Solutions to Problem Set 1 (Due September 22)

EC 228 02, Fall 2010 — Prof. Baum, Ms Hristakeva

Maximum number of points for Problem set 1 is: 36

Problem A.7.

(i) (2 pts.) By exponentiating the left and right sides of the equation we get:

$$Salary = e^{10.6+.027exper}$$

Therefore, for $exper = 0$, we have $Salary_1 = e^{10.6} \approx 40134.84$, and for $exper = 5$, $Salary_2 = e^{10.6+.027*5} = e^{10.735} \approx 45935.80$.

(ii) (2 pts.) $\frac{Salary_2 - Salary_1}{Salary_1} \approx \ln Salary_2 - \ln Salary_1 = .735 - .6 = .135$

(iii) (2 pts.) $\frac{Salary_2 - Salary_1}{Salary_1} = \frac{45935.80-40134.84}{40134.80} \approx .144 > .135$

Problem B.2.

(i) (2 pts.) $\mathrm{P}(X \leq 6) = P[(X-5)/2 \leq (6-5)/2] = P(Z \leq 0.5) \approx 0.692.$, where Z denotes a Normal(0,1) random variable. [We obtain $P(Z) \leq 0.5$ from Table G.1]

(ii) (2 pts.) $P(X > 4) = P[(X-5)/2 > (4-5)/2] = P(Z > -0.5) = P(Z \leq 0.5) \approx 0.692.$

(iii) (2 pts.) $P(|X-5| > 1) = P(X-5 > 1) + P(X-5 < -1) = P(X > 6) + P(X < 4) \approx (1-0.692) + (1-0.692) = 0.616$, where we have used answers from parts (i) and (ii).

Problem B.5.

(i) (3 pts.)As stated in the hint, if X is the number of jurors convinced of Simpson's innocence, then $X \sim$ Binomial(12,.20). We want $P(X \geq 1) = 1 - P(X=0) = 1 - (.8)^{12} \approx .931$.

(ii) (3 pts.) To find $P(X \geq 2)$, notice that $P(X \geq 2) = 1 - P(X=0) - P(X=1)$, or in words, the probability that at least two people believe innocence is equal to 1 minus the probability that exactly zero people or exactly one person believes innocence. We know from part (i) that $1 - P(X=0) \approx .931$. Then we compute that $P(X=1) = 12*(.2)(.8)^{11} \approx .206$ using (B.14) where n=12, x=1, and $\theta = .2$. So $P(X \geq 2) \approx .931 - .206 = .725$. Therefore, there is almost a 75 percent chance that the jury had at least two members convinced of Simpson's innocence prior to the trial.

Problem C.1.

(i) (2 pts.) This is just a special case of what we covered in the text, with $n = 4 : E(\bar{Y}) = \mu$ and $Var(\bar{Y}) = \sigma^2/4$.

(ii)(2 pts.) $E(W) = E(Y_1)/8 + E(Y_2)/8 + E(Y_3)/4 + E(Y_4)/2 = \mu[(1/8) + (1/8) + (1/4) + (1/2)] = \mu(1 + 1 + 2 + 4)/8 = \mu$, which shows that W is unbiased.
Because the Y_i are independent with the same variance σ^2 and because $Var(aY) = a^2 Var(Y)$ for any constant a and variable Y,

$$\begin{aligned} Var(W) &= Var(Y_1)/64 + Var(Y_2)/64 + Var(Y_3)/16 + Var(Y_4)/4 = \\ &= \sigma^2((1/64) + (1/64) + (4/64) + (16/64)) = \sigma^2(22/64) = \sigma^2(11/32). \end{aligned} \tag{1}$$

(iii) (2 pts.) Because $11/32 > 8/32 = 1/4, Var(W) > Var(\bar{Y})$ for any $\sigma^2 > 0$, so $\bar{Y}$ is preferred to W because each is unbiased.

Problem C.6.

(i) (1 pt.) $H_0 : \mu = 0$

(ii) (1 pt.) $H_1 : \mu < 0$

(iii) (2 pts.) The standard error of $\bar{y}$ is $s/\sqrt{n} = 466.4/30 \approx 15.55$. Therefore, the t statistic for testing $H_0 : \mu = 0$ is $t = \bar{y}/se(\bar{y}) = -32.8/15.55 \approx -2.11$. We obtain the p-value as $P(Z \leq -2.11)$, where $Z \sim$ Normal(0,1). These probabilities are in Table G.1: p-value=.0174. Because the p-value is below .05, we reject H_0 against the one-sided alternative at the 5 percent level. We do not reject at the 1 percent level because p-value = .0174 > .01.

(iv) (1 pt.) The estimated reduction, about 33 ounces, does not seem large for an entire year's consumption. If the alcohol is beer, 33 ounces is less than three 12-ounces cans of beer. Even if this is hard liquor, the reduction seems small. (On the other hand, when aggregated across the entire population, alcohol distributors might not think the effect is so small.)

(v) (1 pt.) The implicit assumption is that other factors that affect liquor consumption - such as income, or changes in price due to transportation costs, are constant over the two years.

Problem C1.1

(i) (1 pt.) summ educ. The mean education level is 12.5627 years with min 0 and max 18.

(ii) (1 pt.) summ wage. The mean hourly wage is 5.89 with min .53 and max 24.98.

(iii) (1 pt.) CPI for 2003 is 184.0 and for 1976 it is 56.9, respectively.

(iv) (2 pts.) Therefore, mean salary in 2003 dollars is $5.896 \cdot \frac{184.0}{56.9} = 19.066$

(v) (1 pt.) tabulate female There are 252 women (indicated by a 1 in the female column) and 274 men (indicated by a zero in the female column).