BOSTON COLLEGE
Department of Economics
EC 771: Econometrics
Spring 2012
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PROBLEM SET 1: SOLUTIONS

Point Distribution:
1) to 8): 10 points each
9) a), c), d): 3 points each
9) b), e), f): 2 points each
9) g): 5 points

1) Model: $y = \alpha + \beta x + \epsilon$

a) $\mathbf{y} \equiv \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$, $\mathbf{X} \equiv \begin{pmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix}$, $\mathbf{b} \equiv \begin{pmatrix} a \\ b \end{pmatrix}$

The normal equations for this model are given by

$$(\mathbf{X}'\mathbf{X})\mathbf{b} - \mathbf{X}'\mathbf{y} = 0,$$

which implies that

$$\mathbf{X}'(\mathbf{X}\mathbf{b} - \mathbf{y}) = 0 \implies \mathbf{X}'(-\mathbf{e}) = \mathbf{0}$$

Thus, $\sum_i x_i e_i = 0$. Also, since the first column consists of 1s, $\sum_i e_i = 0$.

b) Since the first normal equation is

$$na + \sum_i x_i b = \sum_i y_i$$

we immediately have that

$$a = \bar{y} - b\bar{x}$$

c) The second normal equation is

$$\sum_i x_i a + \sum_i x_i^2 b = \sum_i x_i y_i.$$

Substituting a from above, we have

$$\bar{y}\sum_i x_i - b\bar{x}\sum_i x_i + b\sum_i x_i^2 = \sum_i x_i y_i \implies b = \frac{\sum_i x_i y_i - \bar{y}\sum_i x_i}{\sum_i x_i^2 - \bar{x}\sum_i x_i}$$

Then,

$$b = \frac{\sum_i x_i y_i - n\bar{x}\bar{y}}{\sum_i x_i^2 - n\bar{x}^2} = \frac{\sum_i x_i y_i - n\bar{x}\bar{y} - n\bar{x}\bar{y} + n\bar{x}\bar{y}}{\sum_i x_i^2 - 2n\bar{x}^2 + n\bar{x}^2} = \frac{\sum_i x_i y_i - \bar{x}\sum_i y_i - \bar{y}\sum_i x_i + n\bar{x}\bar{y}}{\sum_i x_i^2 - 2\bar{x}\sum_i x_i + n\bar{x}^2}$$
$$= \frac{\sum_i (x_i y_i - \bar{x} y_i - \bar{y} x_i + \bar{x}\bar{y})}{\sum_i (x_i^2 - 2\bar{x} x_i + \bar{x}^2)} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

d) $S(\mathbf{b}) = \mathbf{y}'\mathbf{y} - 2\mathbf{y}'\mathbf{X}\mathbf{b} + \mathbf{b}'\mathbf{X}'\mathbf{X}\mathbf{b}$ and so

$$\frac{\partial^2 S(\mathbf{b})}{\partial \mathbf{b}\mathbf{b}'} = 2\mathbf{X}'\mathbf{X} = 2\begin{pmatrix} n & \sum_i x_i \\ \sum_i x_i & \sum_i x_i^2 \end{pmatrix}$$

Now, $n > 0$ and $\sum_i x_i^2 > 0$, since the full rank condition requires that $x_i \neq x_j \forall i \neq j$. Then,

$$\begin{aligned}
\left|\frac{\partial^2 S(\mathbf{b})}{\partial \mathbf{b}\mathbf{b}'}\right| &= 4n\sum_i x_i^2 - 4\left(\sum_i x_i\right)^2 \\
&= 4n\left(\sum_i x_i^2 - n\bar{x}^2\right) \\
&= 4n\left[\sum_i (x_i^2 - 2\bar{x}x_i + \bar{x}^2) + 2\sum_i \bar{x}x_i - n\bar{x}^2 - n\bar{x}^2\right] \\
&= 4n\left[\sum_i (x_i - \bar{x})^2\right]
\end{aligned}$$

