BOSTON COLLEGE
Department of Economics
EC 771: Econometrics
Spring 2009
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Problem Set 2: Solutions

Point Distribution:
1), 4), 5): 5 points each
2), 3): 10 points each
6): 7 points
7): 18 points

1) Notice that the three vectors x_1, x_2, x_3 are not linearly independent. In fact, $4x_1 - x_2 - 4x_3 = 0$. First, solve for x_2 to give $x_2 = 4x_1 - 4x_3$. Then,

$$z = b_1x_1 + b_2x_2 = b_1x_1 + b_2(4x_1 - 4x_3) = (b_1 + 4b_2)x_1 - 4b_2x_3$$

Thus, we have z as a linear combination of x_1 and x_3, and so $z \in \delta(x_1, x_3)$. Next, solve the initial linear dependence equation for x_1 to obtain $x_1 = \frac{1}{4}x_2 + x_3$. Then,

$$z = b_1x_1 + b_2x_2 = b_1(\frac{1}{4}x_2 + x_3) + b_2x_2 = (\frac{1}{4}b_1 + b_2)x_2 + b_1x_3$$

Thus, we have z as a linear combination of x_2 and x_3, and so $z \in \delta(x_2, x_3)$.

2) We have the following system of equations:

$$\begin{aligned} z_1 &= x_1 - 2x_2, \\ z_2 &= x_2 + 4x_3, \\ z_3 &= 2x_1 - 3x_2 + 5x_3, \end{aligned}$$

where x_i, z_j are vectors. Clearly, we can write this system as follows:

$$Z = XA$$

where

$$Z = \begin{pmatrix} z_1 & z_2 & z_3 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 0 & 2 \\ -2 & 1 & -3 \\ 0 & 4 & 5 \end{pmatrix}$$

Next, we need to solve for x_i in terms of z_j. Using some linear algebra technique (substitution, Gaussian elimination, Cramer's rule), one obtains the following system:

$$\begin{aligned} x_1 &= 17z_1 + 10z_2 - 8z_3, \\ x_2 &= 8z_1 + 5z_2 - 4z_3, \\ x_3 &= -2z_1 - z_2 + z_3 \end{aligned}$$

Thus, we have that

$$A^{-1} = \begin{pmatrix} 17 & 8 & -2 \\ 10 & 5 & -1 \\ -8 & -4 & 1 \end{pmatrix}$$

Now, recall the solution to question #3 in Problem Set 1. We determined there that the residuals one obtains from a regression will not be changed if the regressors are linearly transformed by an invertible matrix. Thus, the two regressions (on x_i and on z_j) produce the same residuals, and hence the same predicted values.

Finally, since the fitted values are the same for the two regressions, $X\hat{\beta} = Z\hat{\alpha} = XA\hat{\alpha} \implies \hat{\beta} = A\hat{\alpha}$. Thus, $\hat{\beta}_1 = \hat{\alpha}_1 + 2\hat{\alpha}_3$. It is also the case that $\hat{\alpha} = A^{-1}\hat{\beta} \implies \hat{\alpha}_1 = 17\hat{\beta}_1 + 8\hat{\beta}_2 - 2\hat{\beta}_3$.

