## EC771: Econometrics, Spring 2004

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Chapters 6, 7, 8:
Inference, Prediction, Functional Form, Structural Change, Specification Error

Inference in the regression model

We may readily test hypotheses which involve "nested models": that is, the comparison of a general model with a special case of that model, in which one or more hypotheses on the coefficient vector are assumed to hold with certainty. If we work with purely linear restrictions on $\beta$, we may write the problem as:

$$
y=X \beta+\epsilon,
$$

subject to $R \beta=q$. The matrix $R$, with $J$ rows, must be of full row rank, implying that $J \leq K$.

Realistically, we should also rule out the case of equality, since in that case $\beta=R^{-1} q$, and no estimation is necessary.

We can follow two approaches with this constrained system: first, we may estimate the unconstrained model, and test the degree to which the constraints would be satisfied by the data. Second, we may estimate the model subject to constraints, which must involve a loss of fit, and evaluate how serious that loss might be in relation to the unconstrained alternative.

For the first approach, consider testing the null hypothesis $R \beta=q$ versus the alternative of inequality. Several special cases:

- ANOVA " F ": $R=[0: I]$ contains a (K-1) order identity matrix with a zero vector for the constant term, with $q$ as a (K-1) row
null vector. The null hypothesis is that all slopes are jointly zero.
- A single coefficient is zero (or a particular value): $R$ is a zero vector with a single 1 corresponding to that coefficient's position in the $\beta$ vector, $q=0$ (or the particular value).
- Two of the coefficients are equal: $R$ is a zero vector with 1 and -1 in positions corresponding to those coefficients' positions in the $\beta$ vector, $q=0$.
- The ratio of two coefficients is $a: R$ is a zero vector with 1 and $a$ in appropriate locations, and $q=0$.
- Several coefficients sum to 1: $R$ is a zero vector with 1 s in positions corresponding to those coefficients, and $q=1$.
- A set of coefficients are jointly zero: $R=$ [ $I$ : O] where the first $J$ coefficients are assumed to equal zero, and $q=0$.

More complicated sets of linear restrictions on $\beta$ may be expressed in terms of $R$. In any case, the restrictions must be linearly independent, so that $R$ is of full row rank.

Given the least squares estimator of the unconstrained model, $b$, we are interested in the discrepancy vector $m=R b-q$. Although it is unlikely that $m=0$, how far from the null vector might $m$ be (in terms of its norm) before we will conclude that the null hypothesis
should be rejected? Under that null hypothesis, $m$ has a mean vector 0 and a covariance matrix
$\operatorname{Var}[R b-q \mid X]=R \operatorname{Var}[b \mid X] R^{\prime}=\sigma^{2} R\left(X^{\prime} X\right)^{-1} R^{\prime}$.
We can thus base a test of $H_{0}$ on the Wald statistic

$$
\begin{gathered}
W=m^{\prime}(\operatorname{Var}[m \mid X])^{-1} m \\
W=\sigma^{-2}(R b-q)^{\prime}\left[R\left(X^{\prime} X\right)^{-1} R^{\prime}\right]^{-1}(R b-q)
\end{gathered}
$$

which under the null hypothesis will have a $\chi^{2}(J)$ distribution. To make this test statistic operational, we must replace the unknown $\sigma^{2}$ with a consistent estimate thereof, $s^{2}$. Most commonly, we express this test statistic as an $F$-statistic,

$$
\hat{W}=J^{-1}(R b-q)^{\prime}\left[R\left(s^{2} X^{\prime} X\right)^{-1} R^{\prime}\right]^{-1}(R b-q)
$$

which under the null hypothesis will have $J$ and $(n-K)$ degrees of freedom. If $J=1$,
then this $F$-statistic is just the square of a $t$ statistic with ( $N-K$ ) degrees of freedom. Indeed, any of the hypotheses listed above that involve a single row in $R(J=1)$ are commonly expressed as t-tests, no matter how many coefficients may be involved. In contrast, tests involving $J>1$ are joint tests, and can only be expressed as an $F$-statistic (or $\chi^{2}$ statistic). Note that any of these subset hypothesis tests may be performed in Stata via the test command; that command's accum option may be used to build up a set of restrictions into a single joint $F$-statistic.

