BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
May 27, 2011

Directions: There are four questions to this exam. Questions 1 and 2 have separate parts to them. Answer all parts to each question. Please follow the instructions for each question carefully. Write the answer to Questions 1 and 2 in one bluebook and then Questions 3 and 4 in a separate bluebook.

Write your Alias, Question Number(s) and Part(s) on the front of each blue book.

Please read the entire exam before writing anything.

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Question 1

Part A

Consider the following model with physical and human capital. Assume that the production function is :

$$Y = [(1-a_K)K]^{\alpha}[(1-a_H)H]^{1-\alpha}$$

with $0<\alpha<1, \quad 0<a_K<1, \quad 0<a_H<1$

where Y is output, K physical capital, and H human capital. The production function for net human capital is:

$$\dot{H} = \mathrm{B}[a_K K]^{\gamma}[a_H H]^{\phi}[AL]^{1-\gamma-\phi} - \delta_H H$$

with: $0<\gamma<1, \quad 0<\phi<1, \quad 0<\phi+\gamma<1$

Note, therefore, that human capital is produced in its own sector with its own production function. The equations of motion for K, L and A are:

$$\dot{K} = \mathrm{s}\,\mathrm{Y} - \delta_K K$$

$$\dot{L} = nL$$

$$\dot{A} = gA$$

a) Define *k=K/(AL), h=H/(AL)*. Derive equations for $\dot{k}$ and $\dot{h}$.

b) Find an equation describing the combinations of *k* and *h* for which $\dot{k}=0$. Draw this locus in *(k,h)* space. Do the same for $\dot{h}=0$ and draw the corresponding locus.

c) Does a balanced growth path exist for this economy? Is it unique (for positive values of *h* and *k*)? Is it stable? What are the growth rates of *Y/L, K/L and H/L* on the balanced growth path?

d) Assume the economy is initially on the balanced growth path and there is a permanent increase in s. How does this change affect the path of *Y/L*?

Part B

Assume now (an entirely different model) the production function , with increasing returns, is:

$$Y = K^{\alpha}\left[(1-a_H)H\right]^{\beta}$$

with: $0<\alpha<1, \qquad 0<\beta<1, \qquad \alpha+\beta>1, \qquad 0<a_K<1$

$$\dot{H} = \mathrm{Ba}_{\mathrm{H}}H$$

$$\dot{K} = \mathrm{s\ Y}$$

a) does the economy converge to a balanced growth path? What are the growth rates of K and Y on the balanced growth path? (Hint: first obtain the growth rate for K , $\dot{K}/K = g_K$; second, take the time derivative of the log of the resulting equation in order to get a non linear first order differential equation for g_K. Plot the equation in a graph with $\dot{g}_K$ on the vertical axis and g_K on the horizontal one)

b) What is/are the essential difference(s) between the model of Part A and the model of Part B?

Question 2

Part A

Assume log income per capita in country i in period t ($\log y_{it}$) depends upon the quality of institutions (R_{it}). In turn institutions depends upon the level of development summarized by $\log y_{it}$ and a set of other exogenous variables Z_{it} so that:

$$\log y_{it} = a + b * R_{it} + \varepsilon_{it}$$
$$R_{it} = c + d * \log y_{it} + f * Z_{it} + \eta_{it}$$

ε_{it} and η_{it} are mean zero error terms, uncorrelated over time and with each other.

a) Describe why using Ordinary Least Squares to estimate the first equation (for $\log y_{it}$) on a cross section of countries would not provide you with the right answer about the causal effect of institutions on economic performance. In order to do that:

b) Derive the reduced form for R: that is write R as a function of the Z's and of the error terms.

c) Write down the OLS estimator of b and explain why show that, even if the sample size goes to infinity, the OLS estimator of b does not converge in probability to the true value of b, i.e. the OLS estimator is inconsistent ($p\lim \hat{b}_{OLS} \neq b$). Comment on the likely direction of the bias.

d) Do you have any suggestion on how you could address this issue?

Part B

Assume that individuals live for 2 periods and have utility function:

$U_t = \log(c_{1t}) + 1/(1+\delta) \log (c_{2t+1})$

Assume N_t individuals are born in period t and that $N_t = N_{t-1}(1+n)$. Each individual is endowed with A units of the economy single good. Each unit stored in period t yields $x > 0$ units in the following period. Moreover, in the initial period (time 0) there are $(1/(1+n))$ N_0 old individuals (in addition to the young), each one of them endowed with an amount z of the good.

a) Describe the decentralized equilibrium of this economy.

b) Is the equilibrium Pareto efficient?