3)

```
. use http://fmwww.bc.edu/ec-p/data/greene2008/tbrate

. regress D.r L.pi LD.y LD.r L2D.r

      Source |       SS       df       MS              Number of obs =     185
-------------+------------------------------           F(  4,   180) =    6.99
       Model |  22.1971507     4  5.54928768           Prob > F      =  0.0000
    Residual |  142.934504   180  .794080577           R-squared     =  0.1344
-------------+------------------------------           Adj R-squared =  0.1152
       Total |  165.131655   184  .897454645           Root MSE      =  .89111

------------------------------------------------------------------------------
         D.r |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          pi |
         L1. |   .0160647   .0200335     0.80   0.424     -.023466    .0555955
           y |
         LD. |   18.38055   5.758924     3.19   0.002     7.016859    29.74423
           r |
         LD. |   .2374557   .0740703     3.21   0.002     .0912979    .3836135
        L2D. |  -.1540175   .0725383    -2.12   0.035    -.2971523   -.0108828
       _cons |  -.2319403   .1256143    -1.85   0.066    -.4798063    .0159256
------------------------------------------------------------------------------

. predict rhat
(option xb assumed; fitted values)
(3 missing values generated)

. predict uhat, residuals
(3 missing values generated)

. twoway (connected rhat yq, msize(vsmall)) (line uhat yq)
```

```
. reg uhat rhat

      Source |       SS       df       MS              Number of obs =     185
-------------+------------------------------           F(  1,   183) =    0.00
       Model |  5.6843e-14     1  5.6843e-14           Prob > F      =  1.0000
    Residual |  142.934505   183  .781062871           R-squared     =  0.0000
-------------+------------------------------           Adj R-squared = -0.0055
       Total |  142.934505   184  .776817964           Root MSE      =  .88378

------------------------------------------------------------------------------
        uhat |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        rhat |   7.53e-09   .1875834     0.00   1.000    -.3701043    .3701043
       _cons |   1.93e-10   .0650252     0.00   1.000    -.1282955    .1282955
------------------------------------------------------------------------------
```

Since the residuals are by construction orthogonal to the fitted values, we verify via the above OLS regression that the mean of the residuals is zero and that the fitted values are uncorrelated with the residuals.

```
. reg rhat uhat

      Source |       SS       df       MS              Number of obs =     185
-------------+------------------------------           F(  1,   183) =    0.00
       Model |           0     1           0           Prob > F      =  1.0000
    Residual |  22.1971507   183  .121295905           R-squared     =  0.0000
-------------+------------------------------           Adj R-squared = -0.0055
       Total |  22.1971507   184  .120636688           Root MSE      =  .34828

------------------------------------------------------------------------------
        rhat |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        uhat |   1.17e-09   .0291309     0.00   1.000    -.0574757    .0574757
       _cons |   .0133946   .0256057     0.52   0.602    -.0371258     .063915
------------------------------------------------------------------------------
```

As explained earlier, the residuals and the fitted values are uncorrelated, which is borne out by the above OLS regression results. Additionally, we obtain a non-zero constant that is the mean of the fitted values, which is also the mean of the dependent variable in the original regression, since the residuals are constructed to be mean-zero.

4)

```
. regress D.r LD.y LD.r L2D.r

      Source |       SS       df       MS              Number of obs =     185
```

```
-------------+------------------------------           F(  3,   181) =    9.12
       Model |  21.6865324     3  7.22884414           Prob > F      =  0.0000
    Residual |  143.445122   181  .792514488           R-squared     =  0.1313
-------------+------------------------------           Adj R-squared =  0.1169
       Total |  165.131655   184  .897454645           Root MSE      =  .89023

------------------------------------------------------------------------------
         D.r |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         LD. |   17.47536   5.641639     3.10   0.002     6.343518     28.6072
           r |
         LD. |   .2437294   .0735833     3.31   0.001     .0985381    .3889208
        L2D. |  -.1471644    .071962    -2.05   0.042    -.2891568   -.0051721
       _cons |   -.158009   .0852318    -1.85   0.065    -.3261847    .0101668
------------------------------------------------------------------------------

. predict ehat, residuals
(3 missing values generated)

. regress L.pi LD.y LD.r L2D.r

      Source |       SS       df       MS              Number of obs =     185
-------------+------------------------------           F(  3,   181) =    3.89
       Model |  127.410931     3  42.4703102           Prob > F      =  0.0101
    Residual |  1978.56248   181  10.9312844           R-squared     =  0.0605
-------------+------------------------------           Adj R-squared =  0.0449
       Total |  2105.97341   184  11.4455077           Root MSE      =  3.3062

------------------------------------------------------------------------------
        L.pi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         LD. |  -56.34641   20.95257    -2.69   0.008    -97.68912   -15.00371
           r |
         LD. |   .3905278   .2732821     1.43   0.155    -.1487006    .9297562
        L2D. |   .4265952   .2672608     1.60   0.112    -.1007524    .9539427
       _cons |   4.602094   .3165437    14.54   0.000     3.977503    5.226684
------------------------------------------------------------------------------

. predict vhat, residuals
(3 missing values generated)

. regress ehat vhat

      Source |       SS       df       MS              Number of obs =     185
```

```
-------------+------------------------------           F(  1,   183) =    0.65
       Model |  .510618184     1  .510618184           Prob > F      =  0.4198
    Residual |  142.934503   183  .781062856           R-squared     =  0.0036
-------------+------------------------------           Adj R-squared = -0.0019
       Total |  143.445121   184  .779593048           Root MSE      =  .88378

------------------------------------------------------------------------------
        ehat |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        vhat |   .0160647   .0198686     0.81   0.420    -.0231363    .0552658
       _cons |   4.07e-11   .0649766     0.00   1.000    -.1281996    .1281996
------------------------------------------------------------------------------
```

