Solutions to Problem Set 6 (Due December 8)

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Maximum number of points for Problem set 8 is: 220

Problem 10.6

(i) (5 pts) Given the formula for $\delta_j = \gamma_0 + \gamma_1 j + \gamma_2 j^2$, we substitute into the original equation to get
$y_t = \alpha_0 + \gamma_0 z_t + (\gamma_0 + \gamma_1 + \gamma_2)z_{t-1} + (\gamma_0 + 2\gamma_1 + 4\gamma_2)z_{t-2} +$
$(\gamma_0 + 3\gamma_1 + 9\gamma_2)z_{t-3} + (\gamma_0 + 4\gamma_1 + 16\gamma_2)z_{t-4}$
which we can rearrange in terms of the γ's as follows:
$y_t = \alpha_0 + \gamma_0(z_t + z_{t-1} + z_{t-2} + z_{t-3} + z_{t-4}) +$
$\gamma_1(z_{t-1} + 2z_{t-2} + 3z_{t-3} + 4z_{t-4}) + \gamma_2(z_{t-1} + 4z_{t-2} + 9z_{t-3} + 16z_{t-4})$

(ii) (5 pts) Now we simply define 3 new variables x_0, x_1, x_2 as follows:
$x_0 = z_t + z_{t-1} + z_{t-2} + z_{t-3} + z_{t-4}$
$x_1 = z_{t-1} + 2z_{t-2} + 3z_{t-3} + 4z_{t-4}$
$x_2 = z_{t-1} + 4z_{t-2} + 9z_{t-3} + 16z_{t-4}$
Then we can substitute these into our equation from part (i) and run OLS on:
$y_t = \alpha_0 + \gamma_0 x_0 + \gamma_1 x_1 + \gamma_2 x_2$

(iii) (5 pts) To see how many restrictions we have imposed, notice that our original equation had six parameters: $\alpha_0, \delta_0, \delta_1, \delta_2, \delta_3, \delta_4$ while our new equation from part (ii) has 4 parameters: $\alpha_0, \gamma_0, \gamma_1, \gamma_2$. So we are imposing 6-4=2 restrictions. You could test the validity of these restrictions using an F test with 2 and n-6 degrees of freedom.

Problem 10.7

(i) (5 pts) Since pe_t is increasing, but $(pe_{t-1} - pe_t)$ and $(pe_{t-2} - pe_t)$ are fixed, then we know that both pe_{t-1} and pe_{t-2} must also be increasing at the same amount.

(ii) (5 pts) The long-run propensity is defined as the change in gfr when pe increases permanently. So we can examine this change by increasing pe_{t-2}, pe_{t-1}, pe_t by the same amount. Then we can see from the original equation, that gfr is predicted to increase by $\delta_0 + \delta_1 + \delta_2 = \theta_0$, which is our LRP.

Problem C10.1

(i) (15 pts) Define y1979 if year > 1979; so it is a dummy with 0 for years 1948-1979 and 1 for 1980-2003. Then run regression from eq. 10.15. The coefficient on y1979 is 1.559 which is significant at the 1 percent level. This allows us to conclude that, ceteris paribus, after 1979 the interest rate on 3-month t-bills was 1.55 percent higher. So we can conclude that the regime change towards targeting the short-term interest rate increased this interest rate.

Problem C10.7

(i) (5 pts) The estimated equation is

$$\begin{array}{rcccc} \widehat{gc}_t = & .0081 & + & .571 & gy_t \\ & (.0019) & & (.067) & \end{array}$$

$$n = 36, R^2 = .679.$$

This equation implies that if income growth increases by one percentage point, consumption growth increases by .571 percentage points. The coefficient on gy_t is very statistically significant (t-statistic $\approx$ 8.5).

(ii) (5 pts) Adding gy_{t-1} to the equation gives

$$\begin{array}{rcccccc} \widehat{gc}_t = & .0064 & + & .552 & gy_t + & .096 & gy_{t-1} \\ & (.0023) & & (.070) & & (.069) & \end{array}$$

$$n = 35, R^2 = .695.$$

The t-statistic on gy_{t-1} is only about 1.39, so it is not significant at the usual significance levels. (It is significant at the 20 % level against a two-sided alternative.) In addition, the coefficient is not especially large. At best there is weak evidence of adjustment lags in consumption.