The restricted least squares estimator

Alternatively, we might estimate $b$ subject to the restrictions $R b=q$. This may be done as the Lagrangean expression

$$
L\left(b_{0}, \lambda\right)=\left(y-X b_{0}\right)^{\prime}\left(y-X b_{0}\right)+2 \lambda^{\prime}\left(R b_{0}-q\right)
$$

The necessary conditions are

$$
\frac{\partial L}{\partial b_{0}}=-2 X^{\prime}\left(y-X b_{0}\right)+2 R^{\prime} \lambda=0
$$

and

$$
\frac{\partial L}{\partial \lambda}=2\left(R b_{0}-q\right)=0
$$

Assuming that $X$ is of full column rank, this system may be solved for the solution in terms of the unconstrained estimator $b$ :

$$
b_{0}=b-\left(X^{\prime} X\right)^{-1} R^{\prime}\left[R\left(X^{\prime} X\right)^{-1} R^{\prime}\right]^{-1}(R b-q)
$$

in which the restricted least squares (RLS) estimator may be seen as a corrected version of the OLS estimator in which the correction factor relates to the magnitude of the discrepancy vector $m$. The explicit solution for $\lambda$ likewise depends on the discrepancy vector, indicating the shadow cost of each constraint on the unconstrained solution. The covariance matrix for $b_{0}$ may be written as the covariance matrix of the unrestricted OLS estimator minus
a nonnegative definite matrix. To compare the unconstrained and constrained (RLS) solutions, we may form an $F$-statistic from the expression in the difference of sums of squared residuals:

$$
e_{0}^{\prime} e_{0}-e^{\prime} e=(R b-q)^{\prime}\left[R\left(X^{\prime} X\right)^{-1} R^{\prime}\right](R b-q)
$$

where $e_{0}$ are the RLS residuals. This gives rise to the $F$-statistic

$$
F[J, n-K]=\frac{\left(e_{0}^{\prime} e_{0}-e^{\prime} e\right) / J}{e^{\prime} e /(n-K)}
$$

which can be transformed into

$$
F[J, n-K]=\frac{\left(R^{2}-R_{0}^{2}\right) / J}{\left(1-R^{2}\right) /(n-K)}
$$

In this context, the effect of the restrictions may be viewed as either the loss in the least squares criterion (with the numerator of the first statistic viewed as the average cost per restriction) or the reduction in $R^{2}$ from the restrictions. The numerator of either expression is non-negative, since imposition of the
restrictions cannot increase $R^{2}$ nor decrease the sum of squared residuals. This estimator is provided by Stata as the command cnsreg (constrained regression).

These test statistics are valid for normally distributed disturbances; without normality, we still have an asymptotic justification for the statistics, and $J \times F(J, n-K) \sim \chi^{2}(J)$ in large samples. Since $b$ is asymptotically normal, irrespective of the distribution of the errors, this form of the test statistic will be appropriate for large samples.

Testing nonlinear restrictions

The apparatus we have developed here applies for sets of linear restrictions on $\beta$. What if we have nonlinear restrictions on the parameters: e.g., a constraint involving a product of parameters? Then we might write $H_{0}: c(\beta)=q$, and
we would require an estimate of the variance of the expression $c(b)-q$, a nonlinear function of the LS coefficient vector $b$. This test statistic may be calculated via the delta method, as we have discussed earlier. In the context of Stata, the command testnl performs nonlinear tests on the coefficient vector.

Choosing between non-nested models

The apparatus described above works quite well for classical hypothesis testing, where one model can be viewed as a proper subset of another. Annoyingly, economic theories often are cast in the form of competing hypotheses, where neither may be expressed as a proper subset of the other. Furthermore, no theory may propound the "supermodel" which would encompass all elements of both theories: essentially the artificial nesting of both models. Tests in this context are testing one specific
model versus a hybrid model that contains elements of both, which is not proposed by either theory. Thus, what if we have a setup like

$$
\begin{array}{r}
H_{0}: y=X \beta+\epsilon_{0} \\
H_{1}: y=Z \gamma+\epsilon_{1}
\end{array}
$$

where some of the elements of both $X$ and $Z$ are unique? The Bayesian econometrician would have no difficulty with this, since she would merely ask "which of these hypotheses is more likely to have generated the data"? But examining goodness of fit, and noting that one of these models has a higher $R^{2}$ or $\bar{R}^{2}$, is not likely to be satisfying.