The value of the coefficient on vhat is the same as that of π_{t-1} in the original regression in the previous problem, a consequence of the Frisch-Waugh Theorem (also known as the Frisch-Waugh-Lovell Theorem). See Greene 3.3 for details.

5) That the covariance matrix of **b** is positive semidefinite implies that the determinant of this matrix is non-negative. Given that

$$\text{Var}(\mathbf{b}) = \begin{pmatrix} \text{Var}(b_1) & \text{Cov}(b_1, b_2) \\ \text{Cov}(b_1, b_2) & \text{Var}(b_2) \end{pmatrix}$$

the non-negativity of the matrix implies that

$$\text{Var}(b_1)\text{Var}(b_2) - (\text{Cov}(b_1, b_2))^2 \geq 0 \implies (\text{Cov}(b_1, b_2))^2 \leq \text{Var}(b_1)\text{Var}(b_2)$$

Next, $\rho(b_1, b_2)$, the correlation between b_1 and b_2 is defined by

$$\rho(b_1, b_2) \equiv \frac{\text{Cov}(b_1, b_2)}{\sqrt{\text{Var}(b_1)\text{Var}(b_2)}}$$

Then, from the above inequality, it must be that case that $\rho(b_1, b_2)^2 \leq 1 \implies -1 \leq \rho(b_1, b_2) \leq 1$, as required.

6) The restriction implies that $\beta_3 = 1 - \beta_2$. Hence, $y_t = \beta_1 + \beta_2 x_{t2} + (1 - \beta_2)x_{t3} + u_t \implies y_t - x_{t3} = \beta_1 + \beta_2(x_{t2} - x_{t3}) + u_t$. Then, define the new variables $\tilde{y}_t = y_t - x_{t3}$ and $z_t = x_{t2} - x_{t3}$. The restricted regression is $\tilde{y}_t = \beta_1 + \beta_2 z_t + u_t$. The estimate $\hat{\beta}_2$ is obtained directly, and, since the estimate of β_3 must satisfy the restriction, we see that $\hat{\beta}_3 = 1 - \hat{\beta}_2$.

We can add a third term to the right-hand side of restricted regression equation in order to obtain a model that is equivalent to the original one. This yields

$$\hat{y}_t = \beta_1 + \beta_2 z_t + (\beta_2 + \beta_3 - 1)x_{t3} + u_t$$

Thus, if we run the regression

$$\hat{y}_t = \beta_1 + \beta_2 z_t + \gamma x_{t3} + u_t$$

we will obtain estimates for β_1 and β_2 directly, and we can obtain an estimate of β_3 by using the relation $\hat{\beta}_3 = \hat{\gamma} + 1 - \hat{\beta}_2$. If the restriction held exactly in the data, the estimate of γ would be zero.

Note that we could have eliminated β_2 instead of β_3 in the restricted model, and furthermore obtained an appropriately adjusted counterpart to the regression with a zero coefficient when the restriction holds in the data.