(iii) (5 pts) If we add $r3_t$ to the model estimated in part (i) we obtain

$$\begin{array}{rcccccc} \widehat{gc}_t = & .0082 & + & .578 & gy_t - & .00021 & r3_t \\ & (.0020) & & (.072) & & (.00063) & \end{array}$$

$$n = 36, R^2 = .680.$$

The t-statistic on $r3_t$ is very small. The estimated coefficient is also practically small: a one-point increase in $r3_t$ reduces consumption growth by about .021 percentage points.

Problem C10.9

(i) (5 pts) The sign of β_2 is fairly clear-cut: as interest rates rise, stock returns fall, so $\beta_2 < 0$. Higher interest rates imply that T-bill and bond investments are more attractive, and also signal a future slowdown in economic activity. The sign of β_1 is less clear. While economic growth can be a good thing for the stock market, it can also signal inflation, which tends to depress stock prices.

(ii) (5 pts) The estimated equation is

$$
\begin{array}{rcccccc}
\widehat{rsp500}_t = & 18.84 & + & .036 & pcip_t - & 1.36 & i3_t \\
 & (3.27) & & (.129) & & (.54) &
\end{array}
$$

$$n = 557, R^2 = .012.$$

A one percentage point increase in industrial production growth is predicted to increase the stock market return by .036 percentage points (a very small effect). On the other hand, a one percentage point increase in interest rates decreases the stock market return by an estimated 1.36 percentage points.

(iii) (5 pts) Only $i3$ is statistically significant with t-statistic $\approx$ -2.52.

(iv) (5 pts) The regression in part (i) has nothing directly to say about predicting stock returns because the explanatory variables are dated contemporaneously with $rsp500$. In other words, we do not know $i3_t$ before we know $rsp500_t$. What the regression in part (i) says is that a change in $i3$ is associated with a contemporaneous change in $rsp500$.

Problem C12.2

(i) (10 pts) After estimating the FDL model by OLS, we obtain the residuals and run the regression on $\hat{\mu_t}$ on $\hat{\mu_{t-1}}$ using 272 observations. We get $\hat{\rho} \approx .503$ and $t_{\hat{\rho}} = 9.60$ which is very strong evidence of positive AR(1) correlation.

(ii) (5 pts) When we estimate the model by iterated C-O, the LRP is estimated to be about 1.110.

(iii) (10 pts) We use the same trick as in Problem 11.5, except now we estimate the equation by iterated C-O. In particular, write
$gprice_t = \alpha_0 + \theta_0 gwage_t + \delta_1(gwage_{t-1} gwage_t) + \delta_2(gwage_{t-2} gwage_t) + + \delta_{12}(gwage_{t-12} gwage_t) + u_t$,
Where θ_0 is the LRP and u_t is assumed to follow an AR(1) process. Estimating this equation by C-O gives $\hat{\theta_0} \approx 1.110$ and se($\hat{\theta_0}$) $\approx .191$. The t statistic for testing $H_0 : \theta_0 = 1$ is $(1.1101)/.191 \approx .58$, which is not close to being significant at the 5% level. So the LRP is not statistically different from one.

Problem C12.6

(i) (10 pts) The regression on $\hat{\mu_t}$ on $\hat{\mu_{t-1}}$ (with 35 observations) gives $\hat{\rho} \approx .089$ and se($\hat{\rho}$) $\approx .178$; there is no evidence of AR(1) serial correlation in this equation, even though it is a static model in the growth rates.

(ii) (10 pts) We regress gc_t on gc_{t-1} and obtain the residuals . Then, we regress on gc_{t-1} and (using 35 observations), the F statistic (with 2 and 32 df) is about 1.08. The p-value is about .352, and so there is little evidence of heteroskedasticity in the AR(1) model for gct. This means that we need not modify our test of the PIH by correcting somehow for heteroskedasticity.

Problem C12.8

(i) (10 pts) This is the model that was estimated in part (vi) of Computer Exercise C10.11. After getting the OLS residuals, $\hat{u_t}$, we run the regression $\hat{u_t}$ on $\hat{u_{t-1}}$ t = 2,...,108. (Included an intercept, but that is unimportant.) The coefficient on $\hat{u_{t-1}}$ is $\hat{\rho} = .281$ (se = .094). Thus, there is evidence of some positive serial correlation in the errors ($t \approx 2.99$). A strong case can be made that all explanatory variables are strictly exogenous. Certainly there is no concern about the time trend, the seasonal dummy variables, or *wkends*, as these are determined by the calendar. It is seems safe to assume that unexplained changes in prcfat today do not cause future changes in the state-wide unemployment rate. Also, over this period, the policy changes were permanent once they occurred, so strict exogeneity seems reasonable for *spdlaw* and *beltlaw*. (Given legislative lags, it seems unlikely that the dates the policies went into effect had anything to do with recent, unexplained changes in $prcfat$.

