EC 362 Fall 2000 Exercise 3

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Chapter 5

Q 1.

a. From (5.42), the up and down factors are 1.2394 and 0.8109.

b. The martingale Pr[up] is $\frac{R(h)-D}{U-D}$ where R(h)=exp[0.05 × 0.5]=1.0253, so $\pi = 0.5004$.

c. In the lattice, only the UU state yields positive option value, so $C(0) = \frac{1}{A(2)}\left[\pi^2 11.4445 + 2\pi(1-\pi)0 + (1-\pi)^2 0\right] = 2.7259$ given that $A(2)$ is $\exp[0.05] = 1.0513$.

d. If the stock moves up in period 1,

$$\begin{aligned} m_1 61.4445 + B_1 1.0253 &= 11.4445 \\ m_1 40.2012 + B_1 1/0253 &= 0 \end{aligned}$$

yielding $m_1 = 0.5387$, $B_1 = -21.1234$, and $V(1) = 5.5832$. If the stock moves down in period 1, the option value $V(1)$ is zero. Thus the value in period 0 must compare 5.5832 and zero:

$$\begin{aligned} m_1 49.5760 + B_1 1.0253 &= 5.5832 \\ m_1 32.4360 + B_1 1/0253 &= 0 \end{aligned}$$

yielding $m_0 = 0.3257$, $B_0 = -10.3050$, and $V(0) = 2.7246$, the value of the call option.

Q 3.

a. From (5.42), the up and down factors are 1.1019 and 0.9108.

b. The martingale Pr[up] is $\frac{R(h)-D}{U-D}$ where R(h)=1.0063, so $\pi = 0.5$.

c. $F(1,2) = \pi F(2,2)^{+} + (1-\pi) F(2,2)^{0} = 66.5338$. $F(1,2)^{-} = 54.9950$. $F(0,2) = \pi F(1,2)^{+} + (1-\pi) F(2,2)^{-} = 60.7644$.

Q6.

$S(0) = 100$; $S(1)^+ = 121.0643$; $S(1)^- = 85.0100$. $S(2)^+ = 146.5657 = F(2,2)^+ \cdot S(2)^0 = 102.9167 = F(2,2)^0$. $S(2)^- = 72.2669 = F(2,2)^-$.

The six month investment rate $R(h) = 1.03045$, and the martingale probability is $\pi = 0.50023$. The futures prices ($\pi-$ weighted averages of the terminal futures prices) are $F(1,2)^+ = 124.7513$, $F(1,2)^- = 87.5989$, $F(0,2) = 106.1837$.

To replicate a long stock position using futures and riskless bonds, at $t = 0$, the replicating portfolio is $V(0) = m_0 0 + B_0 = 100$, or $B_0 = 100$. At $t = 1$ if $S(1)^+ = 121.0643$,

$$m_0(124.7517 - 106.1837) + B_0 1.03045 = 121.0643$$

while if $S(1)^- = 85.0100$,

$$m_0(87.5989 - 106.1837) + B_0 1.03045 = 85.0100$$

implying that $m_0 = 0.9704$. We can use similar logic to compute $m_1^+ = 1$ and $B_1^+ = 121.0643$ if the stock goes up, and $m_1^- = 1$ and $B_1^- = 85.01$ if the stock goes down. In other words you replicate the stock by investing the value of one share at all times, and reap the return (+/-) on the corresponding futures contract to receive the capital gain (loss) income on the stock.

Chapter 8

Q 1. A longer term increases the option premium; the effect on δ depends on whether the option is in-the-money or out-of-the-money. If it is out-of-the-money δ rises with the term (and vice versa).

Q 2. (a) decreases with longer term; (b) and (c), increases with longer term. (d) For calls, the effect of longer term is always positive.

Q 4.

(a) You are long a call with strike of \$30 and short a put with strike of \$X.

(b) c = 0.19046.

(c) The critical strike price that (approximately) equates the put premium to the call premium is \$21.70.

(d) If the underlying volatility is higher, the critical strike price is still in the same vicinity, even though the premia are almost twice as large.

Chapter 9 Q 10. Numerical.