EC 362 Fall 2000 Exercise 4 (corrected)

December 2, 2000

Chapter 11
Q 5.
a. The two-period lattice will be

		$S(2)^+ = 0.2009$
	$S(1)^+ = 0.1880$	
S(0)=0.1760		$S(2)^0 = 0.1696$
	$S(1)^- = 0.1587$	
		$S(2)^- = 0.1431$

At $t = 1$,

$$\begin{aligned} c(1)^+ &= \frac{1}{1.0151}\left[0.5 \times 0.0409 + 0.5 \times 0.0096\right] = 0.0249 \\ c(1)^- &= \frac{1}{1.0151}\left[0.5 \times 0.0096 + 0.5 \times 0\right] = 0.0047 \end{aligned}$$

At $t = 0$,

$$c(0) = \frac{1}{1.0151}\left[0.5 \times 0.0249 + 0.5 \times 0.0047\right] = 0.0145$$

Given that the contract size is 250,000 FF the cost of the option will be $3,644.96.

b. At $t = 1$, if $S(1)^+$ =0.1880 the value of the option, if exercised, is 0.0280, so that the value of an American option is

$$c(1)^+ = Max\left[0.0280, 0.0249\right] = 0.0280$$

implying that early exercise is optimal. If $S(1)^- = 0.1587$, exercise would not be optimal. The value of the option at $t = 0$ is

$$c(0) = \frac{1}{1.0151}\left[0.5 \times 0.0280 + 0.5 \times 0.0047\right] = 0.0161$$

which exceeds its exercise value of (0.1760-0.1600). Thus, the value of the American option at $t = 0$ is 0.0161/FF.

Chapter 12

Q1. Profit from the short position is $-(507.30 - 512.15) \times 250$, while profit from the long position is (511.55-516.45)$\times$250. Thus total profit is $250 \times (4.25 - 4.30) = -12.50$. The spread decreases.

Q2. Profit from the long position is $20(183.65 - 186.75) \times 500 = -31,000$ while profit from the short position is $-15(507.30 - 512.15) \times 250 = 18187.50$. Net profit is -12812.50.

Chapter 13

Q1. The price of the T-bill is $1000000\left[1 - 0.03\frac{90}{360}\right] = 992,500$. If the discount rate increases by one bp to 3.01%, the price will be 992,475; a decline of $25.00.

Q2. The value is $1000000 \exp\left[-0.045\right] = \$955,997.48$. The yield expressed as a simple interest rate is

$$955997.48 = \frac{1000000}{1 + \frac{i_s}{100}}, i_s = 4.60\%.$$

Q4. The discretely compounded YTM is given by

$$104.33 = \sum_{j=1}^{4} \frac{6.25}{(1+y)^j} + \frac{100}{(1+y)^4}, y = 5.03\%.$$

The continuously compounded YTM is given by

$$104.33 = \sum_{j=1}^{4} 6.25 \exp\left[-y_c \times j\right] + 100 \exp\left[-4y_c\right], y_c = 4.91\%.$$

Q6. Accrued interest is $\frac{6.375}{2}\frac{131}{182} = 2.2943$. The "flat" price is $105\frac{28}{32} = 105.8750$, so the invoice price is 108.17.

Q8. The futures price is $114\ \frac{23}{32} = 114.8125$. The net return to the short is
bond 1: $114.8125 \times 1.3987 - 162.625 = -2.0368$
bond 2: $114.8125 \times 1.2820 - 139.96875 = 8.2209$
bond 3: $114.8125 \times 1.273 - 131.0625 = 15.0938$
so that bond 3 is C-T-D.(The conversion factor for this bond is a misprint: conversion factors always have 4 decimal digits).

Q9.
a. The market value of the bond is $5\sum_{j=1}^{10} B\left(0, T_j\right) + 100 \times B(0,5) = 117.74$.
b. The YTM is

$$117.74 = \sum_{j=1}^{10} \frac{5}{\left(1+\frac{y_s}{2}\right)^j} + \frac{100}{\left(1+\frac{y_s}{2}\right)^{10}}, y_s = 5.86\%$$

c. The modified duration is

$$d = \frac{1}{2} \frac{\left(\sum_{j=1}^{10} \frac{j \times 5}{\left(1+\frac{y_s}{2}\right)^j} + \frac{10 \times 100}{\left(1+\frac{y_s}{2}\right)^{10}}\right)}{\left(1+\frac{y_s}{2}\right) \times 117.74} = 4.0210.$$