## BOSTON COLLEGE

Department of Economics
EC 228 Econometrics, Prof. Baum, Ms. Yu, Fall 2003

## Problem Set 7 Solutions

Problem sets should be your own work. You may work together with classmates, but if you're not figuring this out on your own, you will eventually regret it.

1. (10.2)

We follow the hint and write

$$
g G D P_{t-1}=\alpha_{0}+\delta_{0} i n t_{t-1}+\delta_{1} i n t_{t-2}+\mu_{t-1},
$$

and plug this into the right hand side of the $i n t_{t}$ equation:

$$
\begin{aligned}
\text { int }_{t} & =\gamma_{o}+\gamma_{1}\left(\alpha_{o}+\delta_{0} i n t_{t-1}+\delta_{1} i n t_{t-2}+\mu_{t-1}-3\right)+v_{t} \\
& =\left(\gamma_{0}+\gamma_{1} \alpha_{0}-3 \gamma_{1}\right)+\gamma_{1} \delta_{0} \text { int }_{t-1}+\gamma_{1} \delta_{1} i n t_{t-2}+\gamma_{1} \mu_{t-1}+v_{t}
\end{aligned}
$$

Now by assumption, $\mu_{t-1}$ has zero mean and is uncorrelated with all right hand side variables in the previous equation, except itself of course. So

$$
\operatorname{Cov}\left(i n t, \mu_{t-1}\right)=E\left(\text { int }_{t} \cdot \mu_{t-1}\right)=\gamma_{1} E\left(\mu_{t-1}^{2}\right)>0
$$

because $\gamma_{1}>0$. If $\sigma_{\mu}^{2}=E\left(\mu_{t}^{2}\right)$ for all $t$ then $\operatorname{Cov}\left(i n t, \mu_{t-1}\right)=\gamma_{1} \sigma_{\mu}^{2}$. This violates the strict exogeneity assumption, TS.2. While $\mu_{t}$ is uncorrelated with $i n t_{t}, i n t_{t-1}$, and so on, $\mu_{t}$ is correlated with $i n t_{t+1}$.
2. (10.5)

The functional form was not specified, but a reasonable one is

$$
\log \left(\text { hsestrts }_{t}\right)=\alpha_{0}+\alpha_{1} t+\delta_{1} Q 2_{t}+\delta_{2} Q 3_{t}+\delta_{3} Q 4_{t}+\beta_{1} i n t_{t}+\beta_{2} \log \left(\text { pcinc }_{t}\right)+\mu_{t}
$$

Where $Q 2_{t}, Q 3_{t}$, and $Q 4_{t}$ are quarterly dummy variables (the omitted quarter is the first) and the other variables are self-explanatory. The inclusion of the linear time trend allows the dependent variable and $\log \left(\right.$ pcinc$\left._{t}\right)$ to trend over time (int $t_{t}$ robably does not contain a trend), and the quarterly dummies allow all variables to display seasonality. The $\beta_{2}$ is an elasticity and $100 \cdot \beta_{1}$ is a semi-elasticity.
3. (10.8)
(i)
(i) . use http://fmwww.bc.edu/ec-p/data/wooldridge/BARIUM

| Source | SS | df | MS | Number of obs | $=131$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | F( 7, 123) | 9.95 |
| Model | 23.0142638 | 7 | 3.28775197 | Prob > F | $=0.0000$ |
| Residual | 40.637988 | 123 | . 330390146 | R-squared | $=0.3616$ |


