EC 362 Fall 2000
Exercise Two Key

October 2, 2000

Chapter 3. #4. The combination of a long put and a long call, with the same strike price and maturity date, is a long straddle. The profit diagram consists of two 45 degree lines emanating from the strike price (one at 45 degrees, one at 135 degrees). The position pays off as long as the underlying price moves away from the strike price, since each of the options are at-the-money.

If instead the put strike is below the call strike, we have a long strangle. This position will not be in-the-money until the underlying price rises (or falls) by a greater amount, but since each of the options are out-of-the-money, the combined premium (cost of the strategy) is lower than that of the long straddle.

#9. The company has purchased a call with a strike of $30 and sold a put with a strike of $22. The payoff will be:

	$S_T < 22$	$22 < S_T < 30$	$30 < S_T$
Long call	0	0	$S_T - 30$
Short put	$-[22 - S_T]$	0	0
Total	$-[22 - S_T]$	0	$S_T - 30$

This contract gives price protection to the pension fund manager, so that she will not have to pay more than $30 per share for the stock. The cost of the protection is reduced by the premium received on selling the put, which is netted against the premium paid for the call.

#10. a) The cost of the discount bond is $B(0,100) = 1 - 0.05\left(\frac{100}{360}\right) = 0.9861$. Using put-call parity, the value of the put option is:

$$\begin{aligned} p\,[50, 47.5, 100] &= 4.375 + 47.5\,(0.9861) - 50 \\ &= 1.21 \end{aligned}$$

Since the option is quoted at the considerably higher price of \$2.125, there is an arbitrage opportunity.

b) Since the put is overpriced, you want to sell it at this price, and take advantage of put-call parity to reduce the risk of this position. Buy the call option, lend the present value of the strike price, and short the stock. At time 0, you will receive \$2.125 from the sale of the put, purchase the call for \$4.375, lend \$47.50(0.9861), and short the stock, receiving \$50.00. The net proceeds are \$2.125-\$1.21=\$0.915.

After 100 days:

	$S < 47.5$	$S > 47.5$
Short put	$-[47.5 - S]$	0
Long call	0	$S - 47.5$
Riskless investment	47.50	47.50
Close short position	$-S$	$-S$
Total	0	0

Since you receive \$0.915 today and a net of zero after 100 days, irregardless of the stock price, this is riskless arbitrage.

Chapter 4. #4. a. $\Pr[S = 121] = 0.75\ (0.75) = 0.5625$

b.$\Pr[S = 99] = 0.75(0.25){+}0.25(0.75){=}0.375$

c. $\Pr[S = 81] = 0.25(0.25) = 1.0\text{-}\Pr[S = 121] - \Pr[S = 99] = 0.0625$

d. $E[S_1] = 0.75(110) + 0.25(90) = 105$, and $E[S_2] = 0.5625(121) + 0.375(99) + 0.0625(81) = 110.25$.

e. $Var[S_1] = (110 - 105)^2(0.75) + (90 - 105)^2(0.25) = 75$

$Var[S_2] = (121{-}110.25)^2(0.5625){+}(99{-}110.25)^2(0.375){+}(81{-}110.25)^2(0.0625) = 165.9375$. The variance is increasing over time.

#7. The drift term is $\mu h = 0.15\left(\frac{7}{365}\right) = 0.002877$ while the volatility term is $\sigma h = 0.30\left(\frac{7}{365}\right) = 0.041545$. Therefore

$$\begin{aligned} U &= \exp\left[\mu h + \sigma\sqrt{h}\right] = 1.0454 \\ D &= \exp\left[\mu h - \sigma\sqrt{h}\right] = 0.9621 \end{aligned}$$

a.The lattice is then

$$
\begin{array}{cccc}
 & & & 114.25 \\
 & & 109.29 & \\
 & 104.54 & & 105.14 \\
100 & & 100.58 & \\
 & 96.21 & & 96.77 \\
 & & 92.56 & \\
 & & & 89.06
\end{array}
$$

b. The expected stock price after one week is
0.5(104.54)+0.5(96.21)=100.38.
c. The expected stock price after two weeks is
0.25(109.29)+0.5(100.58)+0.25(92.56)=100.75.
d. The expected stock price after three weeks is
0.125(114.25)+0.375(105.14)+0.375(96.77)+0.125(89.06)=101.13.
e. Using equation 4.7,

$$
\begin{aligned}
\mu T + \sigma^2 T/2 &= (0.15 + 0.09(0.5)) \left(\frac{21}{365}\right) = 0.011219, so \\
E\left[S_3|S_0\right] &= 100 \exp(0.011219) = 101.128
\end{aligned}
$$

essentially the same as that arising from part d.