A solution to this problem was proposed by Davidson and MacKinnon as their $J$ test: not to be confused with Hansen's $J$ test in GMM. It relates to a very simple approach: if model 0 has explanatory power over and above that of model 1 , then model 0 is superior, and vice
versa. The $J$ test therefore is performed by generating the predicted values of each series, and including them in an augmented regression of the other model. So, for instance, include the $\hat{y}$ from the alternative hypothesis above in the null hypothesis' model. If that $\hat{y}$ is significant, then we reject the model of the null hypothesis. We now reverse the definitions of the two models, and include the $\hat{y}$ from the null hypothesis in the alternative hypothesis' model. Unfortunately, all four possibilities can arise: $H_{0}$ may stand against $H_{1}, H_{1}$ may stand against $H_{0}$, both hypotheses may be rejected, or neither hypothesis may be rejected. Only in the first two cases does the $J$ test deliver a definitive verdict. The Stata user-contributed command nnest performs the $J$ test.

A separate test along these lines is that of Cox (1961, 1962) extended by Pesaran (1974) and Pesaran and Deaton (1978). These tests are
based on likelihood ratio tests that may be constructed from the fitted values and sums of squared residuals of the nonnested models. The Cox-Pesaran-Deaton test is also performed by the nnest package.

## Prediction

We often use regression methods to generate predictions from the estimated model. These predictions may be in-sample: that is, for the values of $X$ used in the estimation, or out-ofsample, for arbitrary values of $X: X_{0}$. If we wish to predict $E\left[y_{0} \mid X_{0}\right]$, the Gauss-Markov theorem indicates that the evaluation of the regression surface is the BLUP (best linear unbiased predictor):

$$
E\left[y_{0} \mid X_{0}\right]=\widehat{y}_{0}=X_{0} b .
$$

In this context, we are predicting the value of $y$ that would be expected for every observation with its regressors equal to $X_{0}$. We must
distinguish this from predicting the value of $y$ that would apply for a specific $X_{0}$ : for instance, the value of an individual house with a particular set of characteristics. When we do the former, we may use our assumptions on the distribution of the error term to take the expected value of $\epsilon$ over the subpopulation as zero. When we want to make a specific prediction, we must acknowledge that this predicted value contains an $\epsilon_{0}$.

The prediction error for the latter case is

$$
y_{0}-\widehat{y}_{0}=(\beta-b) X_{0}+\epsilon_{0},
$$

an expression that includes both the sampling error of $b$ and the specific $\epsilon$ associated with that value of $y$. To generate a standard error for $E\left[y_{0} \mid X_{0}\right]$, we may form the expectation of this error, which only involves the first term, giving rise to the standard error of prediction. For the prediction of a specific value
of $y$ (rather than the expected value from the subpopulation), we consider the variance of the prediction error, which gives rise to the standard error of forecast (including the variance of $\epsilon_{0}$ ).

The variance of $\epsilon_{0}$ in the former case may be written, for a simple $y$ on $X$ regression, as

$$
\sigma^{2}\left[n^{-1}+\frac{\left.\left(X_{0}-\bar{X}\right)^{2}\right)}{\sum x^{2}}\right]
$$

where $x$ is the deviation from mean of $X$. The square root of this quantity (with $\sigma^{2}$ replaced with our consistent estimate $s^{2}$ ) is then utilized in the prediction interval estimate:

$$
\hat{y}_{0} \pm t_{\lambda / 2} \operatorname{se}\left(\epsilon_{0}\right)
$$

The interval prediction is then defined as a pair of parabolas, with the narrowest interval at $\bar{X}$, widening as we diverge from the multivariate point of means.