| Total | 63.6522517 | 130.4 | 489632706 |  | Adj R-squared <br> Root MSE | $\begin{aligned} & =0.3252 \\ & =\quad .5748 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| lchnimp | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| lchempi | -. 6862247 | 1.239712 | -0.55 | 0.581 | -3.140158 | 1.767709 |
| lgas | . 4656256 | . 8761836 | 0.53 | 0.596 | -1.268726 | 2.199977 |
| lrtwex | . 0782138 | . 4724404 | 0.17 | 0.869 | -. 8569531 | 1.013381 |
| befile6 | . 0904704 | . 2512888 | 0.36 | 0.719 | -. 4069404 | . 5878812 |
| affile6 | . 0970053 | . 2573132 | 0.38 | 0.707 | -. 4123303 | . 6063409 |
| afdec6 | -. 351502 | . 2825419 | -1.24 | 0.216 | -. 9107763 | . 2077723 |
| t | . 0127058 | . 0038443 | 3.31 | 0.001 | . 0050963 | . 0203153 |
| _cons | -2.366326 | 20.78231 | -0.11 | 0.910 | -43.50364 | 38.77099 |

Adding a linear time trend to (10.22) gives

$$
\begin{aligned}
& \log (\widehat{\text { chnimp })}=\underset{(20.78)}{-2.37} \underset{(1.240)}{.686} \log (\text { chempi })+\underset{(.876)}{.466 \log (\text { gas })}+\underset{(.472)}{.078 \log (\text { rtwex })} \\
& +.090 \text { befile } 6+.097 \text { af fine } 6-.351 \text { afdec } 6+.013 t \\
& \text { (.251) (.257) (.282) (.004) } \\
& n=131, R^{2}=.362, \overline{R^{2}}=.325 \text {. }
\end{aligned}
$$

Only the trend is statistically significant. In fact, in addition to the time trend, which has a $t$ statistic over three, only afdec6 has a $t$ statistic bigger than one in absolute value. Accounting for a linear trend has important effects on the estimates.
(ii) . test lchempi lgas lrtwex befile6 affile6 afdec6
( 1) lchempi $=0.0$
(2) $\mathrm{lgas}=0.0$
( 3) lrtwex $=0.0$
( 4) befile6 $=0.0$
(5) affile6 = 0.0
( 6) afdec6 $=0.0$

$$
\begin{array}{rll}
F(6, \quad 123) & = & 0.54 \\
\text { Prob }>F & =0.7767
\end{array}
$$

The F statistic for joint significance of all variables except the trend and intercept, of course, is about .54. The $d f$ in the F distribution are 6 and 123 . The $p$-value is about .78 , and so the explanatory variables other than the time trend are jointly very insignificant. We would have to conclude that once a positive linear trend is allowed for, nothing else helps to explain $\log ($ chnimp $)$. This is a problem for the original event study analysis.
(iii) . regress lchnimp lchempi lgas lrtwex befile6 affile6 afdec6 t feb mar apr may jun
jul aug sep oct nov dec

. test feb mar apr may jun jul aug sep oct nov dec

```
    ( 1) feb = 0.0
    (2) mar = 0.0
    ( 3) apr = 0.0
    (4) may = 0.0
    (5) jun = 0.0
    (6) jul = 0.0
    (7) aug = 0.0
    (8) sep = 0.0
    ( 9) oct = 0.0
    (10) nov = 0.0
    (11) dec = 0.0
        F(11, 112) = 0.85
        Prob > F = 0.5943
```

Nothing of importance changes. In fact, the $p$-value for the test of joint significance of all variables except the trend and monthly dummies is about .79 . The 11 monthly dummies themselves are not jointly significant: $p$-value $\approx .59$.
4. (10.9)
. use http://fmwww.bc.edu/ec-p/data/wooldridge/PRMINWGE
. regress lprepop lmincov lusgnp lprgnp t

| Source | SS | df | MS |
| :---: | :---: | :---: | :---: |
| Model | . 284429802 | 4 | . 071107451 |
| Residual | . 035428549 | 33 | . 001073592 |
| Total | . 319858351 | 37 | . 00864482 |


| Number of obs | $=38$ |  |
| :--- | ---: | ---: |
| F 4, | $33)$ | $=66.23$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.8892 |
| Adj R-squared | $=$ | 0.8758 |
| Root MSE | $=.03277$ |  |


