BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
May 25, 2012

Directions: There are four questions to this exam. Please follow the instructions for each question carefully. Write the answer to all questions separately in a bluebook.

Write your Alias and Question Number on the front of each blue book.

Please read the entire exam before writing anything.

Question 1

Assume in each period a fixed number of individuals, L, live for one period, are endowed with a unit of labor, derive utility from consumption only, and are risk neutral. People consume a homogeneous final good, Y_t that is produced competitively using the production function:

$$Y_t = (A_t L)^{1-\alpha} x_t^{\alpha}$$

with $0 < \alpha < 1$. The intermediate good is produced by a monopolist using one unit of the final good. The monopolist maximizes her profit measured in units of the final good:

$$\Pi_t = p_t x_t - x_t$$

where p_t is the price of the intermediate in terms of the final good. GDP is defined as final output minus the part of it that is used in intermediate good production:

$$GDP_t = Y_t - x_t$$

a) Find the profit maximizing quantity of x_t and price p_t and the optimal level of profits Π_t. Moreover, substituting into the production function and using the definition of GDP, find the equilibrium quantity of Y_t and GDP_t and show that Π_t, Y_t and GDP_t are proportional to $A_t L$. Find the constants of proportionality.

Assume that each period there is one person (the inventor/entrepreneur)who can create a new version of the intermediate good with a productivity $A_t = \gamma A_{t-1}$ with $\gamma > 1$ if successful. If she fails $A_t = A_{t-1}$. The probability of success, μ_t, depends upon the amount R_t of final good spent according to the following equation:

$$\mu_t = \lambda n_t^{\sigma}$$

where $n_t = R_t / A_t^*$ and $A_t^* = \gamma A_{t-1}$ is the level of technology achieved if research is successful. $\lambda > 0$ represents the productivity of the research sector and $0 < \sigma < 1$. R_t is chosen by the inventor to maximize expected profits:

$$\lambda \left(R_t / A_t^*\right)^{\sigma} \Pi_t^* - R_t$$

b) Find the optimal level of $n_t = R_t / A_t^*$ (using the expression for optimal Π_t derived under a)) and μ_t and show they are constant over time.

c) Find the expected growth rate of A_t and hence the expected growth rate of Y_t and GDP_t and discuss its main determinants (note that the growth rate of technology between $t-1$ and t, denoted by $g_t = \frac{A_t - A_{t-1}}{A_{t-1}}$ can take two values depending upon whether the innovation is successful or not and that the probability of success is μ_t whose value you should have found in answering to b)).

d) Assume that instead of monopoly pricing, firm uses limit pricing, so that it cannot charge more that the cost of production of a competitive fringe that can produce at a cost equal to χ (assumed to be less than $1/\alpha$). Find the equilibrium x_t and Π_t and show that profits at an optimum are an increasing function of χ. Using χ as an inverse measure of competition, what is the effect of an increase in competition on the steady state growth rate of GDP?

Question 2

(All the necessary tables and figures are reported after Part B. Note that Part A and Part B are unrelated)

Part A

Assume that the production function is (Mankiw, Romer and Weil (1992)):

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}$$

with:

$$\alpha > 0, \beta > 0, \alpha + \beta < 1$$

where Y is output, K physical capital, H human capital, and A the level of technology. L and A grow at the constant rate n and g respectively. Both types of capital depreciate at the rate δ. Assume that gross investment in physical capital is a fraction s_K of output and gross investment in human capital a fraction s_H of output. Write the production function in per units of effective labor (AL).

a) Derive the laws of motion for physical and human capital (each per unit of effective labor) starting from

$$\dot{K} + \delta K = s_K Y$$

$$\dot{H} + \delta H = s_H Y$$

b) Derive the steady state values of physical capital, human capital and output as a function of the saving rates, of δ, n and g.

c) Write out the equation for the log of output per worker for country i, assuming that $A(t) = A\varepsilon_i \exp(gt)$, where ε_i is a random shock. Assume that in a given year t (say 1985) actual and steady state output per capita are identical and assume you estimate the equation using a cross section of countries. The results from Mankiw, Romer and Weil (QJE, 1992) are reported below. Focus on the restricted regressions for the non-oil producing countries. Are these results supportive of the augmented Solow model with human capital? (Additional information: assume that 50% to 70% of total labor income represents the return to human capital). Explain why or why not.