2) Prove: $(\mathbf{y} - \mathbf{X}\mathbf{c})'(\mathbf{y} - \mathbf{X}\mathbf{c}) - (\mathbf{y} - \mathbf{X}\mathbf{b})'(\mathbf{y} - \mathbf{X}\mathbf{b}) = (\mathbf{c} - \mathbf{b})'\mathbf{X}'\mathbf{X}(\mathbf{c} - \mathbf{b})$
Proof:

$$\begin{aligned}
(\mathbf{y} - \mathbf{X}\mathbf{c})'(\mathbf{y} - \mathbf{X}\mathbf{c}) - (\mathbf{y} - \mathbf{X}\mathbf{b})'(\mathbf{y} - \mathbf{X}\mathbf{b}) &= \mathbf{y}'\mathbf{y} - \mathbf{y}'\mathbf{X}\mathbf{c} - \mathbf{c}'\mathbf{X}\mathbf{y} + \mathbf{c}'\mathbf{X}'\mathbf{X}\mathbf{c} - \mathbf{y}'\mathbf{y} + \mathbf{y}'\mathbf{X}\mathbf{b} + \mathbf{b}'\mathbf{X}'\mathbf{y} - \mathbf{b}'\mathbf{X}'\mathbf{X}\mathbf{b} \\
&= \mathbf{b}'\mathbf{X}'\mathbf{X}(\mathbf{b} - \mathbf{c}) + (\mathbf{b}' - \mathbf{c}')\mathbf{X}'\mathbf{X}\mathbf{b} + \mathbf{c}'\mathbf{X}'\mathbf{X}\mathbf{c} - \mathbf{b}'\mathbf{X}'\mathbf{X}\mathbf{b} \\
&= -\mathbf{b}'\mathbf{X}'\mathbf{X}\mathbf{c} + \mathbf{b}'\mathbf{X}'\mathbf{X}\mathbf{b} - \mathbf{c}'\mathbf{X}'\mathbf{X}\mathbf{b} + \mathbf{c}'\mathbf{X}'\mathbf{X}\mathbf{c} \\
&= (\mathbf{c}' - \mathbf{b}')\mathbf{X}'\mathbf{X}\mathbf{c} - (\mathbf{c}' - \mathbf{b}')\mathbf{X}'\mathbf{X}\mathbf{b} \\
&= (\mathbf{c} - \mathbf{b})'\mathbf{X}'\mathbf{X}(\mathbf{c} - \mathbf{b})
\end{aligned}$$

where we use $\mathbf{X}'\mathbf{y} = \mathbf{X}'\mathbf{X}\mathbf{b}$ in the third line.

3) Proof: Let $M_Z = (I - Z(Z'Z)^{-1}Z') \implies M_Z y = e_Z$. Similarly, define M_X, so that $M_X y = e_X$. Now, notice that $M_Z = I - Z(Z'Z)^{-1}Z' = I - XP(P'X'XP)^{-1}P'X'$. But, since P, P', and $(X'X)$ are invertible, $(P'X'XP)^{-1} = P^{-1}(X'X)^{-1}(P')^{-1}$. Then, $M_Z = I - XPP^{-1}(X'X)^{-1}(P')^{-1}P'X' = I - X(X'X)^{-1}X' = M_X$. Thus, $e_Z = M_Z y = M_X y = e_X$.

Thus, we can conclude that changing the units of measurement of the independent variables (i.e. postmultiplying by a diagonal P matrix) has no effect on the fit.

4) Proof: $\left(\begin{pmatrix}\mathbf{x_n}\\ \mathbf{x_s}\end{pmatrix}'\begin{pmatrix}\mathbf{x_n}\\ \mathbf{x_s}\end{pmatrix}\right)^{-1}\begin{pmatrix}\mathbf{x_n}\\ x_s\end{pmatrix}'\begin{pmatrix}\mathbf{y_n}\\ y_s\end{pmatrix} = (\mathbf{x_n}'\mathbf{x_n} + \mathbf{x_s}\mathbf{x_s}')^{-1}\begin{pmatrix}\mathbf{x_n}\\ x_s\end{pmatrix}'\begin{pmatrix}\mathbf{y_n}\\ y_s\end{pmatrix} =$

$$= \left[(\mathbf{x_n}'\mathbf{x_n})^{-1} - \frac{1}{1+\mathbf{x_s}'(\mathbf{x_n}'\mathbf{x_n})^{-1}\mathbf{x_s}}(\mathbf{x_n}'\mathbf{x_n})^{-1}\mathbf{x_s}\mathbf{x_s}'(\mathbf{x_n}'\mathbf{x_n})^{-1}\right](\mathbf{x_n}'\mathbf{y_n} + \mathbf{x_s}y_s)$$