| lprepop \| | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| lmincov \| | -. 2122611 | . 0401525 | -5.29 | 0.000 | -. 293952 | -. 1305703 |
| lusgnp \| | . 4860416 | . 2219838 | 2.19 | 0.036 | . 0344121 | . 937671 |
| lprgnp \| | . 2852399 | . 0804923 | 3.54 | 0.001 | . 1214771 | . 4490027 |
| t \\| | -. 0266632 | . 0046267 | -5.76 | 0.000 | -. 0360764 | -. 01725 |
| _cons \| | -6.663407 | 1.257838 | -5.30 | 0.000 | -9.222497 | -4.104317 |

Adding $\log (p r g n p)$ to equation (10.38) gives

$$
\begin{aligned}
\log \left(\text { prepop }_{t}\right)= & \underset{(1.26)}{-6.66-.212 \log \left(\text { mincov }_{t}\right)}+\underset{(.222)}{.486} \log \left(\text { usgnp }_{t}\right)+\underset{(.080)}{.285} \log \left(\text { prgnp }_{t}\right) \\
& -.027 t \\
& (.005) \\
n= & 38, R^{2}=.889, \overline{R^{2}}=.876 .
\end{aligned}
$$

The coefficient on $\log \left(p r g n p_{t}\right)$ is very statistically significant $(t$ statistic $\approx 3.54)$. Because the dependent and independent variable are in logs, the estimated elasticity of prepop with respect to $\operatorname{prgnp}$ is .285 . Including $\log (p r g n p)$ actually increases the size of the minimum wage effect: the estimated elasticity of prepop with respect to mincov is now -.212, as compared with -. 169 in equation (10.38).
5. (10.13)
(i) . use http://fmwww.bc.edu/ec-p/data/wooldridge/CONSUMP
. regress gc gy

| Source | SS | df | MS |
| :---: | :---: | :---: | :---: |
| Model | . 003793616 | 1 | . 003793616 |
| Residual | . 001796085 | 34 | . 000052826 |
| Total | . 005589701 | 35 | . 000159706 |


| Number of obs | $=$ | 36 |
| :--- | ---: | ---: |
| F ( 1, 34) | $=71.81$ |  |
| Prob $>$ F | $=0.0000$ |  |
| R-squared | $=0.6787$ |  |
| Adj R-squared | $=0.6692$ |  |
| Root MSE | $=.00727$ |  |


| gc l | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| gy I | . 5707806 | . 0673545 | 8.47 | 0.000 | . 4338998 | . 7076613 |
| _cons \| | . 0080792 | . 0018991 | 4.25 | 0.000 | . 0042197 | . 0119386 |

The estimated equation is

$$
\begin{aligned}
\widehat{g c_{t}} & =\begin{array}{l}
0081+.571 g y_{t} \\
(.0019)(.067)
\end{array} \\
n & =36, R^{2}=.679 .
\end{aligned}
$$

This equation implies that if income growth increases by one percentage point, consumption growth increases by .571 percentage points. The coefficient on $g y_{t}$ is very statistically significant ( $t$ statistic $\approx 8.5$ ).
(ii) . regress gc gy gy_1

| Source \| | SS | df MS |  |  | Number of obs $=35$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | F ( 2, 32) | $=36.51$ |
| Model \| | . 003855812 | 2.00 | 7906 |  | Prob > F | $=0.0000$ |
| Residual \| | . 001689785 | 32.00 | 8806 |  | R-squared | $=0.6953$ |
|  |  |  |  |  | Adj R-squared | $=0.6762$ |
| Total \| | . 005545597 | 34.00 | 3106 |  | Root MSE | $=.00727$ |
| gc । | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| gy 1 | . 5522502 | . 0696507 | 7.93 | 0.000 | . 4103763 | . 6941241 |
| gy_1 \| | . 0962134 | . 0690192 | 1.39 | 0.173 | -. 0443741 | . 236801 |
| _cons \| | . 0063567 | . 0022616 | 2.81 | 0.008 | . 0017499 | . 0109634 |

