_3 - {}_{t+6}R_3)] = a + b\,[4\,({}_{t}R_{12} - {}_{t}R_3)] + u_t$$

where the regressions represents respectively:
- the change in the three-month interest rate after three months;
- the change in the six-month interest rate after six months;
- the change in the three-month interest rate in the next four quarters.

If the Expectation theory is consistent the estimated value of b should not different from one, meaning that the Fisher theory is valid when we consider only one interest rate.
We estimate equations (2) and (3) on the period 1980:1-1998:1 using the monthly data of the Italian zero coupon bonds and German zeros (Table1).
The estimation is done by the OLS estimator and the results are reported in the Appendix (Table2).

2.2 Evidence from the Italian market

The method of issuing Italian Treasury Bonds changed in 1975, and until 1982 the movements in the interest rates seem to forecast changes in the inflation rate, but after that period the volatility of the interest rates becomes higher and unpredictable and the relation between interest rate and inflation rate is affected by external factors.
In the 1992, the Italian Lira left the exchange rate agreement of the EMS. In the same period, concern about the sustainability of the fiscal situation came to the fore in policy discussion. The conduct of the monetary policy was affected by the diminished role of the exchange rate as a nominal actor. The Lira suffered a rapid and dramatic depreciation.
There was a sharp tightening of monetary policy in the summer of the 1992 during the currency crisis, a gradual but substantial easing in 1993 and the first half of 1994, followed by a renewed tightening from June 1994 to the end of the 1995.
In the new floating exchange rate regime, the transmission mechanism of monetary policy to inflation and to the interest rates received greater attention, not only for the new scenario but also because of the large fiscal imbalance of the Italian Government.
During the described period the interest rates are characterised by high volatility.

The main findings may be summarised as follows. With regard to the regression of equation (2) the parameter b is statistically significant and not differnt from one, and the constant is equal to zero at confidence level of 99%, which means that there is not evidence of the existence of the risk premium. However when the equation is estimated over the sample period up to around 1992, in correspondence with the suspension of the Italian currency from the European Monetary System the premium risk became visible and the parameter b is far from unity (Table 2a in Appendix).
In equation (3) there is strong evidence of the existence of a risk premium and the residuals are strongly correlated and characterised by high volatility. It is possible to assume that the difference behaviour of the risk premium is caused by the impossibility of changing the terms of long term contracts when something happens, and therefore the long-term interest rates are not able to capture the shocks.
Further, the Expectation Theory is rejected and this means:
(a) that the real interest rate does not follow a stable path. This is a first conclusion, which takes us to suppose that the real interest rate follows a stochastic process. Indeed in what follows we assume that this process is affected by the variability if the unobservable premium risk.
(b) the forward rate is a biased predictor of the expected short term interest rate.

2.3 Evidence from the German market

The German interest rates seem to follow a different process.
Their volatility is higher in the sample period between the 1985 and the 1990 reflecting the Re-unification and the monetary policy changes decided by the Bundesbank. On the whole, however, there is not evidence of the existence of a variable premium risk and in particular the Expectation Theory holds quite well: the forward interest rate implied by the yield curve is an unbiased forecaster of the changes in the expected short term interest rates. In both the equations equals the parameter b not different from one both on the whole sample period as well as in the sub-sample 1989:1-1993:12 (Table 2b in Appendix).
In the German case we try to impose some restrictions on the regressions (2) and (3) just to be sure that the parameter b is equal to one.
We take the regression
$Y_t=\alpha_0+bX_t$
where α_0 is a constant, $X_t=2({}_tR_6-{}_tR_3)$ in the case of the regression (a), and $X_t=2({}_tR_{12}-{}_tR_6)$ in the case of the regression (b). We impose the constraint $R\beta=r$ where "b" is a vector of coefficients $b=[\alpha_0, b]$, R is a matrix (n*K), with "n" number of constraints and "K" is the number of the regressors, "r" represents the constant.
The Wald test tell us that the restriction is accepted and it allows us to conclude that in the German Treasury bonds market the premium risk doesn't exist and the term structure can be explained by the "Expectations Theory".

3. The Kalman Filter

The potential relevance of an unobservable premium risk induced us to apply the state space model in order to capture the influence of this variable on the process followed by the interest rates.
The application of the Kalman filter to the estimation of the term structure model allows us firstly to recover an unobservable factor that affects the interest rate, the premium risk, and secondly to estimate it as a time-varying variable, under the assumption of adaptive agents'expectations.