(ii) (10 pts) Remember, we are still estimating the β_j by OLS, but we are computing different standard errors that have some robustness to serial correlation. Using Stata 7.0, we get $\widehat{\beta_{spdlaw}} = .0671$, $se(\widehat{\beta_{spdlaw}}) = .0267$, and $\widehat{\beta_{beltlaw}} = -.0295$, $se(\widehat{\beta_{spdlaw}}) =$ .0331 . The t statistic for *spdlaw* has fallen to about 2.5, but it is still significant. Now, the t statistic on *beltlaw* is less than one in absolute value, so there is little evidence that *beltlaw* had an effect on $prcfat$.

(iii) (5 pts) For brevity, I do not report the time trend and monthly dummies. The final estimate of $\hat{\rho} = .289$. Regressing $prcfat$ on the time trend, monthly dummies, $wkends, unem, spdlaw, beltlaw$ yields similar results as part (ii). Both policy variable coefficients get closer to zero, and the standard errors are bigger than the incorrect OLS standard errors [and, coincidentally, pretty close to the Newey-West standard errors for OLS from part (ii)]. So the basic conclusion is the same: the increase in the speed limit appeared to increase $prcfat$, but the seat belt law, while it is estimated to decrease $prcfat$, does not have a statistically significant effect.

Problem 15.6

(i) (10 pts) Plugging (15.26) into (15.22) and rearranging gives
$y_1 = \beta_0 + \beta_1(\pi_0 + \pi_1 z_1 + \pi_2 z_2 + v_2) + \beta_2 z_1 + u_1 =$
$(\beta_0 + \beta_1\pi_0) + (\beta_1\pi_1 + \beta_2)z_1 + \beta_1\pi_2 z_2 + u_1 + \beta_1 v_2,$
and so $\alpha_0 = \beta_0 + \beta_1\pi_0, \alpha_1 = \beta_1\pi_1 + \beta_2$, and $\alpha_2 = \beta_1\pi_2$.

(ii) (5 pts) From the equation in part (i), $v_1 = u_1 + \beta_1 v_2$.

(iii) (5 pts) By assumption, u_1 has zero mean and is uncorrelated with z_1 and z_2, and v_2 has these properties by definition. So v_1 has zero mean and is uncorrelated with z_1 and z_2, which means that OLS consistently estimates the α_j. [OLS would only be unbiased if we add the stronger assumptions $E(u_1 \mid z_1, z_2) = E(v_2 \mid z_1, z_2) = 0$.]

Problem C15.2

(i) (10 pts) The equation estimated by OLS is

$$\begin{array}{rcccccccc} \widehat{children} = & -4.138 & - & .0906 & educ+ & .332 & age- & .00263 & age^2 \\ & (.241) & & (.0059) & & (.017) & & (.00027) & \end{array}$$

$$n = 4.361, R^2 = .569$$

Another year of education, holding age fixed, results in about .091 fewer children. In other words, for a group of 100 women, if each gets another of education, they collectively are predicted to have about nine fewer children.

(ii) (5 pts) The reduced form for *educ* is
$educ = \pi_0 + \pi_1 age + \pi_2 age^2 + \pi_3 frsthalf + v$,
and we need $\pi_3 \neq 0$. When we run the regression we obtain $\hat{\pi}_3 = -.852$ and $se(\hat{\pi}_3) = .113$. Therefore, women born in the first half of the year are predicted to have almost one year less education, holding *age* fixed. The t statistic on $frsthalf$ is over 7.5 in absolute value, and so the identification condition holds.

(iii) (10 pts) The structural equation estimated by IV is

$$\begin{array}{rcccccccc} \widehat{children} = & -3.388 & - & .1715 & educ+ & .324 & age- & .00267 & age^2 \\ & (.548) & & (.0532) & & (.018) & & (.00028) & \end{array}$$

$$n = 4.361, R^2 = .550$$

The estimated effect of education on fertility is now much larger. Naturally, the standard error for the IV estimate is also bigger, about nine times bigger. This produces a fairly wide 95% CI for β_1.