Adding $g y_{t-1}$ to the equation gives

$$
\begin{aligned}
\widehat{g c_{t}} & =.0064+\underset{(.0023)}{.552 g y_{t}+} \quad(.070)\left(.096 g y_{t-1}\right. \\
n & =35, R^{2}=.695
\end{aligned}
$$

The $t$ statistic on $g y_{t-1}$ is only about 1.39 , so it is not significant at the usual signiicance levels. (It is significant at the $20 \%$ level against a two-sided alternative.) In addition, the coefficient is not especially large. At best there is weak evidence lags in consumption.
(iii) . regress gc gy r3

| Source \| | SS | df MS |  |  | Number of obs $=36$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | F( 2, 33) | $=$ | 35.03 |
| Model \| | . 00379999 | 2.00 | 99995 |  | Prob > F |  | 0.0000 |
| Residual \| | . 001789711 | 33.00 | 54234 |  | R -squared |  | 0.6798 |
|  |  |  |  |  | Adj R-squared |  | 0.6604 |
| Total \| | . 005589701 | 35.00 | 59706 |  | Root MSE | = | . 00736 |
| gc । | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. |  | terval] |
| gy \| | . 5781102 | . 0715164 | 8.08 | 0.000 | . 432609 |  | . 7236114 |
| r3 \| | -. 0002148 | . 0006265 | -0.34 | 0.734 | -. 0014895 |  | . 0010599 |
| _cons \| | . 0082181 | . 0019665 | 4.18 | 0.000 | . 0042173 |  | . 012219 |

If we add $r 3_{t}$ to the model estimated in part (i) we obtain

$$
\begin{aligned}
\widehat{g c_{t}} & =\begin{array}{l}
.0082+.578 g y_{t}+.00021 r 3_{t} \\
(.0020)(.072) \\
n
\end{array}=36, R^{2}=.680 .
\end{aligned}
$$

The $t$ statistic on $r 3_{t}$ is very small. The estimated coefficient is also practically small: a one-point increase in $r 3_{t}$ reduces consumption growth by about .021 percentage points.

## 6. (10.17)

(i) The variable beltlaw becomes one at $t=61$, which correspnods to January, 1986. The variable spdlaw goes from zero to one at $t=77$, which corresponds to May, 1987.
(ii) . use http://fmwww.bc.edu/ec-p/data/wooldridge/TRAFFIC2
. regress ltotacc $t$ feb mar apr may jun jul aug sep oct nov dec

| Source \| | SS | df MS |  |  | Number of obs $=108$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | F ( 12, 95) | $=31.06$ |
| Model \| | 1.00244222 | 12.08 | 36851 |  | Prob > F | $=0.0000$ |
| Residual \| | . 255498294 | 95.00 | 89456 |  | R -squared | $=0.7969$ |
|  |  |  |  |  | Adj R-squared | $=0.7712$ |
| Total \| | 1.25794051 | 107.01 | 56453 |  | Root MSE | $=.05186$ |
| ltotacc \| | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| t l | . 0027471 | . 0001611 | 17.06 | 0.000 | . 0024274 | . 0030669 |
| feb | -. 042684 | . 0244476 | -1.75 | 0.084 | -. 0912186 | . 0058505 |
| mar | . 0798279 | . 0244491 | 3.27 | 0.002 | . 0312902 | . 1283656 |
| apr | . 0184875 | . 0244518 | 0.76 | 0.451 | -. 0300555 | . 0670304 |
| may I | . 0320994 | . 0244555 | 1.31 | 0.192 | -. 0164509 | . 0806497 |
| jun 1 | . 0201944 | . 0244603 | 0.83 | 0.411 | -. 0283653 | . 0687542 |
| jul \| | . 037584 | . 0244661 | 1.54 | 0.128 | -. 0109874 | . 0861553 |


