BOSTON COLLEGE
Department of Economics
EC 316A Advanced Scientific Computation, Prof. Baum, Fall 2003

Problem Set 1
Due at classtime, Tuesday 16 Sep 2003

Problem sets should be your own work. Please hand in listings of any MATLAB programs used to generate the answers to these problems.

1. a. Consider the three–state Markov transition probability matrix

$$\begin{pmatrix} 0.7 & 0.0 & 0.3 \\ 0.0 & 0.4 & 0.6 \\ 0.5 & 0.0 & 0.4 \end{pmatrix}$$

Is this a stationary Markov chain? Why or why not? Explain.

Answer: Since the elements of the third row do not sum to unity, this is not a proper Markov transition probability matrix. Even if they did (e.g., if the [3,1] element was 0.6), the matrix would be stationary (since its elements are not time–dependent) but would not be irreducible, since State 2 can only be reached from State 2. Once the system leaves State 2, it will never return to that state, violating the irreducibility condition.

b. Now consider the transition probability matrix

$$\begin{pmatrix} 0.7 & 0.0 & 0.3 \\ 0.0 & 0.4 & 0.6 \\ 0.5 & 0.5 & 0.0 \end{pmatrix}$$

Compute the steady–state distribution π of this Markov chain, and explain what each of the elements of that distribution imply.

```
>> type mark

p=[0.7 0 0.3
0 0.4 0.6
0.5 0.5 0]

i=eye(3);
aa=[i-p'];
aa
aaa=[aa(1:2,1:3)
     1 1 1];
r=[0
   0
   1];
```

```
pi = aaa \ r
```

```
>> mark

p =

    0.7000         0    0.3000
         0    0.4000    0.6000
    0.5000    0.5000         0


aa =

    0.3000         0   -0.5000
         0    0.6000   -0.5000
   -0.3000   -0.6000    1.0000


pi =

    0.4762
    0.2381
    0.2857
```

The elements of the π vector represent the steady–state probabilities of this system being in each state: 47.6% for state 1, etc.

2. The Cournot two–firm (duopoly) model presented in class (and on p. 35–36 of the text) considers firms with cost parameters 0.6 and 0.8 and resulting industry (total) output of 1.5284 units.

a. Compute the Cournot outputs for two firms with cost parameters 0.1 and 0.9. What happens to industry output? Discuss.

b. Compute the Cournot outputs for three firms with cost parameters 0.1, 0.5 and 0.9. What happens to industry output vis-a-vis the result of case (a)? Why would industry output be larger or smaller with three competing firms than with two? Discuss.

Answer: For part a, the program and output is:

```
% COURNOT Evaluates the equilibrium condition in the Cournot model
function [fval,fjac] = cournot2(q)
```

```
 c = [0.1; 0.9 ];
 eta = 1.6;
 e = -1/eta;
 fval = sum(q)^e + e*sum(q)^(e-1)*q - diag(c)*q;
 fjac = e*sum(q)^(e-1)*ones(2,2) + e*sum(q)^(e-1)*eye(2) ...
          + (e-1)*e*sum(q)^(e-2)*q*[1 1] - diag(c);

>> q=newton('cournot2',[0.2;0.2]);
>> q

q =

    2.4486
    0.5044

>> sum(q)

ans =

    2.9530
```

With the first firm having a much greater cost advantage, it produces a much larger share of industry output—but the industry output is considerably higher, since the more efficient firm produces at a much lower absolute cost as compared with the baseline.

When a three–firm industry is modeled, we have the program and results:

```
>> type cournot3.m

% COURNOT Evaluates the equilibrium condition in the Cournot model
function [fval,fjac] = cournot3(q)
 c = [0.1; 0.5; 0.9 ];
 eta = 1.6;
 e = -1/eta;
 fval = sum(q)^e + e*sum(q)^(e-1)*q - diag(c)*q;
 fjac = e*sum(q)^(e-1)*ones(3,3) + e*sum(q)^(e-1)*eye(3) ...
          + (e-1)*e*sum(q)^(e-2)*q*[1 1 1] - diag(c);

>> q=newton('cournot3',[0.2;0.2;0.2]);
>> q

q =

    2.5308
    0.7644
    0.4502
```

```
>> sum(q)

ans =

    3.7454
```

The first firm still has a very sizable share of the market, and in fact increases its level of output. Total industry output rises yet again, since the presence of the second firm—producing at an intermediate level of cost—makes it possible to generate a higher industry output at a competitive price.

3. Compute the solution to the mine management problem (`demddp01`), followed by the statements needed to generate the expected trajectory of S:

```
sinit=max(S); nyrs=15;
spath=ddpsimul(pstar,sinit,nyrs); spath'
```

where the spath vector will give S for each period. Redo the model over this same 15–year horizon with the assumption that the state imposes a 20% extraction tax on each unit of output. How does this change the expected trajectory of the mine's contents? What is the tax revenue earned each year? The total revenue over the 15–year horizon? (Assume that the tax is paid at the end of the year based on last year's output).

Answer: The model is modified to allow for a tax rate, which enters the reward matrix by increasing the cost of production:

```
 tau = 0.2;  % extraction tax rate
 f = (price-(1+tau)*XX./(1+SS)).*XX;
```

Which gives rise to the optimal sequence of S:

```
ans =

   100
    79
    62
    49
    39
    31
    24
    19
    15
    12
    10
     8
     6
```

```
     5
     4
     3
```

Showing that at each point in time, the mine is being exploited less vigorously than in the no–tax regime. A program to generate the desired computations, and its output (ignore the first element of `extr`):

```
 >> type mine.m

demddp01a;
tau=0.2;
sinit=max(S); nyrs=15;
spath=ddpsimul(pstar,sinit,nyrs);
spathl=[0 spath(1,1:nyrs)]
optpath=spath'
extr = (spathl'-spath')
extrtax= (extr>0).*extr.*tau
totrev=sum(extrtax)

extr =

  -100
    21
    17
    13
    10
     8
     7
     5
     4
     3
     2
     2
     2
     1
     1
     1


extrtax =

         0
    4.2000
    3.4000
    2.6000
```

```
    2.0000
    1.6000
    1.4000
    1.0000
    0.8000
    0.6000
    0.4000
    0.4000
    0.4000
    0.2000
    0.2000
    0.2000


totrev =

   19.4000
```