The technique consists in applying the Kalman filter to the stochastic process which we suppose the interest rate follows, by transforming the term structure model in a discrete time state space form and then estimating it using maximum likelihood and the Kalman filter algorithm to compute the unobservable variable.
Consider the term structure of the zero coupon bond:
$Y_t(\tau)= a + b\, r_t + v(\tau)$
where $Y(\tau)$ is the vector containing the yields at time t of the zcb with maturity t, and r is the interest rate.
The equation of the yield has been transformed into a discrete time state space system.
We are interested in testing the volatility of the interest rate affected by the risk premium.
The existence of a risk premium, defined as that part of the return that exceeds the expected returns on short-term bonds, implies that the information system on which expectations are based is incomplete; this may result in two distinct cases: i) the unexpected shocks are random, serially uncorrelated and hence unpredictable under both rational and adaptive expectations; ii) the shocks are serially correlated hence unpredictable only under adaptive expectations. It follows that it is only in one case, i.e. when expectation errors are not innovations, that the existence of the risk premium is sufficient to infer the type of expectation process used by the agents.

In what follows we assume that agents update their expectations when new information on spot and future returns becomes available (as it could happen in case i. above) as well as when they realise their expectations to be wrong and proceed to correct them (as it could happen in case ii above).

We estimate the stochastic process that generates the rate of interest by means of a variable parameter model, in particular we use a Kalman filter model and maximum likelihood estimation.
The process describing the evolution of the rate of interest is defined by Kalman's State Space Form which, as usual, comprises a structural equation (the so-called measurement equation) and an equation describing the variation of the permanent component in the structural equation (the transition equation). The Kalman filter starts the estimation using some initial conditions given by the information at time t=0.
The measurement and the transition equations are defined as follows:

(5) $r_t = \vartheta_t + \varepsilon_t$ "measurement equation"
(6) $\vartheta_t = \vartheta_{t-1} + \lambda_{t-1} + \xi_t$ "transition equation"
(7) $\lambda_t = \lambda_{t-1} + \omega_t$

where r_t is the real interest rate, ϑ_t is the permanent component of the real interest rate (the trend rate of interest), and λ_t is the variable component of the ϑ_t, i.e. the risk premium.
The error term ε_t of the interest rate is characterised by heteroscedasticity, it is normal distributed with zero mean and volatility that depends on the past information $\varepsilon_t \sim N(0, V_t \mid \Omega_{t-1})$.
We suppose that the innovation follows a GARCH(1,1) process in which

(8) $V_t = \alpha_0 + \alpha_1 V_{t-1} + \beta_1 \varepsilon_{t-1}^2$

We estimate the model under the assumption that the conditional variance is not constant. The errors terms, ξ_t, ω_t are assumed to be white noise (zero mean and constant variance).
Using compact matrix notation, equations (5)-(7) become:

(5') $r_t = x' \beta_t + \varepsilon_t$

(6') $\beta_t = T \beta_{t-1} + \eta_t$

where:

$x' = [1,0]$

$\beta_t = [\vartheta_t , \lambda_t]'$

$T = [1\ 1, 0\ 1]$

$\eta_t = [\xi_t , \omega_t]'$

The risk premium is given by the difference between the forward interest rate implied in the term structure and the observed interest rates.
The transition equation, which captures, in our case, the way in which the agents update the evolution of the permanent component is specified as follows:

$\vartheta_t = \vartheta^e_{t \mid t-1} + k_t (r_t - r^e_{t \mid t-1})$

where $k_t = (\sigma^2_0 + \sigma^2_\xi)/ \sigma^2_\varepsilon + (\sigma^2_0 + \sigma^2_\xi)$ is the "Kalman gain", σ^2_ε is the variance of ε and $var(\vartheta_{t|t-1}) = \sigma^2_0 + \sigma^2_\xi$ where σ^2_0 is the variance of the initial estimate of ϑ and σ^2_ξ is the variance of the sampling error.

Hence the hypothesis is that the agents update the permanent component of the real interest rate by adding, to the previous period estimate of ϑ, a part of the expectation error. How much of the expectation error is used, i.e. the value of k_t, depends on the "Kalman gain".