Part C

Consider the production function:

$Y(t) = F(K(t), A(t)L(t))$, where Y, K, L, and A denote output, capital, workers and technical progress respectively.

a) Using the standard growth accounting techniques, decompose the growth rate of output per capita in a fraction "due" to growth in K/L ratio and a fraction "due" to technological progress.

b) How can you reconcile your results under a) with the fact that the Solow model implies that the growth rate of output per worker on the balanced growth path is determined solely by the rate of technological progress?

Question 3:

Rietz (1988) and Barro (2005) have suggested that a number of asset-pricing facts can be explained by assuming a time-varying "disaster risk." This question asks you to investigate some of the implications of this assumption.

Suppose there is a fixed amount of capital in the economy, $\bar{K}$. There is no investment or depreciation. A disaster—for example, a war or an earthquake—cuts the amount of capital in half for N periods, after which it goes back to the non-disaster level, $\bar{K}$. Output depends on capital input (which varies with the state of the world), and on labor input:

$$Y_t = \begin{cases} \bar{K}^{\alpha} H_t^{1-\alpha} & \text{if no disaster} \\ (\bar{K}/2)^{\alpha} H_t^{1-\alpha} & \text{if disaster} \end{cases}$$

A representative competitive firm uses this technology to produce output. Output is used only for consumption:

$$Y_t = C_t.$$

A representative consumer maximizes the present value of utility, given by

$$V = E_t \sum_{s=0}^{\infty} \beta^s \left[\ln\left(C_{t+s}\right) + V\left(\bar{H} - H_{t+s}\right) \right],$$

subject to a standard budget constraint. V is a strictly concave function.

The consumer can invest in two assets. One asset is firm equity, E. In each period, equity pays a dividend, D.

$$D_t = Y_t - W_t H_t \ ,$$

where W is the real wage. The price of equity at time t is P_t. (Note that the consumer does not invest in capital directly. Capital is owned by the firm.) Normalize so that there is one unit of equity for every unit of capital in the non-disaster state ($E = \bar{K}$).

The other asset is a risk-free bond, B, which households trade amongst themselves. The gross rate of return on the bond is $1 + r_t$. In equilibrium, the stock of risk-free debt is zero.

The holdings of bonds and equity need to be decided one period in advance. That is, at time t the household chooses how much of each asset to hold at time $t+1$.

(a) Write down the budget constraint and set up the consumer's optimization problem. Derive three first-order conditions: a static tradeoff between work and leisure, and two dynamic Euler equations for bonds and equity.

(b) Prove that consumption is lower in a disaster state than in a non-disaster state. What can you say about relative hours worked in the two states?

For the remaining parts, assume that the economy is in a non-disaster state at time t, but new information has just been received that the probability of a disaster next period (at time t+1) has increased.

(c) In response to this news, what happens to the risk-free real interest rate between time t and time t+1? Why?

(d) In response to this news, what happens to C_t and H_t? Why?

(e) What can you say about the response of the current price-dividend (P_t/D_t) ratio for equities to the news of increased disaster risk? Does the P_t/D_t ratio fall unambiguously? Explain the economics at work. [Hint: Think about what would happen if risk aversion were either lower or higher than 1.]

Question 4:

Suppose that economic fluctuations take place in an economy where many identical consumers maximize utility from consumption and leisure:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\ln\left(C_{t+i}\right) + \nu_{t+i} \ln\left(\bar{H} - H_{t+i}\right) \right],$$

where ν_t is a random variable that is always positive. Consumers are subject to a series of budget constraints:

$$K_{t+1} = W_t H_t + \left(R_t + (1-\delta)\right) K_t + \Pi_t - C_t.$$

K is physical capital (the only asset); R is the rental rate of capital; δ is the depreciation rate; W is the real wage; and Π is economic profit (if any).

Production in this economy is done by a representative, competitive firm with the production function:

$$Y_t = K_t^{\alpha} (ZH_t)^{1-\alpha},$$

where Z is the constant level of technology.

(a) Suppose the economy is in a steady state with a constant level of ν. Suddenly ν decreases permanently. What can you say about the levels of Y, C, H and K in the new steady state with a lower ν relative to their levels before the shock?

(b) Define a business cycle as positive comovement between Y, C, H and I, where I is investment. Suppose there are transitory but persistent shocks to ν_t. Can such shocks create positive comovement among these four key economic variables?

(c) Conditional on the model above, how would you measure the volatility and persistence of shocks to ν? Explain clearly, using equations.

(d) Are there other facts you know about the business cycle that are inconsistent with a model of business cycles driven only by shocks to ν? Are these facts conditional or unconditional correlations?

(e) If you suggested any inconsistent facts in part (d), can you modify the model in a way that makes it consistent with the cited facts? Again, be clear whether you are trying to match conditional or unconditional correlations. (For this part only, you can modify the stated assumptions about preferences, technology, and competitive behavior by households and firms.)