$$= \mathbf{b_n} + (\mathbf{x_n}'\mathbf{x_n})^{-1}\mathbf{x_s}y_s - \frac{1}{1+\mathbf{x_s}'(\mathbf{x_n}'\mathbf{x_n})^{-1}\mathbf{x_s}}(\mathbf{x_n}'\mathbf{x_n})^{-1}\mathbf{x_s}\mathbf{x_s}'(\mathbf{x_n}'\mathbf{x_n})^{-1}(\mathbf{x_n}'\mathbf{y_n})$$

$$-\frac{1}{1+\mathbf{x_s}'(\mathbf{x_n}'\mathbf{x_n})^{-1}\mathbf{x_s}}(\mathbf{x_n}'\mathbf{x_n})^{-1}\mathbf{x_s}\mathbf{x_s}'(\mathbf{x_n}'\mathbf{x_n})^{-1}(\mathbf{x_s}y_s)$$

$$= \mathbf{b_n} + \frac{1}{1+\mathbf{x_s}'(\mathbf{x_n}'\mathbf{x_n})^{-1}\mathbf{x_s}}(\mathbf{x_n}'\mathbf{x_n})^{-1}\mathbf{x_s}(y_s - \mathbf{x_s}'\mathbf{b_n})$$

5) Let $x_i = \begin{pmatrix}1 & Y_i & P_{d,i} & P_{n,i} & P_{s,i}\end{pmatrix} \implies X = \begin{pmatrix}1 & Y & P_d & P_n & P_s\end{pmatrix}$. Then, $E_j = Xb_j + e_j$, where $j \in \{d, n, s\}$, so $b_j = (X'X)^{-1}X'E_j$. Now, $Y = E_d + E_n + E_s$. Therefore, $\sum_j b_j = \sum_j (X'X)^{-1}X'E_j = (X'X)^{-1}X'\sum_j E_j = (X'X)^{-1}X'Y$. But since Y is a column in X, we will have an exact fit if we ran a regression of Y on X i.e. $b_Y = (X'X)^{-1}X'Y = \begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$, where the coefficient for the regressor Y is 1 and all the others are 0. Then, since $b_Y = \sum_j b_j$, the sum of the expenditure coefficients is 1 and all other coefficients sum to 0.

6) We have $E[N] = E[D] = E[Y] = 0$ and $var(N) = var(D) = var(Y) = 1$. Also, $var(C) = var(N + D) = var(N) + var(D) + 2cov(N, D) = 2(1 + cov(N, D))$. In the regression of D on Y, the slope is 0.4 which implies $cov(D, Y)/var(Y) = cov(D, Y) = 0.4$. In the regression of C on Y, the slope is 0.8 which implies $cov(C, Y)/var(Y) = cov(C, Y) = 0.8$. Note that $cov(C, Y) = cov(N + D, Y) = cov(N, Y) + cov(D, Y) = cov(N, Y) + 0.4 = 0.8 \Rightarrow cov(N, Y) = 0.4$. In the regression of C on N the slope is 0.5 which implies that $cov(C, N)/var(N) = cov(C, N) = 0.5$. Note that $cov(C, N) = cov(N + D, N) = var(N) + cov(N, D) = 1 + cov(N, D) = 0.5 \Rightarrow cov(N, D) = -0.5$. We can also compute $cov(C, D) = cov(N + D, D) = cov(N, D) + var(D) = -0.5 + 1 = 0.5$ as well as $var(C) = 2(1 + cov(N, D)) = 2(1 - 0.5) = 1$. Now, in the regression of C on D, the sum

of squared residuals is given by:

$$\sum_i e_i^2 = \sum_i (C_i - \bar{C})^2 - b^2 \sum_i (D_i - \bar{D})^2$$

We can rewrite the above expression (using the fact that all moments are computed using $1/(n-1)$ as the divisor) as:

$$= (n-1)\left(var(C) - (cov(C,D)/var(D))^2\, var(D)\right)$$

$$= 20(1 - (0.5)^2) = 20(0.75) = 15$$

7) For the estimator to be unbiased, it must be that $c_1 + c_2 = 1$, since $\mathrm{E}[\hat{\theta}] = c_1\mathrm{E}[\hat{\theta_1}] + c_2\mathrm{E}[\hat{\theta_2}] = (c_1 + c_2)\theta$, where θ is the true parameter value.