The credibility of the German moneatry policy, which ensue from the conduct of the German monetary authorities throughout the years, was enhanced by the fact that Deutsche Bundesbank policy assurged to the reference point for the single monetary policy adopted by the EU countries.
Yield differentials have acquired a specific relevance with the Maastricht Treaty the Protocol on convergence conditions for eligibility to the Economic and Monetary Union states that long term nominal interst rate on bond issued by a member state should not exceed the average level of the rates of the three best performing best members.
Germany yields have been and are the lowest on all maturities. The German interest rates are stable and they are not characterized by high volatility, the only period in which the interest rates seem to have peak is in corrispondence with the 1990 when occured the Re-unification.

Given the series of the optimal linear estimator (minimum MSE) of the unobservable state variable conditional on the past information, and given the variance of the estimation errors in t, we can calculate the log-likelihood function, which depends on the parameters of the model.
Maximising the function we obtain the estimates of the parameters of the model. The procedure is iterative, as the estimated model parameters are used to recalculate the unobservable variable with the Kalman filter, the procedure continues until that we reach the convergence.

By following this approach the empirical model describes the nominal interest rate as dependent on the risk premium, i.e. the variable component of the real interest rate extracted by the term structure of the interest rate and measured by difference of returns between actual and expected rates (expected rates calculated as forward rates implied in term structure given by the spot rate) and on the expected rate of inflation.

Hence, with respect to the original Fisher equation in addition to expected inflation we include in the empirical equation the nominal interest rate, the variable component of the real interest rate on the assumption that the real interest rate is not stationary.

3.1 Data and estimation

To estimate we use monthly data of the Italian and German bonds'returns (BOT and BUNDS respectively) with maturity three, six and twelve months; the sample period is 1980:1-1998:1.
The rate of inflation is computed as the log change of the consumer price index, and we replace the expected inflation rate (p_t) with the realised rate, under the assumption of rational expectations.
Before proceed to the variable parameter model the instability of the coefficients has been tested using RLS. It turns out that Italian bonds show structural instability problems on the whole sample period whereas German bonds have some problems only between 1985-90 in conjunction with the political changes which eventually determined the re-unification of Germany (the full set of results on RLS are available on request from the authors).
Table 1 below reports the results obtained using the time varying model on three and six-month returns. The results are different for Italy and for Germany.

The path followed by time varying parameter on the Italian zero coupon bonds with maturity three-month and six-month (Figure 3) suggests that the premium risk can be well defined using a state space model. The Kalman Filter allows describing the time variation better in the case of Italian zcb than in the case of the German zcb (Figure 4).
The results for Italy suggest that the premium risk is variable on all the sample period and in particular in the 1992 and in the 1993 where the graph shows sharply peaks.
The German results clearly don't show the presence of a variable premium risk even if the path followed by the return are unstable before the 1980.

Tab.1
Estimated State Space model by the Kalman Filter: risk premium unobservable variable

Country	Bond	Parameters	Estimates
Italy	Three-month ZCB (BOT)	p_t	2,96***
			(0,04)
		λ_t	-0,85**
			(0,30)
Log. Likelihood=-53,01			
Germany	Three-month ZCB (BOT)	p_t	1,85***
			(0,10)
		λ_t	0,20
			(0,55)
Log. Likelihood=-183,65			
Italy	Six-month ZCB (BOT)	p_t	2,90***
			(0,05)
		λ_t	-0,81**
			(0,36)
Log. Likelihood=-57,45			
Germany	Six-month ZCB (BOT)	p_t	1,82***
			(0,09)
		λ_t	0,4
			(0,43)
Log. Likelihood=-151,06			

*Statistically significant at the confidence level of 10%
**Statistically significant at the confidence level of 5%
***Statistically significant at the confidence level of 1%

The use of the Kalman filter allows us to recursively update the coefficients β_i and their variance, using new information on i_t and on the regressors p_t, λ_t. Thus we can evaluate how agents' expectations affect the variability of interest rate and we can test how the risk premium causes the returns.
Table 1 reports the results for Italy and Germany. The risk premium variable is statistically significant and this confirms our previous analysis.