In contrast, if we wish to predict the specific value for $y_{0}$ corresponding to a value $X_{0}$, we have the forecast variance

$$
\sigma^{2}\left[1+n^{-1}+\frac{\left.\left(X_{0}-\bar{X}\right)^{2}\right)}{\sum x^{2}}\right]
$$

Since the latter expression contains a $\sigma^{2}$ term which does not go to zero as $n \rightarrow \infty$, the forecast interval is wider than the prediction interval, and the $\sigma^{2}$ represents an irreducible minimum for the accuracy of the forecast.

For a multiple regression problem, the prediction variance is

$$
\operatorname{Var}\left[(\beta-b) X_{0} \mid X\right]=X_{0}^{\prime}\left[\sigma^{2}\left(X^{\prime} X\right)^{-1}\right] X_{0}
$$

whereas the forecast variance for a specific value of $y_{0}$ is

$$
\operatorname{Var}\left[(\beta-b) X_{0} \mid X\right]=\sigma^{2}+X_{0}^{\prime}\left[\sigma^{2}\left(X^{\prime} X\right)^{-1}\right] X_{0} .
$$

These two constructs, the standard error of prediction and standard error of forecast, may
be computed following a regression in Stata via predict newvar, stdp and predict newvar, stdf respectively.

Binary variables

One of the most useful tools in applied econometrics is the binary or dummy variable. We use dummies for many purposes, and there are numerous commands in any econometrics language that work with dummy variables. We often want to use a set of dummy variables to reflect the values of a qualitative variable: one which takes on several distinct values. We may also use dummies to reflect the values of an ordinal variable: one which takes on discrete values, but has no numerical significance. E.g., a Likert scale with values "Strongly disagree", "Disagree", "Neutral", " Agree", "Strongly Agree" might be coded as 1,2,3,4,5, but we would
never want to enter those values into a regression equation-that would treat them as cardinal measurements, which they are not.

We often use dummies to indicate membership in a set. If we are defining a dummy for male survey respondents, we can use generate male = (sex == "M") if we know that there are no missing values in the sex variable. Otherwise, we should use generate male = (sex == "M") if sex !="" to ensure that missing values become missing in the dummy. If sex was coded as (1,2), we would use generate male = (sex == 2) if sex <. to accomplish the same task.

For a numerically coded qualitative variable (sex=(1,2), or region $=(11,21,31,41)$ ) we could use the tab varname, gen(stub) command. For instance, tab sex, gen(gender) would create gender1 and gender2. This will automatically take
care of missing values, and becomes particularly handy if we have a large number of categories (e.g. the 50 states). What if the variable is a string variable, e.g., state=AK, AL, $A Z$, etc.? Then we may use the encode command. For instance, encode state, gen(stid) would generate a new variable stid which appears identical to state. In reality, it now takes on numeric values of $1-50$, with value labels linking each numeric code and the original string. The stid variable may now be used in any command that requires numeric values (e.g., tsset). Note that the mean of a dummy variable is the sample proportion of 1 s for that category.

We define a complete set of dummy variables as mutually exclusive and exhaustive. If $D$ is a $n \times d$ matrix of dummies, $D \iota_{d}=\iota_{n}$. Note that the mean of a dummy variable is the sample proportion of 1 s for that category, and those proportions must sum to unity for a complete
set. Since a complete set of dummies sums to an $\iota$ vector, it cannot normally be included in a regression equation; we must drop one (any one) of the dummies and include the rest. The coefficients on the included dummies are then with reference to the excluded class.

For instance, consider region, with four values. A regression of income on any three of the region dummies will yield estimates of the differences in mean income levels between those regions and the excluded region (whose income level is given, in point and interval form, by the constant term). The ANOVA "F" statistic for this regression is aptly named, for this model is a one-way analysis of variance: does the qualitative factor region affect income? That null hypothesis is tested by the ANOVA "F", and all "slopes" being equal to zero is equivalent to all regions having statistically indistinguishable income levels. (This is analogous to the
$t$-test performed by the ttest command, but that test can only be used on two groups (e.g., $M$ and $F$ )). What if we wanted point and interval estimates of the regional income levels? Since we are performing one-way ANOVA, we can include all dummies and suppress the constant term. This form of the equation will not be useful for testing differences in mean income levels, though.

