The DF-GLS unit root test

Although common practice in time series modelling has involved the application of (augmented) Dickey-Fuller and Phillips-Perron tests to determine whether a series possesses a unit root, improved tests with much better statistical properties are now available. In their 1996 *Econometrica* article, Elliott, Rothenberg and Stock (ERS) proposed an efficient test, modifying the Dickey-Fuller test statistic using a generalized least squares (GLS) rationale. They demonstrate that this modified test has the best overall performance in terms of small-sample size and power, conclusively dominating the ordinary Dickey-Fuller test. In particular, Elliott et al. find that their "DF-GLS" test "has substantially improved power when an unknown mean or trend is present." (1996, p.813)

The test proceeds as follows (Stock, 1994, p. 2768).

Let $z_t = (1, t)$. For the timeseries y_t, regress $[y_1, (1 - \alpha L)\, y_2, ..., (1 - \alpha L)\, y_T]$ on $[z_1, (1 - \alpha L)\, z_2, ..., (1 - \alpha L)\, z_T]$ yielding $\tilde{\beta}_{GLS}$ where $\alpha = 1 + \bar{c}/T$, $u_0 = 0$, and $\bar{c}$ = -13.5 for the detrended statistic. Detrended $\tilde{y}_t = y_t - z_t^{'}\tilde{\beta}_{GLS}$ is then employed in the (augmented) Dickey-Fuller regression, with no intercept nor time trend. The $t-$statistic on $\tilde{y}_{t-1}$ is the DF-GLS statistic. For the demeaned case, the t is omitted from z_t, and $\bar{c}$ = -7.0.

Just as the standard Dickey-Fuller test may be run with or without a trend term, there are two forms of DF-GLS: GLS detrending and GLS demeaning. With GLS detrending, the series to be tested is regressed on a constant and linear trend, and the residual series is used in a standard Dickey-Fuller regression. With GLS demeaning, only a constant appears in the first stage regression; the residual series is then used as the regressand in a Dickey-Fuller regression. In the *Stata* implementation of the DF-GLS test (Baum, 2000), GLS detrending is the default, and GLS demeaning is selected by the *notrend* option.

Any test involving an augmented Dickey-Fuller regression is sensitive to the lag length (number of lagged differences with which the regression is augmented). In the *Stata* implementation of the DF-GLS test, a maximum lag order may be specified, or the default value (calculated from the sample size using a rule provided by Schwert (1989) may be used. If the maximum lag exceeds one, the test is executed for each lag, with the sample size held constant at the maximum feasible for that maximum lag order. An estimate of the optimal lag order as chosen by the Ng-Perron (1995) sequential $t-$test criterion is provided. This criterion selects the appropriate lag order, starting with the maximum lag and testing the highest lag's coefficient for significance. When the $p-$value falls below 0.10, that lag is retained and the optimal lag is indicated. The lag producing the optimal Schwarz criterion (SIC or BIC) is also printed; it should be noted that Ng and Perron have shown that the SIC-selected lag may lead to a test with very low power in the presence of a large negative moving average component in the error process. Thus, the lag length chosen by the Ng-Perron criterion is generally to be preferred.

Critical values for the GLS detrending test are given in ERS, and interpolated values are provided by the DF-GLS routine. Critical values for the GLS demeaning test are those applicable to the no-constant, no-trend Dickey-Fuller test; interpolated values (from Stata's *dfuller* routine) are provided by DF-GLS.

References

Baum, C F., 2000. DFGLS: Stata module to compute Dickey-Fuller/GLS unit root test. IDEAS-SSC:

`http://ideas.uqam.ca/ideas/data/Softwares/bocbocodeS410001.html`

Elliott, Graham, Rothenberg, Thomas J., and James H. Stock. Efficient Tests for an Autoregressive Unit Root. *Econometrica*, 64:4, 1996, 813-836.

Ng, Serena and Pierre Perron. Unit Root Tests in ARMA Models with Data-Dependent Methods for the Selection of the Truncation Lag. *Journal of the American Statistical Association*, 90:429, 1995, 268-281.

Schwert, G.W. Tests for Unit Roots: A Monte Carlo Investigation. *Journal of Business and Economic Statistics*, 7, 1989, 147-160.

Stock, James H. Unit Roots, Structural Breaks and Trends. Chapter 46 in *Handbook of Econometrics, Vol. IV*, Engle, R. and McFadden, D., eds. 1994: Elsevier, Amsterdam.

EC771B Spring 2000

The KPSS unit root test

Most of the tests in the unit root literature have as a null hypothesis the nonstationarity of the series being tested: that is, $H_0 = y\tilde{\ }I(1)$. The Kwiatkowski, Phillips, Schmidt and Shin test (KPSS, 1992) has the opposite (and perhaps more intuitive) null: that the series being tested is stationarity, $H_0 = y\tilde{\ }I(0)$. Like the Dickey-Fuller, DF-GLS and Phillips-Perron tests, the KPSS test may be conducted under the assumption that the series is trend stationary or level stationary; in the former case, the trend is removed. The KPSS test is viewed as complementary to the more commonly employed tests, since it may be used to verify their results: if, say, the DF-GLS test fails to reject its null of a unit root, and the KPSS test rejects, then the evidence from both tests is supportive of a unit root in the series.

For the trend stationary case, the test statistic is produced as follows. Let $\{e_t\}_1^T$ be the residuals from the regression of y_t on an intercept and time trend. Define $S_t = \sum^t e_i$, $t = 1, ..., T$ as the partial sum process of the residuals. Then $\eta = T^{-2} \sum^T S_t^2 / s_l^2$ is the KPSS statistic for the null hypothesis of trend stationarity, where s_l^2 is a consistent estimate of the long-run variance of the series (usually computed as the Newey-West robust estimate of the variance, using l lags of the series).

The KPSS test has also been employed as a test for fractional integration. If conventional unit root tests reject their null, and the KPSS test also rejects the null, the evidence may be taken as supporting a fractionally integrated model for the series, $(1 - L)^d y_t = \epsilon_t$, with d a real number. See Lee and Schmidt (1996) and the *Stata* commands *gphudak*, *modlpr*, and *roblpr*, available from SSC-IDEAS.

As with other unit root tests, a number of lags of the series may be used to ensure that the test captures any short-term dynamics in the series. In the *Stata* implementation of the KPSS test (Baum, 2000), the maximum lag order for the test is calculated by default from the sample size, using a rule provided by Schwert (1989). The test is performed for each lag, with the sample size held constant at the maximum feasible for the maximum lag order. The level stationarity hypothesis may be tested by specifying the *notrend* option.

Approximate critical values for the test are given in KPSS, and reported by the KPSS procedure.

References

Baum, C F., 2000. KPSS: Stata module to compute Kwiatkowski-Phillips-Schmidt-Shin test for stationarity. IDEAS-SSC:

`http://ideas.uqam.ca/ideas/data/Softwares/bocbocodeS410401.html`

Elliott, Graham, Rothenberg, Thomas J., and James H. Stock. Efficient Tests for an Autoregressive Unit Root. *Econometrica*, 64:4, 1996, 813-836.

Kwiatkowski, D., Phillips, P.C.B., Schmidt, P. and Y. Shin. Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics*, 54, 1992, 159-178.

Lee, D. and P. Schmidt. On the power of the KPSS test of stationarity against fractionally-integrated alternatives. *Journal of Econometrics*, 73, 1996, 285-302.

Schwert, G.W. Tests for Unit Roots: A Monte Carlo Investigation. *Journal of Business and Economic Statistics*, 7, 1989, 147-160.

EC771B Spring 2000