The volatility of the interest rates is higher in correspondence with significant premium risk. The premium risk could be considered as a proxy of the agents'expectations given that it's highly volatile when the agents make forecasting errors.
The agents realised the computed errors in their estimation insert the new information in the decisional process affecting the stochastic process followed by the interest rate.
It's possible to test that the computed errors are a function of the time: the longer the maturity considered the higher is the forecaster error. Higher volatility causes unpredictable changes in the expected interest.
For the three-month Italian Bonds the risk premium is of almost one percentage point (-0,847), and it doubles for the annual bonds (-2,031). We reach a different conclusion when we analyse the German bonds. As already mentioned when we describe the term structure of "Bund", in this case as well it is evident that the risk premium does not seem significant as proxy of the market expectations and as variable of the expected rate. This allow us to conclude that in the German market the "Expectations Theory" holds and with it the Fisher hypothesis.

The estimation of the GARCH model is done using the same data as before, three-month, six-month and one-year interest rates on the Italian and German zero coupon bond in the sample period 1980:1-1998:1.

Here we report the results of the estimated GARCH(1,1) model for the six-month interest rates, the results for the three-month zero coupon bonds are reported in the Table A3 in Appendix.

Tab.2 Estimated GARCH(1,1) model of the six-month zero coupon bonds

Bond	Parameters	Estimates	Standard Errors
Italian ZCB (BOT)	c_0	0.17	0.19
	c_1	0,99***	0.01
	a_0	0,08*	0.06
	a_1	0,12*	0.06
	b_1	0,66***	0.21
Log. Likelihood=-180,22			
DW=1,89			
Adjusted R-squared = 0,97			
German ZCB (Bund)	c_0	0.05	0.08
	c_1	0,99***	0.01
	a_0	0,01***	0,49E-02
	a_1	0,08***	0.04
	b_1	0,84***	0.37
Log. Likelihood=-137,82			
DW=1,92			
Adjusted R-squared= 0,96			

*Statistically significant at the confidence level of 10%
**Statistically significant at the confidence level of 5%
***Statistically significant at the confidence level of 1%

The long term variance is convergent and the sum of the coefficients ($\alpha_1 + \beta_1$) is less than one, therefore the unconditional variance is well defined:

$\sigma^2_{nc} = \alpha_0 / (1-\alpha_1 -\beta_1)$.

Using the Italian data we observe an increase in volatility in correspondence of the shocks in the monetary policy, these shocks cause a negative difference between the expected interest rates and the forward interest rates. The volatility is very high when the risk premium is high, which means that there exists a correlation between the volatility of the errors in the stochastic process and the variance of the premium risk.
There is strong evidence of this phenomenon in correspondence of the 1992 when the Italian lira was suspended from the European Monetary System and when in the same period occurred the monetary restriction that caused an increasing in the interest rates at each maturity. After an initial increasing in the interest rates followed an expanding monetary policy that caused an unpredictable fluctuation in the interest rates and in their variance.
From the German data high volatility is observed during the 1989 in correspondence of the Re-unification but it is not possible to define the existence of a variable premium risk in all the sample period.
It could be possible to suppose that credible monetary policy, like the German one, makes agents' forecasts to be more precise than in the case of not credible or unexpected monetary policy.
The slope of the term structure and the stochastic process followed by the real interest rate give us some information on the agents' expectations on the returns and on the expected inflation.

4. Conclusion

Our main conclusion is that there is a very close relationship between time-varying parameter on the premium risk, high volatility in interest rates and agents' expectations.

Where agents are unable to capture the shocks and insert them in the information set, the interest rates seem to follow a stochastic process influenced by the structure of the innovation term.

The credibility of the monetary policy plays an important role in evaluating the shocks and the changes of the interest rates and this is particular evident in the Treasury bonds market. We find different behaviours of Italian and German interest rates; in particular, whereas interest rates on Italian bonds are affected by time-varying risk premium the latter does not show up in the German bond market.

It follows that in the Italian case the relationship between nominal interest rate, real interest rate and expected inflation rate can not be explained by the Fisher equation and more generally, the forward interst rate cannot be taken as an unbiased estimator of the expected short interest rate.

The Expectation theory in a generalised framework of the Fisher theory well applies to German bond market.

The process followed by the premium risk is the primary key in the understanding such differences between markets.

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Figure 1.