Thus, we need to minimize the variance of $c_1\hat{\theta_1} + (1-c_1)\hat{\theta_2}$. Now, $v \equiv var[\hat{\theta}] = var[c_1\hat{\theta_1} + (1-c_1)\hat{\theta_2}] = c_1^2 v_1 + (1-c_1)^2 v_2 + c_1(1-c_1)cov(\hat{\theta_1}, \hat{\theta_2})$, where $v_i = var[\hat{\theta_i}]$. Since, $\hat{\theta_1}$ and $\hat{\theta_2}$ are independent, the covariance term is equivalent to 0. Thus, $v = c_1^2 v_1 + (1-c_1)^2 v_2$. Then, $\frac{\partial v}{\partial c_1} = 2c_1 v_1 - 2(1-c_1)v_2 = 0$, which implies that $c_1 = \frac{v_2}{v_1+v_2}$ and $c_2 = \frac{v_1}{v_1+v_2}$.

8) Let $q = \mathrm{E}[Q|P]$. Then, the expected profit $\Pi = Pq - Cq = P(a+bP) - C(a+bP)$, where C is the constant marginal cost. Profit is maximized when $\frac{\partial \Pi}{\partial p} = 0$ i.e. $a + 2bP - bC = 0$. Thus, $P^* = \frac{C}{2} - \frac{a}{2b}$. Given that $C = 10$, we have $P^* = 5 - \frac{a}{2b}$ and so the optimal quantity is given by $\frac{a}{2} + 5b$.

```
. regress Q P

      Source |       SS       df       MS              Number of obs =      15
-------------+------------------------------           F(  1,    13) =   12.52
       Model |  197.088735     1  197.088735           Prob > F      =  0.0036
    Residual |  204.644598    13  15.7418922           R-squared     =  0.4906
-------------+------------------------------           Adj R-squared =  0.4514
       Total |  401.733333    14  28.6952381           Root MSE      =  3.9676

------------------------------------------------------------------------------
           Q |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           P |  -.8405832   .2375627    -3.54   0.004    -1.353806   -.3273602
       _cons |   20.76912   2.821568     7.36   0.000     14.67349    26.86475
------------------------------------------------------------------------------

. lincom _cons / 2 + 5 * P
```

```
 ( 1)  5 P + .5 _cons = 0

------------------------------------------------------------------------------
           Q |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   6.181644   .5276531    11.72   0.000     5.041719     7.32157
------------------------------------------------------------------------------
```

Thus, the expected value of the profit-maximizing output is 6.18, with the 95% confidence interval [5.042, 7.322].

9) a)

```
. tsset Year,yearly
       time variable:  Year, 1953 to 2004

. // per capita gas consump, income
. gen gaspc = GasExp/(Gasp*(Pop/1e6))

. // logs
. gen lngaspc = log(gaspc)

. local allreg  Income Gasp PNC PUC PPT PD PN PS

. // reg of part a
. reg gaspc ‘allreg’ Year

      Source |       SS       df       MS              Number of obs =      52
-------------+------------------------------           F(  9,    42) =  530.82
       Model |  56.7083042     9  6.30092268           Prob > F      =  0.0000
    Residual |   .49854905    42  .011870215           R-squared     =  0.9913
-------------+------------------------------           Adj R-squared =  0.9894
       Total |  57.2068532    51    1.121703           Root MSE      =  .10895

------------------------------------------------------------------------------
       gaspc |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      Income |   .0002157   .0000518     4.17   0.000     .0001113    .0003202
        Gasp |  -.0110838   .0039781    -2.79   0.008     -.019112   -.0030557
         PNC |   .0005774   .0128441     0.04   0.964    -.0253432    .0264979
```

```
         PUC |  -.0058746   .0048703    -1.21   0.234    -.0157033    .0039541
         PPT |   .0069073   .0048361     1.43   0.161    -.0028524     .016667
          PD |   .0012289   .0118818     0.10   0.918    -.0227495    .0252072
          PN |   .0126905    .012598     1.01   0.320    -.0127333    .0381142
          PS |  -.0280278   .0079962    -3.51   0.001    -.0441649   -.0118907
        Year |   .0725037   .0141828     5.11   0.000     .0438816    .1011257
       _cons |  -140.4213   27.19985    -5.16   0.000    -195.3128    -85.5298
------------------------------------------------------------------------------
```

One would expect the coefficient of the price of gasoline (Gasp) to be negatively correlated, since demand should be downward sloping, which it is. For income (Income) one would expect a positive coefficient because of the income effect; the regression produces this expected result. One might expect that the coefficient of the price of new cars (PNC) to be negative, since cars and gasoline are complements, but the regressions suggests otherwise (note however that the coefficient isn't significantly different from 0). It is possible that better fuel efficieny of newer cars more than offset the increased price of the newer cars. The coefficient of the price of public transportation (PPT) is sensible, since public transportation and gasoline are substitutes. Cars are durables, so (PD) poses the same puzzle as (PNC); the same explanation above for this puzzle might apply.

