BOSTON COLLEGE
Department of Economics

EC771: Econometrics
Spring 2010
Prof. Baum, Mr. Dmitriev

PROBLEM SET 4: DUE TUESDAY 30 MARCHL 2010 AT CLASSTIME

1. In sampling from a Normal distribution $N(\mu, \sigma^2)$, both the mean $\bar{x}_n$ and median M_n are consistent estimators of μ. Their asymptotic properties are

$$\begin{aligned} \bar{x}_n &\sim N[\mu, \sigma^2/n] \\ M_n &\sim N[\mu, (\pi/2)\sigma^2/n] \end{aligned}$$

Perform a Monte Carlo simulation exercise in which you draw samples ($N = 500$) from the Student t distribution for 3, 6, 10 and 100 d.f. Use 1000 replications for each d.f. to estimate the ratio of the asymptotic variance of the median to the asymptotic variance of the mean. Compute mean square errors (MSE) for each d.f. for the ratio of variances.

2. Consider the linear regression model

$$y_i = \alpha + \beta x_i + \gamma z_i + \varepsilon,\ \varepsilon \sim N[0, 1]$$

Does the Wald statistic reject the null too often? Construct a Monte Carlo study on the behavior of the Wald statistic for testing the hypothesis $\gamma = 0$ in this model. Draw a sample of size 50 on (x_i, z_i) from independent $N[0, 1]$ distributions. For each of 1000 replications, draw a sample of 50 values of ε, following the alternative distributions:

- distributed as above
- distributed as Student t_5
- distributed with conditional variance $Var[\varepsilon|(x_i, z_i)] = [\exp(0.2x_i)]^2$

The Wald statistic is the square of the t ratio on the parameter in question. Reject the null hypothesis if the Wald statistic exceeds 3.84, the critical value from $\chi^2(1)$ for a nominal size of 0.05. Evaluate the performance of the Wald statistic for values of $\gamma = (-0.5, 0.2, 0.6, 0.9)$.

(over)

3. Repeat exercise 2 considering the LM statistic

$$LM = \frac{\mathbf{e}_0'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{e}_0}{\mathbf{e}_0'\mathbf{e}_0/n}$$

where $\mathbf{X} = (1, x, z)$ and $\mathbf{e}_0$ is the vector of least squares residuals obtained from the regression of y on $(1, x)$ and not z. That is, z is mistakenly omitted from the model. Under the assumptions of the model, the asymptotic distribution of the LM statistic is χ^2 with one d.f., so the null of $\gamma = 0$ should be rejected if LM exceeds the critical value of 3.84 (for nominal size 0.05). Evaluate the performance of the LM statistic under the three assumptions about the error distribution and four possible values for γ.

Please construct a do-file (within the Stata do-file editor or any text editor) and run the do-file to produce a log file. (`log using ps1,replace` at the top of the do-file and `log close` at the foot will do that. Hand in a printout of the resulting .smcl file (you may View the logfile in the Stata Viewer and print it out from there).