Term Structure of the Italian Interest Rates

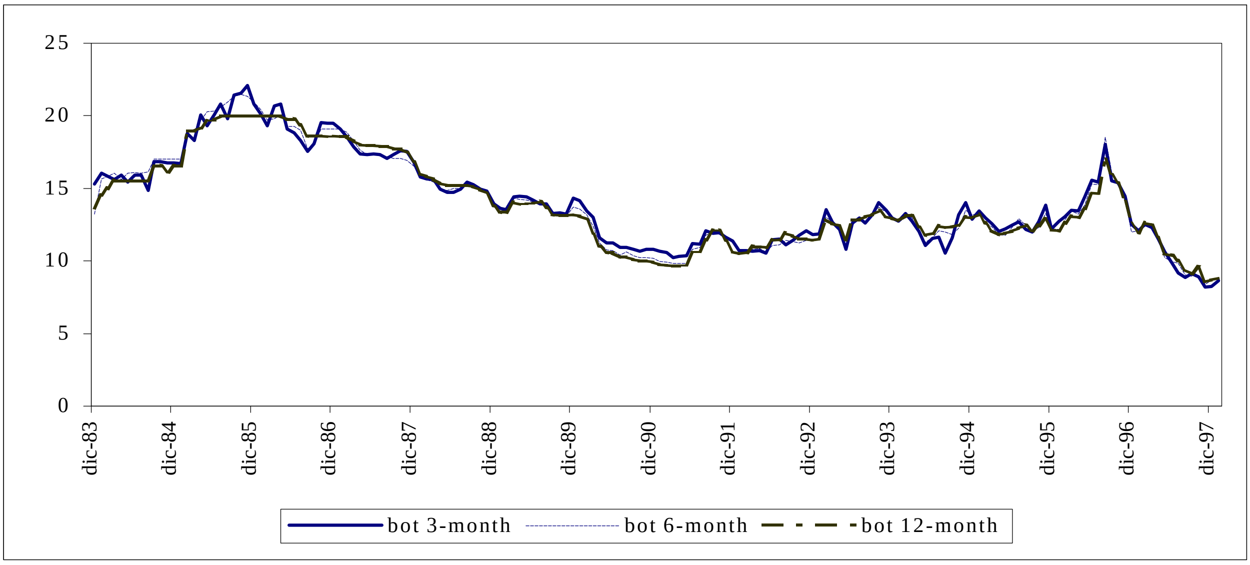


Figure 2.

Term Structure of the German Interest Rates

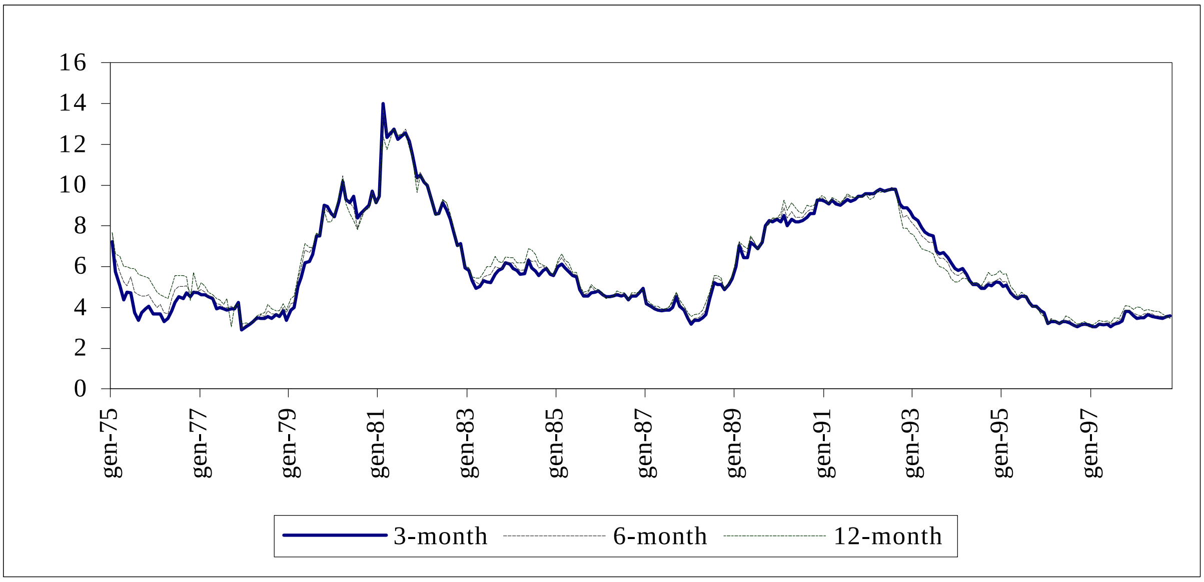


Table A2.