b) Notice that the 95% confidence interval for PUC is a subset of the 95% confidence interval of PNC. Thus, the null hypothesis that the true parameter value of the coefficients of PUC and PNC are the same cannot be rejected.

```
. test PNC = PUC

 ( 1)  PNC - PUC = 0

       F(  1,    42) =    0.24
            Prob > F =    0.6233
```

c)

```
. est store a

. // elasticities: compute at t=2004
. mean ‘allreg’ Year if Year==2004

Mean estimation                     Number of obs    =       1
```

```
-------------------------------------------------------------
             |       Mean   Std. Err.     [95% Conf. Interval]
-------------+-----------------------------------------------
      Income |      27113          0             .           .
        Gasp |    123.901          0             .           .
         PNC |      133.9          0             .           .
         PUC |      133.3          0             .           .
         PPT |      209.1          0             .           .
          PD |      114.8          0             .           .
          PN |      172.2          0             .           .
          PS |      222.8          0             .           .
        Year |       2004          0             .           .
-------------------------------------------------------------

. mat x2004 = e(b)

. est restore a
(results a are active now)

. mfx compute, eyex at(x2004)

Elasticities after regress
      y  = Fitted values (predict)
         =  6.1726971
------------------------------------------------------------------------------
variable |      ey/ex    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
  Income |   .9476599       .2263    4.19   0.000   .504127  1.39119     27113
    Gasp |  -.2224796      .08093   -2.75   0.006  -.381102 -.063857   123.901
     PNC |   .0125245       .2786    0.04   0.964  -.533521   .55857     133.9
     PUC |  -.1268632      .10488   -1.21   0.226  -.332432  .078706     133.3
     PPT |   .2339837      .16441    1.42   0.155   -.08826  .556228     209.1
      PD |   .0228545      .22098    0.10   0.918  -.410256  .455965     114.8
      PN |   .3540265      .35281    1.00   0.316  -.337474  1.04553     172.2
      PS |  -1.011648      .29332   -3.45   0.001  -1.58654 -.436759     222.8
    Year |   23.53872     4.63929    5.07   0.000   14.4459  32.6316      2004
------------------------------------------------------------------------------
```

The own price elasticity is -0.2225 (and significantly different from 0), the income elasticity is 0.9477 (and significantly different from 0) and the cross- price elasticity with PPT is 0.2340 (but not significantly different from 0).

d)

```
. foreach v of local allreg {
  2.         gen ln`v' = log(`v')
  3.         local logreg "`logreg' ln`v'"
  4. }

. reg  lngaspc `logreg'

      Source |       SS       df       MS              Number of obs =      52
-------------+------------------------------           F(  8,    43) =  249.60
       Model |  2.84726323     8  .355907904           Prob > F      =  0.0000
    Residual |  .061313662    43  .001425899           R-squared     =  0.9789
-------------+------------------------------           Adj R-squared =  0.9750
       Total |   2.9085769    51   .05703092           Root MSE      =  .03776

------------------------------------------------------------------------------
     lngaspc |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lnIncome |   1.883045    .223034     8.44   0.000     1.433254    2.332836
      lnGasp |   .0735984   .0676117     1.09   0.282    -.0627536    .2099504
       lnPNC |   .3772717     .30747     1.23   0.226    -.2428007     .997344
       lnPUC |   -.334021   .0996132    -3.35   0.002    -.5349102   -.1331318
       lnPPT |   .1404593   .1683464     0.83   0.409    -.1990435    .4799621
        lnPD |   .6422717   .1817908     3.53   0.001     .2756555    1.008888
        lnPN |   -.492239   .3269502    -1.51   0.139    -1.151597     .167119
        lnPS |  -.6288652   .4383016    -1.43   0.159    -1.512785    .2550542
       _cons |  -15.79148    2.35185    -6.71   0.000    -20.53443   -11.04852
------------------------------------------------------------------------------
```

The own price elasticity is 0.07360, the income elasticity is 1.883, and the cross- price elasticity with PPT is 0.1405.