7) a)

```
. xi i.year
i.year            _Iyear_66-73        (naturally coded; _Iyear_66 omitted)

. ivreg2 lw expr s (iq = age kww med)

IV (2SLS) estimation
--------------------

                                                      Number of obs =      758
                                                      F(  3,   754) =   105.26
                                                      Prob > F      =   0.0000
Total (centered) SS     =  139.2861498                Centered R2   =   0.2886
Total (uncentered) SS   =  24652.24662                Uncentered R2 =   0.9960
Residual SS             =   99.0915462                Root MSE      =    .3616

------------------------------------------------------------------------------
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          iq |  -.0012932   .0047482    -0.27   0.785    -.0105995    .0080132
        expr |   .0442341   .0065777     6.72   0.000     .0313421     .057126
           s |   .1107632   .0157675     7.02   0.000     .0798595    .1416668
       _cons |   4.259495   .3124346    13.63   0.000     3.647134    4.871855
------------------------------------------------------------------------------
Anderson canon. corr. LR statistic (underidentification test):          43.846
                                                   Chi-sq(3) P-val =    0.0000
------------------------------------------------------------------------------
Cragg-Donald F statistic (weak identification test):                    14.927
Stock-Yogo weak ID test critical values:  5% maximal IV relative bias    13.91
                                         10% maximal IV relative bias     9.08
                                         20% maximal IV relative bias     6.46
                                         30% maximal IV relative bias     5.39
                                         10% maximal IV size             22.30
                                         15% maximal IV size             12.83
                                         20% maximal IV size              9.54
                                         25% maximal IV size              7.80
Source: Stock-Yogo (2005).  Reproduced by permission.
------------------------------------------------------------------------------
Sargan statistic (overidentification test of all instruments):          84.806
                                                   Chi-sq(2) P-val =    0.0000
------------------------------------------------------------------------------
Instrumented:         iq
```

```
Included instruments: expr s
Excluded instruments: age kww med
------------------------------------------------------------------------------
```

The Anderson canonical correlation test rejects at the 5% level the null hypothesis of underidentification. However, the rejection of the null of the Sargan test suggests that one or more of the instruments is not uncorrelated with the disturbance process.

b)

```
. ivreg2 lw expr s _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71 _Iyear_73 (iq = age kww med)

IV (2SLS) estimation
--------------------

                                                      Number of obs =      758
                                                      F(  9,   748) =    47.13
                                                      Prob > F      =   0.0000
Total (centered) SS     =  139.2861498                Centered R2   =   0.3621
Total (uncentered) SS   =  24652.24662                Uncentered R2 =   0.9964
Residual SS             =  88.85241753                Root MSE      =    .3424

------------------------------------------------------------------------------
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          iq |    .007033   .0040735     1.73   0.084    -.0009509    .0150169
        expr |   .0398175   .0067903     5.86   0.000     .0265086    .0531263
           s |   .0565379   .0139059     4.07   0.000     .0292829    .0837929
   _Iyear_67 |  -.0725177   .0497367    -1.46   0.145    -.1699999    .0249644
   _Iyear_68 |   .0504323   .0465702     1.08   0.279    -.0408436    .1417082
   _Iyear_69 |   .1605229    .045594     3.52   0.000     .0711604    .2498854
   _Iyear_70 |   .2097466    .053631     3.91   0.000     .1046318    .3148614
   _Iyear_71 |    .183241   .0456348     4.02   0.000     .0937985    .2726836
   _Iyear_73 |   .2792134   .0420477     6.64   0.000     .1968014    .3616254
       _cons |   4.013944   .2761018    14.54   0.000     3.472795    4.555094
------------------------------------------------------------------------------
Anderson canon. corr. LR statistic (underidentification test):          54.386
                                                   Chi-sq(3) P-val =    0.0000
------------------------------------------------------------------------------
Cragg-Donald F statistic (weak identification test):                    18.497
Stock-Yogo weak ID test critical values:  5% maximal IV relative bias    13.91
                                         10% maximal IV relative bias     9.08
                                         20% maximal IV relative bias     6.46
                                         30% maximal IV relative bias     5.39
                                         10% maximal IV size             22.30
                                         15% maximal IV size             12.83
```

```
                                           20% maximal IV size             9.54
                                           25% maximal IV size             7.80
Source: Stock-Yogo (2005).  Reproduced by permission.
------------------------------------------------------------------------------
Sargan statistic (overidentification test of all instruments):          91.950
                                                    Chi-sq(2) P-val =    0.0000
------------------------------------------------------------------------------
Instrumented:         iq
Included instruments: expr s _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71
                      _Iyear_73
Excluded instruments: age kww med
------------------------------------------------------------------------------
```