It is useful to note that any form of ANOVA may be expressed as a regression equation with nothing but dummy variables on the RHS. For instance, a "two-way ANOVA" takes account of two qualitative factors: e.g., gender and region. We may include all but one of the dummies for each factor in the equation; the constant term is now the mean income level of the excluded class (e.g., males in region 11). In this form, we consider that the two-way table defined by this equation may be filled out
from the marginal entries, which is equivalent to assuming that the effects of gender and region on income are independent factors. We can imagine cases where this is not so: for instance, unemployment rates may be high for teenagers, high for minorities, but the unemployment rate among minority teens may be even higher than would be predicted by the sum of the two marginal effects. This gives rise to a two-way ANOVA with interactions, in which we consider an additional set of coefficients that interact the factors. (Note that this can be done in a Boolean manner, treating the individual factors' dummies with an AND condition, or algebraically, since the product of two dummies is the AND of those factors). Such a model allows the two factors to have nonlinear effects on the dependent variable. The nonlinearity is readily testable as the condition that all interaction terms have zero coefficients. Note that in the case of two-way

ANOVA, even if we exclude the constant term from the equation, we cannot include more than one complete set of dummies.

We may consider higher-level ANOVA models, but even in a large dataset we often run out of degrees of freedom in populating all of the cells of a three- or four-way design. Since each cell of such a design is a conditional mean, we must have a large enough $n$ in each cell to estimate such a mean reliably. Note also that a dummy variable that is unity for a single observation has the effect of removing that observation from the analysis. This should be kept in mind in a time-series data context when defining dummies which allow for special events (strikes, wars, etc.): if those events last a single period, a dummy has the effect of dropping them from the sample.

In much applied econometric research, we are concerned with what a statistician would call
"ANOCOVA": analysis of covariance, in which the regression equation contains both qualitative and quantitative factors (a combination of measurable Xs and dummies). In such a model, dummies alone serve to shift the constant term for those observations which belong to a particular category. In such a model, dummies may also be interacted with measurable variables, which allows for different slopes as well as intercepts for different categories. We can readily define a set of functions that would apply for, e.g., data containing males/females and blacks/whites. Up to four distinct functions may be defined for these data, and we may consider all of the proper subsets of those functions in which slopes, intercepts, or both are constrained across the categories. Note that estimating the four functions (and allowing both slope and intercept to vary) is essentially the same as estimating separate regression equations for each category, with the
single constraint that all of those equations are forced to have the same $\sigma^{2}$.

A set of seasonal dummies may be used, in the context of time-series data, to deseasonalize the data. If quarterly (monthly) retail sales are regressed on three (eleven) seasonal dummies, the residuals from this model are deseasonalized retail sales. (It is customary to add the mean of the original series to restore the scale of the data). This would be additive seasonal adjustment, in which we assume that the effect of being in Q1 is a certain dollar amount of retail sales. If we wished to apply multiplicative seasonal adjustment, we would regress $\log y$ on the three (eleven) dummies, which would assume that the effect of being in Q1 is a certain percentage deviation in retail sales from the excluded season. This seasonal adjustment may be done as a separate step, or may be applied in the context of a regression model: if we wish to regress retail sales
(NSA) on other data which are SA or SAAR, we may just include the seasonal dummies in the regression equation (as we would a time trend; the notion of partialling off the effect of trend or seasonality is the same if we assume that the regressors are free from those effects. If the regressors might contain a trend or seasonal factors, they should be detrended (or deseasonalized) as well). In any case, we would test for the existence of seasonal factors by an $F$-test on the set of seasonal dummies.

