EC 327 Problem Set 2: Solutions

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Exercise 14.1

First, for each $t > 1$, $var(\Delta u_{it}) = var(u_{it} - u_{i,t-1}) = var(u_{it}) + var(u_{i,t-1}) = 2\sigma_u^2$, where we use the assumptions of no serial correlation in $\{u_t\}$ and constant variance. Next, we find the covariance between Δu_{it} and $\Delta u_{i,t+1}$. Because these each have a zero mean, the covariance is $E(\Delta u_{it} \cdot \Delta u_{i,t+1}) = E[(u_{it} - u_{i,t-1})(u_{i,t+1} - u_{it})] = E(u_{it}u_{i,t+1}) - E(u_{it}^2) - E(u_{i,t-1}u_{i,t+1}) + +E(u_{i,t-1}u_{it}) = -E(u_{it}^2) = -\sigma_u^2$ because of the no serial correlation assumption. Because the variance is constant across t, by Problem 11.1, $corr(\Delta u_{it}, \Delta u_{i,t+1}) = cov(\Delta u_{it}, \Delta u_{i,t+1})/var(\Delta u_{it}) = -\sigma_u^2/(2\sigma_u^2) = -.5$.

Exercise 14.4

(i) Mens athletics are still the most prominent, although womens sports, especially basketball but also gymnastics, softball, and volleyball, are very popular at some universities. Winning percentages for football and mens and womens basketball are good possibilities, as well as indicators for whether teams won conference championships, went to a visible bowl game (football), or did well in the NCAA basketball tournament (such as making the Sweet 16). We must be sure that we use measures of athletic success that are available prior to application deadlines. So, we would probably use football success from the previous school year; basketball success might have to be lagged one more year.

(ii) Tuition could be important: ceteris paribus, higher tuition should mean fewer applications. Measures of university quality that change over time, such as student/faculty ratios or faculty grant money, could be important.

(iii) An unobserved effects model is

$$\log(apps_{it}) = \delta_1 d90_t + \delta_2 d95_t + \beta_1 athsucc_{it} + \beta_2 \log(tuition_{it}) + \ldots + a_i + u_{it}, t = 1, 2, 3.$$

The variable $attsucc_{it}$ is shorthand for a measure of athletic success; we might include several measures. If, for example, $attsucc_{it}$ is football winning percentage, then $100\beta_1$ is the percentage change in applications given a one percentage point increase in winning percentage. It is likely that a_i is correlated with athletic success, tuition, and so on, so fixed effects estimation is appropriate. Alternatively, we could first difference to remove a_i, as discussed in Chapter 13.

Exercise C 14.1

(i) See Computer Exercise 13.5(i).

(ii) See Computer Exercise 13.5(ii).

(iii) See Computer Exercise 13.5(iii).

(iv) The fixed effects estimates, reported in equation form, are

$$\widehat{\log(rent_{it})} = .386y90_y + .072\log(pop_{it}) + .310\log(avginc_{it}) + .0112pctstu_{it},$$

$$N = 64,\ T = 2.$$

(There are $N = 64$ cities and $T = 2$ years.) We do not report an intercept because it gets removed by the time demeaning. The coefficient on $y90_t$ is identical to the intercept from the first difference estimation, and the slope coefficients and standard errors are identical to first differencing. We do not report an R-squared because none is comparable to the R-squared obtained from first differencing.

Exercise C 14.4

(i) Write the equation for times t and $t1$ as

$$\log(uclms_{it}) = a_i + c_i t + \beta_1 ez_{it} + u_{it}$$

$$\log(uclms_{i,t-1}) = a_i + c_i(t-1) + \beta_1 ez_{i,t-1} + u_{i,t-1}$$

and subtract the second equation from the first. The a_i are eliminated and $c_i t c_i(t1) = c_i$. So, for each $t \geq 2$, we have

$$\Delta\log(uclms_{it}) = c_i + \beta_1 \Delta ez_{it} + u_{it}.$$

(ii) Because the differenced equation contains the fixed effect c_i, we estimate it by FE. We get $\hat{\beta}_1 = -.251$, $se(\hat{\beta}_1) = .121$. The estimate is actually larger in magnitude than we obtain in Example 13.8 [where $\hat{\beta}_1 = -1.82$, $se(\hat{\beta}_1) = .078$], but we have not yet included year dummies. In any case, the estimated effect of an EZ is still large and statistically significant.

(iii) Adding the year dummies reduces the estimated EZ effect, and makes it more comparable to what we obtained without $c_i t$ in the model. Using FE on the first-differenced equation gives $\hat{\beta}_1 = -.192$, $se(\hat{\beta}_1) = .085$, which is fairly similar to the estimates without the city-specific trends.