The own price elasticity is quite different from part c) and positive, implying an upward sloping demand curve. However, the parameter estimate is not statistically significantly different from 0. The income elasticity estimate is almost twice as high as in part c). It is likely that the elimination of the time-trend from the log-log regression has resulted in the income growth rate "bleeding" into the estimate for the effect of income. The cross-price elasticity with the price of public transportation (PPT), while somewhat similar in value from the linear regression above, is also not significantly different from 0.

The log model tries to fit a constant elasticity function to the data, whereas the previous calculation of the elasticities was carried out at the mean point of the graph assuming a linear structural equation. If the elasticity varies with the dependent variable, then one should not expect that the two models produce the same elasticities.

It isn't clear which specification is appropriate.

e)

```
. corr 'logreg' Year
(obs=52)

             | lnIncome   lnGasp    lnPNC    lnPUC    lnPPT     lnPD     lnPN     lnPS     Year
-------------+---------------------------------------------------------------------------------
    lnIncome |   1.0000
      lnGasp |   0.9448   1.0000
       lnPNC |   0.9473   0.9667   1.0000
       lnPUC |   0.9599   0.9674   0.9940   1.0000
       lnPPT |   0.9790   0.9665   0.9891   0.9910   1.0000
        lnPD |   0.9536   0.9776   0.9932   0.9945   0.9864   1.0000
        lnPN |   0.9754   0.9839   0.9900   0.9902   0.9942   0.9923   1.0000
        lnPS |   0.9809   0.9742   0.9902   0.9912   0.9985   0.9886   0.9979   1.0000
        Year |   0.9923   0.9471   0.9631   0.9683   0.9878   0.9571   0.9809   0.9885   1.0000
```

It appears that there is a large degree of positive correlation among all the variables. One cannot however conclude that we have a multicollinearity problem, not without further investigation. That the log-log regression produces a positive own-price elasticity is particularly concerning. One can estimate the Variance Inflation Factor (VIF) by regressing the suspect variable (lnGasp) on the other regressors used in the original log-log regression.

```
. regress lnGasp lnIncome lnPNC lnPUC lnPPT lnPD lnPN lnPS Year

      Source |       SS       df       MS              Number of obs =      52
-------------+------------------------------           F(  8,    43) =  400.72
       Model |  23.2012964     8  2.90016205           Prob > F      =  0.0000
    Residual |  .311204941    43  .007237324           R-squared     =  0.9868
-------------+------------------------------           Adj R-squared =  0.9843
       Total |  23.5125013    51  .461029438           Root MSE      =  .08507

------------------------------------------------------------------------------
      lnGasp |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lnIncome |    -1.7125   .6569362    -2.61   0.013    -3.037339   -.3876623
```

```
       lnPNC |   -2.27155   .6694628    -3.39   0.001    -3.621651     -.92145
       lnPUC |   .1843208   .2389148     0.77   0.445    -.2974968    .6661385
       lnPPT |   -.368804   .3811388    -0.97   0.339    -1.137444    .3998356
        lnPD |   .5337328   .7292676     0.73   0.468    -.9369755    2.004441
        lnPN |   2.218312   .6676095     3.32   0.002     .8719495    3.564675
        lnPS |   .7517911   .9898355     0.76   0.452    -1.244402    2.747985
        Year |   .0066669   .0211907     0.31   0.755    -.0360683    .0494021
       _cons |   3.023167   37.03858     0.08   0.935    -71.67225    77.71858
------------------------------------------------------------------------------
```

The R^2 for the regression is very close to 1, implying a very high VIF. However, there is no critical VIF that allows one to classify a regression as suffering from multicollinearity or not.

The easy to use command vif does this for you for all the regressors.

```
. vif

    Variable |       VIF       1/VIF
-------------+----------------------
        lnPS |   4902.30    0.000204
        lnPN |   1566.09    0.000639
       lnPPT |    790.87    0.001264
       lnPNC |    645.15    0.001550
        lnPD |    305.77    0.003270
    lnIncome |    216.20    0.004625
       lnPUC |    192.91    0.005184
      lnGasp |     75.38    0.013266
-------------+----------------------
    Mean VIF |   1086.83
```

The high VIFs strongly suggest that multicollinearity "problems" might exist, which is to say that the estimates are highly sensitive to particular data points. See Greene's discussion for more on this topic.

f) As figured out in problem 3 of this Problem Set, the units of measurement do not affect the fit of the regression, but only the value of the relevant coefficients, which are scaled by the conversion factor between the two units.