| aug \| | .0539858 | .024473 | 2.21 | 0.030 | .0054007 | .1025708 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| sep \| | .042362 | .024481 | 1.73 | 0.087 | -.0062389 | .0909628 |
| oct \| | .0821147 | .02449 | 3.35 | 0.001 | .033496 | .1307334 |
| nov | .07128 | .0245 | 2.91 | 0.005 | .0226413 | .1199187 |
| dec \| | .0961584 | .0245111 | 3.92 | 0.000 | .0474976 | .1448191 |
| _cons \| | 10.46856 | .0190029 | 550.89 | 0.000 | 10.43084 | 10.50629 |

```
. test feb mar apr may jun jul aug sep oct nov dec
    ( 1) feb = 0.0
    ( 2) mar = 0.0
    (3) apr = 0.0
    (4) may = 0.0
    (5) jun = 0.0
    (6) jul = 0.0
    (7) aug = 0.0
    (8) sep = 0.0
    (9) oct =0.0
    (10) nov = 0.0
    (11) dec = 0.0
        F(11, 95) = 5.15
        Prob > F = 0.0000
```

The OLS regression gives

$$
\begin{aligned}
\log (\widehat{t o t a c c})= & \left.\begin{array}{l}
10.469+.00275 t-.0427 \\
(.019) \\
(.00016) \\
(.0244)
\end{array}\right)(.0798 \mathrm{mar}+\underset{(.0244)}{.0185 \mathrm{apr}} \quad \\
& +.0424 \mathrm{sep}+.0821 \text { oct }+.0713 \text { nov }+.0962 \mathrm{dec} \\
& (.0245)(.0245)(.0245) \\
n= & 108, R^{2}=.797 .
\end{aligned}
$$

When multiplied by 100, the coefficient on $t$ gives roughly the average monthly percentage growth in totacc, ignoring seasonal factors. In other words, once seasonality is eliminated, totacc grew by about $.275 \%$ per month over this period, or, $12(.275)=3.3 \%$ at an annual rate.

There is pretty clear evidence of seasonality. Only February has a lower number of total accidents than the base month, January. The peak is in December:roughly, there are $9.6 \%$ more accidents in December than January in the average year. The F statistic for joint significance of the monthly dummies is $\mathrm{F}=5.15$. With 11 and 95 $d f$, this gives a $p$-value essentially equal to zero.
(iii) . regress ltotacc $t$ feb mar apr may jun jul aug sep oct nov dec wkends unem spdlaw
beltlaw
Source | SS Mf MS Number of obs = 108


I will report only the coefficients on the new variables:

$$
\begin{aligned}
& \widehat{\log (\widehat{t o t a c c})}=\underset{(.063)}{10.469}+\cdots+\underset{(.00378)}{.00333} \text { wkends }-\underset{(.0034)}{.0212} \text { unem } \\
& -.0538 \text { spdlaw }+.0954 \text { beltlaw } \\
& \text { (.0126) } \\
& n=108, R^{2}=.910 .
\end{aligned}
$$

The negative coefficient on unem makes sense if we view unem as a measure of economic activity . As economic activity increases - unem decreases - we expect more driving, and therefore more accidents. The estimate is that a one percentage point increase in the unemployment rate reduces total accidents by about $2.1 \%$. A better economy does have costs in terms of traffic accidents.
(iv) At least initially, the coefficients on spdlaw and beltlaw are not what we might expect. The coefficient on spdlaw implies that accidents dropped by about $5.4 \%$ after the highway speed limit was increased from 55 to 65 miles per hour. There are at least a couple of possible explanations. One is that people become safer drivers after the increased speed limiting, recognizing that they must be more cautious. It could also be that some other change - other than the increased speed limit or the relatively new seat belt law - caused a lower total number of accidents, and we have not properly accounted for this change.