Exercise C 14.7

(i) If there is a deterrent effect then $\beta_1 < 0$. The sign of β_2 is not entirely obvious, although one possibility is that a better economy means less crime in general, including violent crime (such as drug dealing) that would lead to fewer murders. This would imply $\beta_2 > 0$.

(ii) The pooled OLS estimates using 1990 and 1993 are

$$\widehat{mrdrte}_{it} = -5.28 - 2.07d93_t + .128exec_{it} + 2.53unem_{it}$$

$$N = 51, T = 2, R^2 = .102.$$

There is no evidence of a deterrent effect, as the coefficient on *exec* is actually positive (though not statistically significant).

(iii) The first-differenced equation is

$$\Delta\widehat{mrdrte}_i = .413 - .104\Delta exec_i - .067\Delta unem_i$$

$$N = 51, R^2 = .110.$$

Now, there is a statistically significant deterrent effect: 10 more executions is estimated to reduce the murder rate by 1.04, or one murder per 100,000 people. Is this a large effect? Executions are relatively rare in most states, but murder rates are relatively low on average, too. In 1993, the average murder rate was about 8.7; a reduction of one would be nontrivial. For the (unknown) people whose lives might be saved via a deterrent effect, it would seem important.

(iv) The heteroskedasticity-robust standard error for $\Delta exec_i$ is .017. Somewhat surprisingly, this is well below the nonrobust standard error. If we use the robust standard error, the statistical evidence for the deterrent effect is quite strong ($t \approx -6.1$).

(v) Texas had by far the largest value of *exec*, 34. The next highest state was Virginia, with 11. These are three-year totals.

(vi) Without Texas in the estimation, we get the following, with heteroskedasticity-robust standard errors in [·]:

$$\begin{array}{lccccc} \widehat{mrdrte}_i = & .413 & - & .067 & \Delta exec_i - & .070 & \Delta unem_i \\ & (0.211) & & (.105) & & (.160) & \\ & [0.200] & & [.079] & & [.146] & \end{array}$$

$$N = 50, R^2 = .013.$$

Now the estimated deterrent effect is smaller. Perhaps more importantly, the standard error on $\Delta exec_i$ has increased by a substantial amount. This happens because when we drop Texas, we lose much of the variation in the key explanatory variable, $\Delta exec_i$.

(vii) When we apply fixed effects using all three years of data and all states we get

$$\widehat{mrdrte}_{it} = 5.82 + 1.55 d90_t + 1.73 d93_t - .138 exec_{it} + .221 unem_{it}$$

$$N = 51, T = 3, R^2 = .073.$$

The size of the deterrent effect is only about half as big as when 1987 is not used. Plus, the t-statistic, about -.78, is very small. The earlier finding of a deterrent effect is not robust to the time period used. Oddly, adding another year of data causes the standard error on the $exec$ coefficient to markedly increase.

Exercise C 14.10

(i) The pooled OLS estimate of β_1 is about .360. If $\Delta concen = .10$ then $\Delta \widehat{lfare} = .360(.10) = .036$, which means air fare is estimated to be about 3.6% higher.

(ii) The 95% CI obtained using the usual OLS standard error is .301 to .419. But the validity of this standard error requires the composite error to have no serial correlation, which effectively means ai is not in the equation. The fully robust 95% CI, which allows any kind of serial correlation over the four years (and any kind of heteroskedasticity), is .298 to .422 quite a bit wider than the usual CI. The wider CI is appropriate, as the neglected serial correlation introduces uncertainty into our parameter estimators.

(iii) The quadratic has a U-shape, and the turning point is about $.902/[2(.103)] \approx 4.38$. This is the value of $\log(dist)$ where the slope becomes positive. The value of $dist$ is $\exp(4.38)$, or about 80. The shortest distance in the data set is 95 miles, so the turning point is outside the range of the data (a good thing in this case). What is being captured is an increasing elasticity of $fare$ with respect to dist as fare increases.

(iv) The RE estimate of β_1 is about .209, which is quite a bit smaller than the pooled OLS estimate. Still, the estimate implies a positive relationship between fare and concentration. The estimate is very statistically significant, too, with $t = 7.88$.

(v) The FE estimate is .169, which is lower yet but not so different from the RE estimate. The value of $\hat{\lambda}$ in the RE estimation is about .900, and so we expect RE and FE to be fairly similar. [Remember, RE uses a quasi-demeaning that depends on the estimate of lamda; see equation (14.11).]

(vi) Factors about the cities near the two airports on a route could affect demand for air travel, such as population, education levels, types of employers, and so on. Of course, each of these can be time-varying, although, over a short stretch of time, they might be roughly constant. The quality of the freeway system and access to trains, along with geographical features (is the city near a river?) would roughly be time-constant. These could certainly be correlated with concentration.