Dummy variables are often used to test for structural change in a regression function in which we specify a priori the timing of the possible structural breakpoints. We may allow for different slopes and/or intercepts for different periods in a time-series regression (e.g., allowing for a consumption function to shift downward during wartime). If we fully interact the regime dummies, we are in essence estimating separate regressions for each regime,
apart from the constraint of a common $\sigma^{2}$. The test that all coefficients associated with regime $2(3, \ldots)$ are equal to zero is often termed a Chow test. One can perform such a test by estimating the two (three,...) regressions separately, and comparing the sum of their SSRs to that of the restricted (singleregime) regression. However, this is generally easier to implement using regime dummies interacted with all regressors. This strategy is also desirable since it allows for a variety of special cases where we assume that some coefficients are stable across regimes while others vary: those interacted with regime dummies will be allowed to vary.

In some cases a regime may be too short to perform this test-i.e. estimate the regime as a separate regression. This applies particularly at the end of a time series, where we may want to ask whether the most recent $n_{2}$ observations are generated by the same model as the
prior $n_{1}$ observations. In this case, one can construct an $F$-test by running the regression over all $n$ observations, then running it over the first $n_{1}$ observations, and comparing their SSRs. The SSR for the full sample will exceed that from the first $n_{1}$ observations unless the regression fits perfectly over the additional $n_{2}$ data points. Therefore, this test has $n_{2}$ d.f. in the numerator:

$$
F\left[n_{2}, n_{1}-K\right]=\frac{\left(e_{n}^{\prime} e_{n}-e_{1}^{\prime} e_{1}\right) / n_{2}}{\left(e_{1}^{\prime} e_{1}\right) /\left(n_{1}-K\right)}
$$

where $e_{n}$ is the residual vector from the full sample. This is often termed the Chow predictive test.

We may also be concerned about the realistic possibility that the $\sigma^{2}$ has changed over regimes. We could deal with this by computing robust standard errors for the regression with regime dummies, but we might want to estimate the differing variances for each regime.

This could be handled by explicitly providing for "groupwise heteroskedasticity", as we will discuss at a later date.

In other cases, we may want to allow a profile (such as an age-earnings profile) to reflect different slopes over time, but force the resulting function to be piecewise continuous. This can be achieved by a linear spline: for three segments, for instance, we wish to constrain the estimated segments to join at the two points, known as knots, of the spline function. Say that we wish to estimate three separate segments of the earnings function, for those less than 25, between 25 and 40, and over 40. Rather than estimating six coefficients (three slopes, three intercepts), we must place two constraints on the system: that the function evaluated at the knots is piecewise continuous. Doing the algebra for this problem will show that three regressors (and a constant term)
are needed: age, (age-25) and (age-40). The latter two are set to zero if they are negative. Regression of earnings on these three regressors and a constant term will result in a piecewise continuous age-earnings profile. This can be automated by Stata's mkspline command.

Splines of higher orders are often useful in applied work; e.g. a quadratic spline will be continuous and differentiable once, and a cubic spline will be continuous and twice differentiable. Both have been widely used in financial analysis: e.g., term structure research.

## Tests of model stability

The dummy variable methods described above are useful when the timing (or location, in cross-sectional terms) of a structural break is known a priori. However, we often are unsure as to whether a change has taken place, or in
a time-series context, when a relationship may have undergone a structural shift. This is particularly problematic when a change may be a gradual process rather than an abrupt and discernable break. A number of tests have been devised to evaluate the likelihood that a change has taken place, and if so, when that break may have occurred. The "cusums" tests of Brown, Durbin and Evans (1975) are based on recursive residuals (or their squares), and the notion that a change will show up in the one-step ahead prediction errors if we consider all possible breakpoints in a series. A formal test may be devised by evaluating the cumulative sum of residuals, and noting that the cumulative sum will stray outside a confidence interval in the vicinity of a structural break. Tests of this nature are available in Stata via the user-contributed routine cusum6. The power of cusums tests may be quite low in practice, but they will be somewhat useful in detecting a structural break in the relationship.