The coefficient on beltlaw also seems counterintuitive at first. But, perhaps people became less cautious once they were forced to wear seatbelts.
(v) . summ prcfat


The average of prcfat is about .886 , which means, on average, slightly less than one percent of all accidents result in a fatality. The highest value of prcfat is 1.217 , which means there was one month where $1.2 \%$ of all accidents resulted in a fatality.
(vi)

```
. regress prcfat t feb mar apr may jun jul aug sep oct nov dec
wkends unem spdlaw
beltlaw
```

| Source | SS | df | MS |
| :---: | :---: | :---: | :---: |
| Model | . 764228341 | 16 | . 047764271 |
| Residual | . 301019813 | 91 | . 00330791 |
| Total | 1.06524815 | 107 | . 00995559 |


| Number of obs | $=$ | 108 |
| :--- | ---: | ---: |
| F (16, 91) | $=14.44$ |  |
| Prob $>$ F | $=0.0000$ |  |
| R-squared | $=0.7174$ |  |
| Adj R-squared | $=0.6677$ |  |
| Root MSE | $=.05751$ |  |


| prcfat \| | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| t l | -. 0022352 | . 0004208 | -5.31 | 0.000 | -. 0030711 | -. 0013993 |
| feb \| | . 0008607 | . 0289967 | 0.03 | 0.976 | -. 0567377 | . 0584592 |
| mar \| | . 0000923 | . 0274069 | 0.00 | 0.997 | -. 0543481 | . 0545327 |
| apr \| | . 0582201 | . 0278195 | 2.09 | 0.039 | . 0029601 | . 1134801 |
| may I | . 0716392 | . 0276432 | 2.59 | 0.011 | . 0167293 | . 1265492 |
| jun \| | . 1012618 | . 0280937 | 3.60 | 0.001 | . 0454571 | . 1570665 |
| jul \| | . 1766121 | . 0272592 | 6.48 | 0.000 | . 122465 | . 2307592 |
| aug \| | . 1926116 | . 0274448 | 7.02 | 0.000 | . 1380958 | . 2471274 |
| sep \| | . 1600165 | . 028203 | 5.67 | 0.000 | . 1039948 | . 2160382 |
| oct \| | . 1010357 | . 0276702 | 3.65 | 0.000 | . 0460722 | . 1559991 |
| nov \| | . 013949 | . 0281436 | 0.50 | 0.621 | -. 0419548 | . 0698528 |
| dec \| | . 0092005 | . 027858 | 0.33 | 0.742 | -. 046136 | . 064537 |
| wkends \| | . 0006259 | . 0061624 | 0.10 | 0.919 | -. 0116151 | . 0128668 |
| unem \| | -. 0154259 | . 0055444 | -2.78 | 0.007 | -. 0264392 | -. 0044127 |
| spdlaw \| | . 0670876 | . 0205683 | 3.26 | 0.002 | . 0262312 | . 107944 |
| beltlaw \| | -. 0295053 | . 0232307 | -1.27 | 0.207 | -. 0756503 | . 0166397 |
| _cons \| | 1.029799 | . 1029524 | 10.00 | 0.000 | . 8252965 | 1.234301 |

As in part (iii), I do not report the coefficients on the time trend and seasonal dummy
variables:

$$
\begin{aligned}
& \widehat{\text { prcfat }}=\underset{(.103)}{1.030}+\cdots+\underset{(.00616)}{.00063} \text { wkends }-\underset{(.0055)}{.0154 \text { unem }} \\
& -.0671 \text { spdlaw }+.0295 \text { beltlaw } \\
& \text { (.0206) (.0232) } \\
& n=108, R^{2}=.717 .
\end{aligned}
$$

Higher speed limits are estimated to increase the percent of fatal accidents, by .067 percentage points. This is a statistically significant effect. The new seat belt law is estimated to decrease the percent of fatal accidents by about .03 , but the two-sided $p$-value is about .21 .

Interestingly, increases economic activity also increases the percent of fatal accidents. This may be because more commercial trucks are on the roads, and these probably increase the chance that an accident results in a fatality.