Part B

a) Consider the plot in Figure 1 (Kremer, QJE, 1993). The world population growth rate is on the vertical axis and populations levels on the horizontal axis. Can you rationalize this figure (or more specifically the upward sloping portion of the figure) with a model with the following characteristics:

Output is produced with labor, L, and land, T, according to the production function:

$Y = AT^{1-\alpha}L^{\alpha}$, with $0<a<1$

Land is fixed.

Output per capita (Y/L) is set to the level y^{M} (Malthusian assumption).

Technological progress depends upon the stock of labor in the following way:

$$\frac{\dot{A}}{A} = gL$$

where g is a positive constant.

b) Assume now the production of knowledge equation is:

$$A = \delta L A^{\phi}.$$

with $0<\phi<1$. What is the relationship between population levels and population growth rate in this case? Is this model "better" in explaining the data in figure one?

c) Can the model under a) help explain why population density is higher in "larger" continents?

d) Continue to assume that the production of knowledge equation is as in b). Assume that the population growth rate is constant and it equals n. What is the steady state growth rate of knowledge?

From Kremer (93)

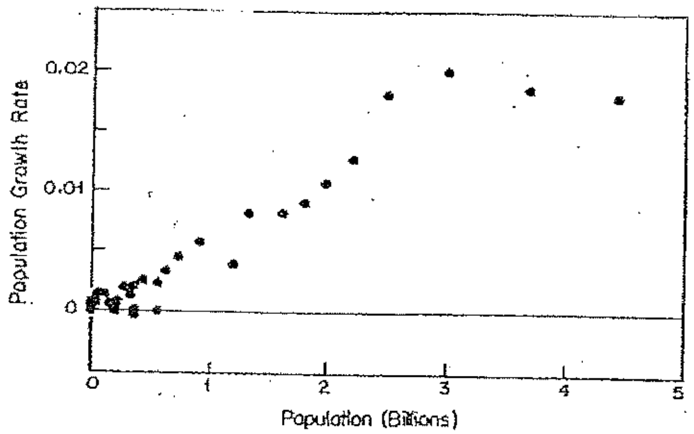

FIGURE I
Population Growth Versus Population

From Mankiw, Romer, and Weil (92)

TABLE II
ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	6.89	7.81	8.63
	(1.17)	(1.19)	(2.19)
ln(I/GDP)	0.69	0.70	0.28
	(0.13)	(0.15)	(0.39)
$\ln(n + g + \delta)$	−1.73	−1.50	−1.07
	(0.41)	(0.40)	(0.75)
ln(SCHOOL)	0.66	0.73	0.76
	(0.07)	(0.10)	(0.29)
$\bar{R}^2$	0.78	0.77	0.24
s.e.e.	0.51	0.45	0.33
Restricted regression:			
CONSTANT	7.86	7.97	8.71
	(0.14)	(0.15)	(0.47)
$\ln(I/GDP) - \ln(n + g + \delta)$	0.73	0.71	0.29
	(0.12)	(0.14)	(0.33)
$\ln(SCHOOL) - \ln(n + g + \delta)$	0.67	0.74	0.76
	(0.07)	(0.09)	(0.28)
$\bar{R}^2$	0.78	0.77	0.28

Question 3

Consider the following Dynamic New Keynesian (DNK) model.

Consumers face the problem:

$$\text{Max} \quad E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{C_{t+j}^{1-\sigma}}{1-\sigma} - a \frac{H_{t+j}^{1+\eta}}{1+\eta} \right]$$

s.t.

$$C_t + \frac{B_t}{P_t} = \frac{W_t}{P_t} H_t + (1+i_{t-1}) \frac{B_{t-1}}{P_t} + \Pi_t$$

with standard notation. C_t is a Dixit-Stiglitz aggregate of a continuum of consumption varieties C_{it} with elasticity of substitution $\theta > 1$. Π are profits rebated lump-sum to consumers. B represents private (inside) debt, and is zero in equilibrium. Assume an *ad-hoc* money demand function:

$$\frac{M_t}{P_t} = Y_t.$$

A firm producing output of type i has the production function:

$$Y_{it} = H_{it}^{1+\chi}.$$

There is no investment, so $\quad Y_t = C_t.$

Due to frictions in changing prices, inflation follows the NKPC:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{\varphi}_t,$$

where φ is marginal cost. Assume $\beta < 1$, $a < 1$, $\sigma \geq 0$, $\eta \geq 0$, and $\chi \geq 0$.