## Specification error

We have worked with regression models under the maintained hypothesis that the model is correctly specified. What if this assumption does not hold? Although there is no necessary relation between the specification employed and the true data generating process, let us consider two alternatives: given the dependent variable $y$, we may omit relevant variables, or include irrelevant variables. First consider omission of relevant variables from the correctly specified model

$$
y=X_{1} \beta_{1}+X_{2} \beta_{2}+\epsilon
$$

with $K_{1}$ and $K_{2}$ regressors in the two subsets. Imagine that we ignore $X_{2}$, so that

$$
\begin{array}{r}
b_{1}=\left(X_{1}^{\prime} X_{1}\right)^{-1} X_{1}^{\prime} y \\
=\beta_{1}+\left(X_{1}^{\prime} X_{1}\right)^{-1} X_{1}^{\prime} X_{2} \beta_{2}+\left(X_{1}^{\prime} X_{1}\right)^{-1} X_{1}^{\prime} \epsilon
\end{array}
$$

Unless $X_{1}^{\prime} X_{2}=0$ or $\beta_{2}=0$, the estimate of $b_{1}$ is biased, so that

$$
E\left[b_{1} \mid X\right]=\beta_{1}+P_{1 \cdot 2} \beta_{2}
$$

where $P_{1 \cdot 2}=\left(X_{1}^{\prime} X_{1}\right)^{-1} X_{1}^{\prime} X_{2}$ is the $K_{1} \times K_{2}$ matrix reflecting the regression of each column of $X_{2}$ on the columns of $X_{1}$. If $K_{1}=K_{2}=1$ and the $X_{2}$ variable is correlated with $X_{1}$, we may derive the direction of bias, but in general we will not be able to make any statements about the coefficients in $b_{1}$ with multiple variables in each set. It appears, then, that the cost of omitting relevant variables is high: the resulting coefficients are biased and inconsistent.

What about the inclusion of irrelevant explanatory variables: for instance, if the true DGP involves only $X_{1}$, but we mistakenly include the $X_{2}$ variables as well? In that case, we are failing to impose the restrictions that $b_{2}=0$, but
since in the population, $\beta_{2}=0$, those restrictions are costless. The inclusion of $X_{2}$ leaves our estimates of $b_{1}$ unbiased and consistent, as is the estimate of $\sigma^{2}$. What is the cost, then, of overfitting the model and including the additional variables? We lose degrees of freedom, of course, and by ignoring the information that the $X_{2}$ variables do not belong in the model, we generate less efficient estimates of $b_{1}$ than we would with the correctly specified model. This is especially apparent if we have $K_{1}=K_{2}=1$ and the correlation between $X_{1}$ and $X_{2}$ is high. Mistakenly including $X_{2}$ will lead to quite imprecise estimates of $b_{1}$.

Although there is some cost to overfitting a model, it would appear that the costs of these two types of specification error are quite asymmetric, and that we would much rather err on the side of caution (including additional variables) to avoid the severe penalties of underfitting the model. Given this, a model selection
strategy that starts with a simple specification and seeks to refine it by adding variables is flawed, and the opposite approach: David Hendry's general-to-specific methodology has much to recommend it. Although a very general specification may be plagued by collinearity, and a model with sufficient variables will run afoul of the 5\% type I error probability, it is much more likely than a recursive simplification strategy will yield a usable model at the end of the specification search.

In dynamic models utilizing time-series data, this advice translates into "do not underfit the dynamics". If the time form of a dynamic relationship is not known with certainty, it would be prudent to include a number of lagged values, and "test down" to determine whether the longer lags are necessary. This will follow a general-to-specific methodology, allowing for the more complex dynamic specification
if it is warranted. Omitting higher-order dynamic terms is a common cause of apparent non-independence of the regression errors, as signalled by residual independence tests.

One particular approach to possible functional form misspecification is offered by Ramsey's RESET (regression specification error) test. This test is simple: after a regression of $y$ on the regressors $X$, one adds polynomials in the fitted values $\hat{y}$ to the regression: for instance, squares and cubes. Under the hypothesis that the relationship between $y$ and $X$ is adequately modeled as linear, the coefficients on these additional regressors should not be significantly different from zero. Since polynomials in the regressors can approximate many functions, a rejection in the RESET test may also suggest the appropriateness of a nonlinear specification: for instance, a log-linear or double-log model. The RESET test may easily be performed in Stata via the ovtest command.