TAB.A2a) Estimated OLS regression: Italian results

Regression: Italian results	Parameters	Parameters value
(2) ${}_{t+3}R_3 - {}_tR_3 = a + b\,[2\,({}_tR_6 - {}_tR_3)] + u_t$	a	8,12E-017
	b	0,99***
DW=2,24		
R^2=1		
(3) ${}_{t+6}R_6 - {}_tR_6 = a + b\,[2\,({}_tR_{12} - {}_tR_6)] + u_t$	a	-2,86E-16
	b	1,99***
DW=1,85		
R^2=1		
Sample period 1989:1-1993:12		
(2) ${}_{t+3}R_3 - {}_tR_3 = a + b\,[2\,({}_tR_6 - {}_tR_3)] + u_t$	a	12,99**
	b	1,86**
DW=0,25		
R^2=0,26		
(3) ${}_{t+6}R_6 - {}_tR_6 = a + b\,[2\,({}_tR_{12} - {}_tR_6)] + u_t$	a	12,54**
	b	1,59**
DW=0,12		
R^2=0,18		

*Statistically significant at the confidence level of 10%
**Statistically significant at the confidence level of 5%
***Statistically significant at the confidence level of 1%

TAB.A2b) Estimated OLS regression: German results

Regression: German results	Parameters	Parameters value
(2) ${}_{t+3}R_3 - {}_tR_3 = a + b\,[2\,({}_tR_6 - {}_tR_3)] + u_t$	a	-2,99E-017
	b	1***
DW=2,02		
R^2=1		
(3) ${}_{t+6}R_6 - {}_tR_6 = a + b\,[2\,({}_tR_{12} - {}_tR_6)] + u_t$	a	-3,11E-17
	b	1***
DW=1,85		
R^2=1		
Sample period 1989:1-1993:12		
(2) ${}_{t+3}R_3 - {}_tR_3 = a + b\,[2\,({}_tR_6 - {}_tR_3)] + u_t$	a	4,21**
	b	1,01**
DW=0,36		
R^2=0,41		
(3) ${}_{t+6}R_6 - {}_tR_6 = a + b\,[2\,({}_tR_{12} - {}_tR_6)] + u_t$	a	19,23**
	b	1,04**
DW=0,25		
R^2=0,54		

*Statistically significant at the confidence level of 10%
**Statistically significant at the confidence level of 5%
***Statistically significant at the confidence level of 1%

Table A3.

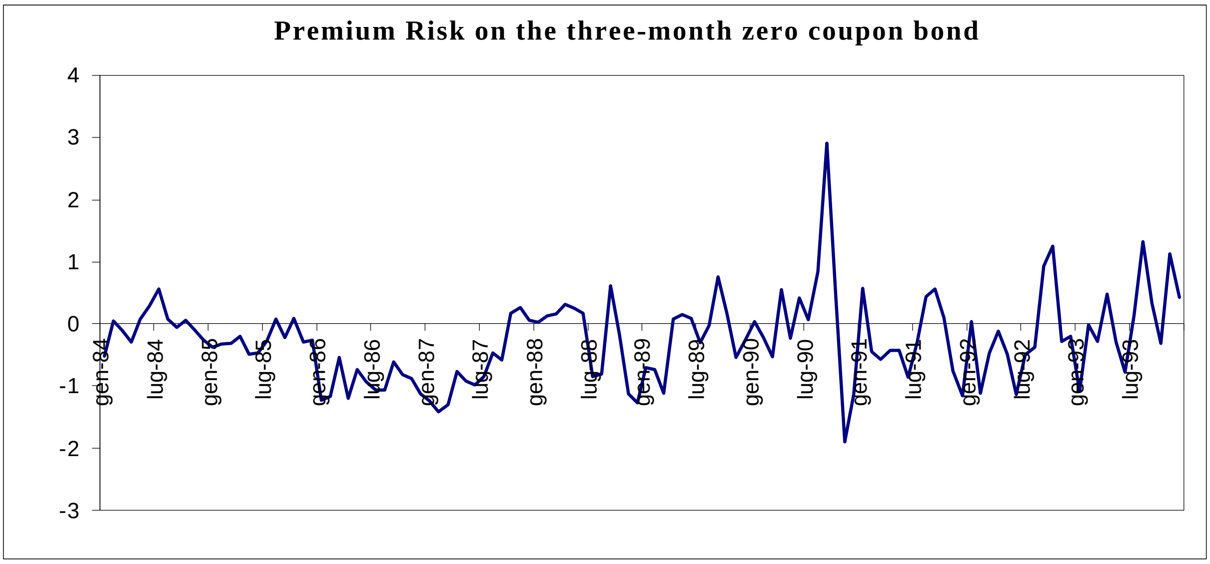
Estimated GARCH(1,1) model of the three-month zero coupon bonds