$$b_X \equiv (X'X)^{-1}X'y$$

$$b_Z \equiv (Z'Z)^{-1}Z'y = (P'X'XP)^{-1}P'X'y = P^{-1}(X'X)^{-1}(P')^{-1}P'X'y = P^{-1}(X'X)^{-1}X'y = P^{-1}b_X$$

However, the log model will have the same coefficients for the regressors regardless of the unit of measurement; only the constant term will be altered by such a change of units, but not the fit. This follows from the simple algebraic fact that $\ln(sx) = \ln(s) + \ln(x)$, where s is some scaling factor (a scalar). Thus, the change in the units will change the constant term in the log-log regression.

g)

```
. gen break = tin(1974,2004)

. ttest lngaspc, by(break)

Two-sample t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       0 |      21    1.334769      .04365    .2000295    1.243717    1.425822
       1 |      31    1.730146     .012755    .0710169    1.704097    1.756195
---------+--------------------------------------------------------------------
combined |      52    1.570475    .0331172    .2388115    1.503989     1.63696
---------+--------------------------------------------------------------------
    diff |           -.3953765    .0389887               -.4736877   -.3170653
------------------------------------------------------------------------------
    diff = mean(0) - mean(1)                                      t = -10.1408
Ho: diff = 0                                     degrees of freedom =       50

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 1.0000
```