(vii) Accounting for an unobserved effect and using fixed effects gives us a positive, statistically significant relationship. I would go with the FE estimate, .169, which allows for concentration to be correlated with all time-constant features that affect costs and demand.

Exercise 11.2

(i) $E(x_t) = E(e_t)(1/2)E(e_{t-1}) + (1/2)E(e_{t-2}) = 0$ for t = 1,2,...Also, because the e_t are independent, they are uncorrelated and so $Var(x_t) = Var(e_t) + (1/4)Var(e_{t-1}) + (1/4)Var(e_{t-2}) = 1 + (1/4) + (1/4) = 3/2$ because $Var(e_t) = 1$ for all t.
(ii) Because x_t has zero mean:

$$Cov(x_t, x_{t+1}) = E(x_t x_{t+1}) = E[(e_t - (1/2)e_{t-1} + (1/2)e_{t-2})(e_{t+1} - (1/2)e_t + (1/2)e_{t-1})] =$$

$$= E(e_t e_{t+1}) - (1/2)E(e_t^2) + (1/2)E(e_t e_{t-1}) - (1/2)E(e_{t-1}e_{t+1}) + (1/4)E(e_{t-1}e_t) - (1/4)E(e_{t-1}^2) +$$

$$+ (1/2)E(e_{t-2}e_{t+1}) - (1/4)E(e_{t-2}e_t) + (1/4)E(e_{t-2}e_{t-1}) = -(1/2)E(e_t^2) - (1/4)E(e_{t-1}^2) = -3/4$$

The third to last equality follows because the e_t are pairwise uncorrelated and $E(e_t^2) = 1$ for all t. Thus:

$$Corr(x_t, x_{t+1}) = -(3/4)/(3/2) = 1/2.$$

Computing $Cov(x_t, x_{t+2})$ is even easier because only one of the nine terms has expectation different from zero: $(1/2)E(e_t^2) = \frac{1}{2}$. Therefore, $Corr(x_t, x_{t+2}) = (1/2)/(3/2) = 1/3$.
(iii) $Corr(x_t, x_{t+h}) = 0$ for $h > 2$ because, for $h > 2$, x_{t+h} depends at most on e_{t+j} for $j > 0$, while x_t depends on e_{t+j}, $j \leqslant 0$.
(iv) Yes, because terms more than two periods apart are actually uncorrelated, and so it is obvious that $Corr(x_t, x_{t+h}) = 0$ as h tends to ∞.

Exercise C11.2

The estimated equation is

$$ghr\hat{w}age_t = -0.010 + .728goutphr_t + .458goutphr_{t-1}$$

$$n = 39, R^2 = .493.$$

The t statistic on the lag is about 2.76, so the lag is very significant.
(ii) We follow the hint and write the LRP as $\theta = \beta_1 + \beta_2$, and then plug $\beta_1 = \theta - \beta_2$ into the original model

$$ghrwage_t = \beta_0 + \theta goutphr_t + \beta_2(goutphr_{t-1} - goutphrt_t) + u_t.$$

Therefore, we regress $ghrwage_t$ onto $goutphr_t$, and $(goutphr_{t-1} - goutphr_t)$ and obtain the standard error for θ. Doing this regression gives 1.186 (as we can compute directly from part (i)) and $se(\theta) = .203$. The t statistic for testing $H_0 : \theta = 1$ is $(1.1861)/.203 \approx .916$, which is not significant at the usual significance levels (not even 20 per cent against a two-sided alternative).
(iii) When $goutphr_{t-2}$ is added to the regression from part (i), and we use the 38 observations now available for the regression, $\hat{\beta}_3 \approx .065$ with a t statistic of about .41. Therefore, $goutphr_{t-2}$ need not be in the model.

Exercise C11.6

The estimated accelerator model is

$$hatDeltainven_t = 2.59 + .152\Delta GDP_t$$

$$n = 36, R^2 = .554.$$

$\hat{\beta}_1$ is very statistically significant, with $t \approx 6.49$.
(ii) When we add $r3_t$, we obtain

$$\Delta inven_t = 3.00 + .159\Delta GDP_t - .895r3_t$$

$$n = 36, R^2 = .562.$$

The sign of $\hat{\beta}_2$ is negative, as predicted by economic theory, and it seems practically large. However, $\hat{\beta}_2$ is not statistically different from zero. (Its t statistic is less than one in absolute value.)
If $\Delta r3_t$ is used instead, the coefficient becomes about -.470, $se = 1.540$. So this is even less significant than when $r3_t$ is in the equation. But, without more data, we cannot conclude that interest rates have a ceteris paribus effect on inventory investment.