A. Derive the consumer's static and dynamic first-order conditions for utility maximization. Using these conditions, express the real wage as a function of C, H, and parameters. Log-linearize this expression to solve for $\widehat{W_t / P_t}$.

B. Assuming initially that $\chi = 0$, solve for the real marginal cost of production for each firm. Use your result from Part A and equilibrium conditions to express $\hat{\varphi}_t$ as a function of $\hat{Y}_t$.

C. Now assume that prices are sticky and $\sigma > 0$, $\eta > 0$, and $\chi = 0$. Suppose there is a 1% permanent increase in M. Explain why this shock has real effects on output in the short run but has no real effect in the long run in this model. Your explanation should include both economic intuition and key equations.

D. Now allow for $\chi > 0$. Consider the same 1% permanent increase in M. Is there any value of χ that would make output, Y, rise by 1% both on impact and permanently, given the admissible ranges for the other parameters? That is, would some value of χ make money non-neutral in the long run in this model?

   Be sure to comment on how the values of σ and η affect your answer. Is there any possible value of χ if $\sigma \geq 1$?

   (*Hint:* Assume the hypothesized result—that money has a one-for-one, permanent effect on output—and see if you can derive a contradiction.)

E. Discuss why the long-run (non-)neutrality of money is important for our overall view of aggregate fluctuations. Be brief, and make reference to specific models and macroeconomic controversies.

Question 4

The representative consumer maximizes utility from consumption, C, and labor supply, N:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\ln\left(C_{t+i}\right) + V\left(\bar{N} - N_{t+i}\right) \right],$$

where V is a concave function. The national income accounts identity implies:

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t - G_t.$$

There is a continuum of identical, competitive firms in the economy, indexed by $i \in [0,1]$. All firms have the production technology:

$$Y_{it} = E_t (U_{it} K_{it})^{\alpha} (A_t N_{it})^{1-\alpha},$$

where U denotes capacity utilization, A is exogenous technical change (common across firms), and E is an external effect that each firm takes as exogenous. E depends on aggregate output:

$$E_t = Y_t^{1-(1/\gamma)}, \ \gamma \geq 1.$$

The penalty of utilizing capital more intensively is that it wears out faster:

$$\delta = \delta\left(U\right), \delta' > 0, \delta'' > 0.$$

Define the important parameter ζ as $\zeta \equiv \delta''(U^*)U^* / \delta'(U^*)$.

Government consumption is financed by lump-sum taxes period by period: $G_t = T_t$.

A. Assume that each firm maximizes revenue minus cost, where cost is $\left(r_t + \delta(U_{it})\right) K_{it} + W_t N_{it}$. Find the profit-maximizing level of $\hat{U}$ as a function of $\hat{H}$, $\hat{E}$, $\hat{A}$ and $\hat{K}$.

B. Solve for the log-linearized aggregate production function and the log-linearized aggregate labor demand curve. Substitute out for $\hat{U}$ using your result in Part A.

C. Is it possible for consumption to rise following an increase in G in this model if $\gamma = 1$? What if $\gamma > 1$? Explain.

D. For a fixed γ, does the existence of variable utilization make it more or less likely that consumption will rise following an increase in G in this model? Be sure to define what you mean by "likely," and provide both an algebraic proof and some economic intuition for your answer.

E. Propose a functional form for $\delta(U)$ that allows you to calibrate ζ as a function of r^* and $\delta(U^*)$ only. (A * denotes a steady-state value.) Under your functional form, what is the relationship between ζ and a different quantity, $\delta'(U^*)U^* / \delta(U^*)$? Comment on the pros and cons of using this functional form assumption to calibrate ζ.