The average value of log per capita gas consumption for period 1 is 1.3348 and for period 2 is 1.7301, with a statistically significant increase in the value from period 1 to period 2 of 0.3954.

```
. // regs for each subset
. gen iota = 1

. reg lngaspc ‘logreg’ Year if ~break

      Source |       SS       df       MS              Number of obs =      21
-------------+------------------------------           F(  9,    11) =  584.71
       Model |  .798567151     9  .088729683           Prob > F      =  0.0000
    Residual |  .001669259    11  .000151751           R-squared     =  0.9979
```

```
-------------+------------------------------           Adj R-squared =  0.9962
       Total |  .800236411    20  .040011821           Root MSE      =  .01232

------------------------------------------------------------------------------
     lngaspc |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lnIncome |   .6648792   .2234255     2.98   0.013     .1731229    1.156636
      lnGasp |   -.202044   .4191071    -0.48   0.639    -1.124492    .7204044
       lnPNC |   .5912882   .3024403     1.96   0.076    -.0743785    1.256955
       lnPUC |   -.294078   .1420679    -2.07   0.063    -.6067674    .0186114
       lnPPT |  -.3584459   .3104284    -1.15   0.273    -1.041694    .3248024
        lnPD |  -.1022751   1.072572    -0.10   0.926    -2.462991    2.258441
        lnPN |  -.0383662   .5780072    -0.07   0.948    -1.310551    1.233819
        lnPS |   .7541618   .7414295     1.02   0.331    -.8777136    2.386037
        Year |   .0090829   .0184495     0.49   0.632    -.0315243    .0496901
       _cons |  -24.22851   35.60422    -0.68   0.510    -102.5929    54.13584
------------------------------------------------------------------------------

. mat bpre = e(b)'

. mat vpre = e(V)

. qui predict ypre if e(sample)

. est store pre

. mean ypre

Mean estimation                     Number of obs    =      21

--------------------------------------------------------------
             |       Mean   Std. Err.     [95% Conf. Interval]
-------------+------------------------------------------------
        ypre |   1.334769   .0436045      1.243812    1.425727
--------------------------------------------------------------

. mean `logreg' Year iota if e(sample)

Mean estimation                     Number of obs    =      21

--------------------------------------------------------------
```

```
             |       Mean   Std. Err.     [95% Conf. Interval]
-------------+------------------------------------------------
    lnIncome |   9.307999   .0382598       9.22819    9.387807
      lnGasp |   2.973497   .0218159       2.92799    3.019005
       lnPNC |   3.919241   .0123054      3.893572    3.944909
       lnPUC |   3.319498   .0322032      3.252324    3.386673
       lnPPT |   3.220735   .0564034       3.10308    3.338391
        lnPD |   3.682407   .0184103      3.644004     3.72081
        lnPN |   3.539391   .0300972      3.476609    3.602173
        lnPS |   3.276916   .0479733      3.176846    3.376987
        Year |       1963   1.354006      1960.176    1965.824
        iota |          1          0             .           .
--------------------------------------------------------------

. mat xpre = e(b)

. reg  lngaspc ‘logreg’ Year if break

      Source |       SS       df       MS              Number of obs =      31
-------------+------------------------------           F(  9,    21) =  104.38
       Model |  .147993657     9  .01644374            Prob > F      =  0.0000
    Residual |   .00330819    21  .000157533           R-squared     =  0.9781
-------------+------------------------------           Adj R-squared =  0.9688
       Total |  .151301846    30  .005043395           Root MSE      =  .01255

------------------------------------------------------------------------------
     lngaspc |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lnIncome |   .5181589   .1498427     3.46   0.002     .2065439    .8297739
      lnGasp |  -.0770111   .0501662    -1.54   0.140    -.1813374    .0273152
       lnPNC |   .6158313   .2687583     2.29   0.032     .0569178    1.174745
       lnPUC |   .2402007   .0938617     2.56   0.018     .0450045    .4353969
       lnPPT |  -.1616701   .0748211    -2.16   0.042    -.3172691   -.0060711
        lnPD |  -.6564543   .3175261    -2.07   0.051    -1.316786    .0038775
        lnPN |   .2370631   .2603476     0.91   0.373    -.3043593    .7784855
        lnPS |  -.2148074   .1814829    -1.18   0.250    -.5922217    .1626069
        Year |   .0007693   .0052182     0.15   0.884    -.0100827    .0116212
       _cons |  -4.904748   9.733856    -0.50   0.620    -25.14741    15.33791
------------------------------------------------------------------------------

. mat bpost = e(b)’
```

```
. mat vpost = e(V)

. qui predict ypost if e(sample)

. est store post

. mean ypost

Mean estimation                    Number of obs    =      31

--------------------------------------------------------------
             |       Mean   Std. Err.     [95% Conf. Interval]
-------------+------------------------------------------------
       ypost |   1.730146   .0126148      1.704383    1.755909
--------------------------------------------------------------

. mean `logreg' Year iota if e(sample)

Mean estimation                    Number of obs    =      31

--------------------------------------------------------------
             |       Mean   Std. Err.     [95% Conf. Interval]
-------------+------------------------------------------------
    lnIncome |   9.918829    .031305      9.854896    9.982762
      lnGasp |     4.2413   .0585685      4.121687    4.360913
       lnPNC |   4.692742   .0483805      4.593936    4.791548
       lnPUC |   4.637867   .0770827      4.480443    4.795291
       lnPPT |   4.765984   .0959479      4.570032    4.961936
        lnPD |   4.616158   .0479135      4.518305     4.71401
        lnPN |   4.709391   .0602159      4.586413    4.832368
        lnPS |   4.783979   .0866703      4.606975    4.960984
        Year |       1989   1.632993      1985.665    1992.335
        iota |          1          0             .           .
--------------------------------------------------------------

. mat xpost = e(b)

.
. // first term B-O decomp: take m as post
. mat t1 = xpost*(bpost-bpre)
```

```
. // cov mtx for first term
. mat vd = vpre+vpost

. // std error for first term
. mat t1var = xpost*vd*xpost'

. scalar t1se = sqrt(t1var[1,1])

. // second term
. mat t2 = (xpost-xpre)*bpre

. // total effect
. mat t3 = t1 + t2

.
. mat list t1, ti("Differential due to change in coeffs")

symmetric t1[1,1]:  Differential due to change in coeffs
            y1
y1  -.50270686

. di "Std error " t1se " approx c.i." t1[1,1]-1.96*t1se " , " t1[1,1]+1.96*t1se
Std error .24585864 approx c.i.-.98458979 , -.02082394

. mat list t2, ti("Differential due to change in regressors")

symmetric t2[1,1]:  Differential due to change in regressors
           y1
y1  .89808339

. mat list t3, ti("Total differential")

symmetric t3[1,1]:  Total differential
           y1
y1  .39537652
```

You can verify the above results by using the decomposition command oaxaca written by Ben Jann.

```
. oaxaca post pre, weight(0)
(high estimates: post; low estimates: pre)

                                                 Mean prediction 1 =  1.730146
                                                 Mean prediction 2 =  1.334769
------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  difference |   .3953765   .0455803     8.67   0.000     .3060407    .4847124
------------------------------------------------------------------------------

Linear decomposition
------------------------------------------------------------------------------
       Total |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
W=0          |
   explained |   .8980834   .2564011     3.50   0.000     .3955465     1.40062
 unexplained |  -.5027069   .2503842    -2.01   0.045    -.9934509   -.0119628
------------------------------------------------------------------------------
```