EC 313 Spring 2000 Problem Set 4

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Due Thursday 6 April 2000 at classtime

Produce a Mathematica notebook containing the answers to these questions, label it with your name, and email it (as an email attachment) to `baum@bc.edu` to submit your problem set. I will be glad to answer any clarifying questions; please stop by or send me email.

The economy contains two consumers and two firms. Each consumer is endowed with 10 barrels of oil, which are worth $1.00 per barrel. Consumer 1 owns firm 1, which produces guns from oil via the production function g = 2 x (where x = number of barrels of oil). Consumer 2 owns firm 2, which produces butter from oil via the production function b = 3 x.

Consumer 1 has a utility function defined over guns and butter of u1(g,b)=g^0.4 b^0.6 while consumer 2 has a utility function of u2(g,b)=10 + 0.5 ln g + 0.5 ln b, where ln is the natural logarithm of its argument.

1. Define the profit function for firm 1 in terms of the price it receives for guns and the cost of production (recall that p(oil)==$1.00). Do the same for firm 2. In general equilibrium, both firms must earn zero profit. Use that condition to solve for the equilibrium prices of guns and butter.

2. Express the economy-wide budget constraint – that both consumers' (unknown) purchases of guns and butter, times their respective prices, must add up to no more than the total endowment of $20.00 (twenty barrels of oil at $1.00/barrel). Use this constraint to express the output of guns as a function of the output of butter.

3. It turns out that consumer 1's demand for butter is (0.6 W)/pb, where W is her endowment of $10.00 and pb is the price of butter. How much butter does she buy? Likewise, consumer 2's demand for butter is (0.5 W)/pb. How much butter does he buy? How many barrels of oil are used to make all this greasy butter?

4. How many guns are produced, and how much oil is used to produce them? Consumer 1's demand for guns is (0.4 W)/pg. How many guns does she buy? How many guns does consumer 2 buy?

5. Calculate the attained levels of utility for each consumer. They have the same endowment (of $10.00 worth of oil). Shouldn't they have the same utility in general equilibrium? Do they? If not, why not?

6. Now say that a technological innovation changes the butter production function to b = 5 x (i.e. one barrel of oil now makes 5 pounds of butter). How do the prices of the two goods change? How much butter is now going to be produced and sold? Does this innovation make the consumers each better off (in terms of attained utility)? For whom does it have the greater impact? Explain your findings.