The year dummies for years after 1968 are all significant and positive, suggesting some unmodeled change in the underlying process determining the wage that isn't captured by the included characteristics of workers. IQ now has a positive coefficient, but one that is still not statistically significantly different from 0 at the 5% level. The Anderson test and the Sargan test produce similar results as in part a).

c)

```
. ivreg2 lw expr s _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71 _Iyear_73 (iq = age kww med),
 robust

IV (2SLS) estimation
--------------------

Statistics robust to heteroskedasticity

                                                      Number of obs =      758
                                                      F(  9,   748) =    42.35
                                                      Prob > F      =   0.0000
Total (centered) SS     =  139.2861498                Centered R2   =   0.3621
Total (uncentered) SS   =  24652.24662                Uncentered R2 =   0.9964
Residual SS             =  88.85241753                Root MSE      =    .3424

------------------------------------------------------------------------------
             |               Robust
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          iq |    .007033    .004181     1.68   0.093    -.0011616    .0152276
        expr |   .0398175   .0068121     5.85   0.000     .0264659     .053169
           s |   .0565379   .0141939     3.98   0.000     .0287185    .0843574
   _Iyear_67 |  -.0725177   .0474303    -1.53   0.126    -.1654794    .0204439
   _Iyear_68 |   .0504323    .046312     1.09   0.276    -.0403376    .1412021
   _Iyear_69 |   .1605229   .0426472     3.76   0.000     .0769361    .2441098
   _Iyear_70 |   .2097466   .0563248     3.72   0.000      .099352    .3201412
```

```
   _Iyear_71 |    .183241   .0433592     4.23   0.000     .0982585    .2682235
   _Iyear_73 |   .2792134   .0420768     6.64   0.000     .1967443    .3616824
       _cons |   4.013944    .285412    14.06   0.000     3.454547    4.573341
------------------------------------------------------------------------------
Anderson canon. corr. LR statistic (underidentification test):          54.386
                                                   Chi-sq(3) P-val =    0.0000
Test statistic(s) not robust
------------------------------------------------------------------------------
Cragg-Donald F statistic (weak identification test):                    18.497
Stock-Yogo weak ID test critical values:  5% maximal IV relative bias    13.91
                                         10% maximal IV relative bias     9.08
                                         20% maximal IV relative bias     6.46
                                         30% maximal IV relative bias     5.39
                                         10% maximal IV size             22.30
                                         15% maximal IV size             12.83
                                         20% maximal IV size              9.54
                                         25% maximal IV size              7.80
Test statistic(s) not robust
Source: Stock-Yogo (2005).  Reproduced by permission.
------------------------------------------------------------------------------
Hansen J statistic (overidentification test of all instruments):        72.328
                                                   Chi-sq(2) P-val =    0.0000
------------------------------------------------------------------------------
Instrumented:         iq
Included instruments: expr s _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71
                      _Iyear_73
Excluded instruments: age kww med
------------------------------------------------------------------------------
```

Using robust standard errors does not seem to affect the standard errors very much, suggesting that heteroskedasticity is not an issue.