Bond	Parameters	Estimates	Standard Errors
Italian ZCB (BOT)	c_0	0,1	0,38
	c_1	0,99***	0,03
	a_0	0,13*	0,06
	a_1	0,21*	0,11
	b_1	0,51***	0,19
Log. Likelihood=-142,02			
DW=2,08			
Adjusted R-squared = 0,89			
German ZCB (Bund)	c_0	0,75E-02	0,06
	c_1	0,99***	0,68E-02
	a_0	0,6***	0,23E-02
	a_1	0,14***	0,04
	b_1	0,84***	0,04
Log. Likelihood=-136,98			
DW=2,01			
Adjusted R-squared= 0,96			

*Statistically significant at the confidence level of 10%
**Statistically significant at the confidence level of 5%
***Statistically significant at the confidence level of 1%

Figure 3.

Italian premium risk on the zero coupon bond



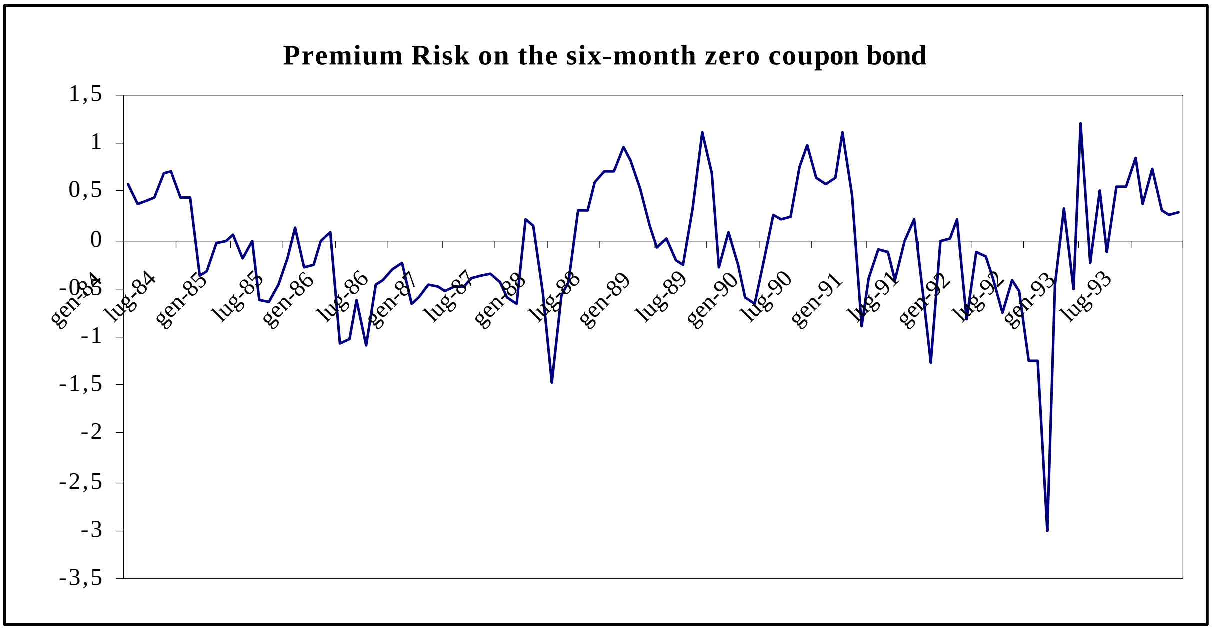


Figure 4.

German premium risk on the zero coupon bond