(iv) (10 pts) When we add $electric, tv$, and $bicycle$ to the equation and estimate it by OLS we obtain

$$\begin{array}{rcccccccc} \widehat{children} = & -4.390 & & - & .0767 & educ+ & .340 & age- & .00271 & age^2 \\ & (.0240) & & & (.0064) & & (.016) & & (.00027) & \\ & -.303 & electric- & & .253 & tv+ & .318 & bicycle & & \\ & (.076) & & & (.091) & & (.049) & & & \end{array}$$

$$n = 4356, R^2 = .576$$

The 2SLS (or IV) estimates are

$$\begin{array}{rcccccccc} \widehat{children} = & -3.591 & - & .1640 & educ+ & .328 & age- & .00272 & age^2 \\ & (.645) & & (.0655) & & (.019) & & (.00028) & \\ & -.107 & electric- & .0026 & tv+ & .332 & bicycle & & \\ & (.166) & & (.2092) & & (.052) & & & \end{array}$$

$$n = 4356, R^2 = .558$$

Adding *electric*, *tv*, and *bicycle* to the model reduces the estimated effect of *educ* in both cases, but not by too much. In the equation estimated by OLS, the coefficient on *tv* implies that, other factors fixed, four families that own a television will have about one fewer child than four families without a TV. Television ownership can be a proxy for different things, including income and perhaps geographic location. A causal interpretation is that TV provides an alternative form of recreation.
Interestingly, the effect of TV ownership is practically and statistically insignificant in the equation estimated by IV (even though we are not using an IV for *tv*). The coefficient on *electric* is also greatly reduced in magnitude in the IV estimation. The substantial drops in the magnitudes of these coefficients suggest that a linear model might not be the best functional form, which would not be surprising since *children* is a count variable. (See Section 17.4.)

Problem C15.6

(i) (5 pts) Sixteen states executed at least one prisoner in 1991, 1992, or 1993. (That is, for 1993, exec is greater than zero for 16 observations.) Texas had by far the most executions with 34.

(ii) (10 pts) The results of the pooled OLS regression are

$$\begin{array}{rccccccc} \widehat{mrdrte} = & -5.28 & - & 2.07 & d93+ & .128 & exec+ & 2.53 \quad unem \\ & (4.43) & & (2.14) & & (.263) & & (.78) \end{array}$$

$$n = 102, R^2 = .102, \bar{R}^2 = .074.$$

The positive coefficient on *exec* is no evidence of a deterrent effect. Statistically, the coefficient is not different from zero. The coefficient on *unem* implies that higher unemployment rates are associated with higher murder rates.

(iii) (5 pts) When we difference (and use only the changes from 1990 to 1993), we obtain

$$\begin{array}{rccccc} \Delta\widehat{mrdrte} = & .413 & - & .104 & \Delta exec- & .067 \quad \Delta unem \\ & (.209) & & (.043) & & (.159) \end{array}$$

$$n = 51, R^2 = .110, \bar{R}^2 = .073.$$

The coefficient on $\Delta exec$ is negative and statistically significant (p-value $\approx .02$ against a two-sided alternative), suggesting a deterrent effect. One more execution reduces the murder rate by about .1, so 10 more executions reduce the murder rate by one (which means one murder per 100,000 people). The unemployment rate variable is no longer significant.

(iv) (5 pts) The regression $\Delta exec$ on $\Delta exec_{-1}$ yields

$$\begin{array}{rcccc} \widehat{\Delta exec} = & .350 & - & 1.08 & \Delta exec_{-1} \\ & (.370) & & (.17) & \end{array}$$

$$n = 51, R^2 = .456, \bar{R}^2 = .444.$$

which shows a strong negative correlation in the change in executions. This means that, apparently, states follow policies whereby if executions were high in the preceding three-year period, they are lower, one-for-one, in the next three-year period.

Technically, to test the identification condition, we should add $\Delta unem$ to the regression. But its coefficient is small and statistically very insignificant, and adding it does not change the outcome at all.

(v) (5 pts) When the differenced equation is estimated using $\Delta exec_{-1}$ as an IV for $\Delta exec$,we obtain

$$\begin{array}{rcccccc} \Delta\widehat{mrdrte} = & .411 & - & .100 & \Delta exec- & .067 & \Delta unem \\ & (.211) & & (.064) & & (.159) & \end{array}$$

$$n = 51, R^2 = .110, \bar{R}^2 = .073.$$

This is very similar to when we estimate the differenced equation by OLS. Not surprisingly, the most important change is that the standard error on $\hat{\beta}_1$ is now larger and reduces the statistically significance of $\hat{\beta}_1$.