d)

```
. ivreg2 lw expr s _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71 _Iyear_73 (iq = age kww med),
 gmm

2-Step GMM estimation
---------------------

Statistics robust to heteroskedasticity

                                                      Number of obs =      758
                                                      F(  9,   748) =    41.49
                                                      Prob > F      =   0.0000
Total (centered) SS     =  139.2861498                Centered R2   =   0.3562
```

```
Total (uncentered) SS   =  24652.24662                Uncentered R2 =   0.9964
Residual SS             =  89.67457928                Root MSE      =     .344

------------------------------------------------------------------------------
             |               Robust
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          iq |   .0077785    .004178     1.86   0.063    -.0004103    .0159673
        expr |   .0457042   .0067758     6.75   0.000      .032424    .0589845
           s |   .0550396   .0141922     3.88   0.000     .0272233    .0828558
   _Iyear_67 |  -.0602284   .0474059    -1.27   0.204    -.1531422    .0326854
   _Iyear_68 |   .0569231   .0463035     1.23   0.219    -.0338301    .1476763
   _Iyear_69 |   .1601399   .0426317     3.76   0.000     .0765833    .2436965
   _Iyear_70 |   .1794522   .0561917     3.19   0.001     .0693184     .289586
   _Iyear_71 |   .1548847     .04323     3.58   0.000     .0701555    .2396139
   _Iyear_73 |   .2763517   .0420029     6.58   0.000     .1940274     .358676
       _cons |   3.940908    .285036    13.83   0.000     3.382248    4.499568
------------------------------------------------------------------------------
Anderson canon. corr. LR statistic (underidentification test):          54.386
                                                   Chi-sq(3) P-val =    0.0000
Test statistic(s) not robust
------------------------------------------------------------------------------
Cragg-Donald F statistic (weak identification test):                    18.497
Stock-Yogo weak ID test critical values:  5% maximal IV relative bias    13.91
                                         10% maximal IV relative bias     9.08
                                         20% maximal IV relative bias     6.46
                                         30% maximal IV relative bias     5.39
                                         10% maximal IV size             22.30
                                         15% maximal IV size             12.83
                                         20% maximal IV size              9.54
                                         25% maximal IV size              7.80
Test statistic(s) not robust
Source: Stock-Yogo (2005).  Reproduced by permission.
------------------------------------------------------------------------------
Hansen J statistic (overidentification test of all instruments):        72.328
                                                   Chi-sq(2) P-val =    0.0000
------------------------------------------------------------------------------
Instrumented:         iq
Included instruments: expr s _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71
                      _Iyear_73
Excluded instruments: age kww med
------------------------------------------------------------------------------
```

None of the results are markedly different from that obtained in part b). In the GMM model we estimate here, we do not maintain the assumption of conditional homoskedasticity, but rather allow arbitrary heteroskedasticity. The GMM model also delivers efficient estimates. The Hansen J statistic allows a test of overidentification similar to that provided by the Sargan statistic in the 2SLS model; the Hansen J is consistent in the presence of heteroskedasticity. The rejection of the null in this test suggests that one or more of the instruments is not uncorrelated with the disturbance process. The Anderson test as before indicates that the model is not underidentified.

e)

```
. ivreg2 lw expr _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71 _Iyear_73 (s iq = age kww med),
 gmm endog(s)

2-Step GMM estimation
---------------------

Statistics robust to heteroskedasticity

                                                      Number of obs =      758
                                                      F(  9,   748) =    37.83
                                                      Prob > F      =   0.0000
Total (centered) SS     =  139.2861498                Centered R2   =   0.0906
Total (uncentered) SS   =  24652.24662                Uncentered R2 =   0.9949
Residual SS             =  126.6665339                Root MSE      =    .4088

------------------------------------------------------------------------------
             |               Robust
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .1993476   .0254187     7.84   0.000     .1495279    .2491674
          iq |  -.0089693   .0054021    -1.66   0.097    -.0195573    .0016187
        expr |   .0630694   .0081395     7.75   0.000     .0471162    .0790225
   _Iyear_67 |  -.0753593   .0560256    -1.35   0.179    -.1851675    .0344488
   _Iyear_68 |    .012483   .0531677     0.23   0.814    -.0917237    .1166897
   _Iyear_69 |   .0967016    .050023     1.93   0.053    -.0013417    .1947449
   _Iyear_70 |   .1450002   .0670161     2.16   0.030      .013651    .2763494
   _Iyear_71 |   .0198738   .0584071     0.34   0.734     -.094602    .1343495
   _Iyear_73 |  -.0100273   .0670913    -0.15   0.881    -.1415238    .1214693
       _cons |    3.81719   .3332255    11.46   0.000      3.16408      4.4703
------------------------------------------------------------------------------
Anderson canon. corr. LR statistic (underidentification test):          45.115
                                                   Chi-sq(2) P-val =    0.0000
Test statistic(s) not robust
------------------------------------------------------------------------------
Cragg-Donald F statistic (weak identification test):                    15.270
Stock-Yogo weak ID test critical values: 10% maximal IV size             13.43
                                         15% maximal IV size              8.18
                                         20% maximal IV size              6.40
                                         25% maximal IV size              5.45
```

```
Test statistic(s) not robust
Source: Stock-Yogo (2005).  Reproduced by permission.
------------------------------------------------------------------------------
Hansen J statistic (overidentification test of all instruments):         0.482
                                                   Chi-sq(1) P-val =    0.4873
-endog- option:
Endogeneity test of endogenous regressors:                              71.528
                                                   Chi-sq(1) P-val =    0.0000
Regressors tested:    s
------------------------------------------------------------------------------
Instrumented:         s iq
Included instruments: expr _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71
                      _Iyear_73
Excluded instruments: age kww med
------------------------------------------------------------------------------
```

The endogeneity test rejects the null hypothesis of exogeneity of the variable s, years of schooling.

f)

```
. ivreg2 lw expr _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71 _Iyear_73 (s iq = age kww med),
 gmm

2-Step GMM estimation
---------------------

Statistics robust to heteroskedasticity

                                                      Number of obs =      758
                                                      F(  9,   748) =    37.83
                                                      Prob > F      =   0.0000
Total (centered) SS     =  139.2861498                Centered R2   =   0.0906
Total (uncentered) SS   =  24652.24662                Uncentered R2 =   0.9949
Residual SS             =  126.6665339                Root MSE      =    .4088

------------------------------------------------------------------------------
             |               Robust
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .1993476   .0254187     7.84   0.000     .1495279    .2491674
          iq |  -.0089693   .0054021    -1.66   0.097    -.0195573    .0016187
        expr |   .0630694   .0081395     7.75   0.000     .0471162    .0790225
   _Iyear_67 |  -.0753593   .0560256    -1.35   0.179    -.1851675    .0344488
   _Iyear_68 |    .012483   .0531677     0.23   0.814    -.0917237    .1166897
   _Iyear_69 |   .0967016    .050023     1.93   0.053    -.0013417    .1947449
   _Iyear_70 |   .1450002   .0670161     2.16   0.030      .013651    .2763494
```

```
   _Iyear_71 |   .0198738   .0584071     0.34   0.734    -.094602    .1343495
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                                         15% maximal IV size              8.18
                                         20% maximal IV size              6.40
                                         25% maximal IV size              5.45
Test statistic(s) not robust
Source: Stock-Yogo (2005).  Reproduced by permission.
------------------------------------------------------------------------------
Hansen J statistic (overidentification test of all instruments):         0.482
                                                   Chi-sq(1) P-val =    0.4873
------------------------------------------------------------------------------
Instrumented:         s iq
Included instruments: expr _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71
                      _Iyear_73
Excluded instruments: age kww med
------------------------------------------------------------------------------
```

Unlike for the previous regression models, the Hansen J test fails to reject the null hypothesis of instruments uncorrelated with the disturbance process, suggesting, together with the successful rejection of underidentification via the Anderson test, that the instrument set and endogenous variables set used are valid.