BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
May 23, 2008

Directions: There are six questions. Please follow the instructions for each question carefully. Write the answer to each question in separate bluebooks.

Write your alias, question number(s) on the front of each blue book

Please read the entire exam before writing anything.

QUESTION #1

Growth

I. An Open-Economy OLG Model

Consider the following small open economy OLG model.

The world interest rate is $\bar{r} > 0$. There is no depreciation or population growth.

A country is perfectly open to the world capital market. The production function in the country is

$$Y_t = K_t^{0.5}(e_t L_t)^{0.5}$$

where Y_t is output at time t, K_t is the capital stock, L_t is labor supply, and e_t is a measure of efficiency units per worker.

e increases at rate g: $$e_{t+1} = (1+g)e_t, \quad g > 0.$$

Every period, a new generation is born. People live two periods: in the first they work and consume. In the second they do not work and they consume their savings plus interest. There are no bequests or intergenerational transfers. People maximize lifetime utility functions:

$$V = \ln(C_1) + \ln(C_2)$$

where C_1 is consumption in the first period of life and C_2 is consumption in the second period of life.

Derive an expression showing whether, in steady state, this country will have positive, negative, or zero net foreign assets.

QUESTION #2

II. Physical and Human Capital

Consider the following model with physical and human capital.

Output is produced with physical capital, human capital, and labor according to the production function (written in per-worker terms):

$$y = k^{\alpha}[(1-u)h]^{1-\alpha}$$

where y is output per worker, h is human capital per worker, k is physical capital per worker, and $(1 - u)$ is the fraction of their time that workers spend producing outputl

There is no population growth. Physical capital accumulates according to

$$\dot{k} = sy - \delta k$$

where the saving rate, s, is exogenous and fixed.

When workers are not producing output, they are producing human capital (there is no leisure). The only input required to produce human capital is time. Human capital depreciates at the same rate δ as physical capital. Thus, the evolution of human capital per capita is given by:

$$\dot{h} = u - \delta h .$$

Assume that $s = \delta$.

A. Describe the steady state of the model. Is it one with a constant level of output or with a constant growth rate of output?

B. Solve for the level of u that maximizes either the level or the growth rate of output (depending on which is constant in steady state).

C. Let u^* be the level of u that you solved for in part B. Suppose that the economy is in steady state with $u < u^*$. Suddenly, u jumps up to u^*. Analyze the behavior of h and k following this change. Draw time series pictures showing how h and k behave during the transition to the new steady state, and justify your answers.

QUESTION #3

Non-Monetary Business Cycles

I. Varying Costs of Variable Capital Utilization

Suppose firms produce output with a Cobb-Douglas production function:

$$Y_t = (U_t K_t)^{\alpha} (Z_t L_t)^{1-\alpha} - \Phi,$$

where $\Phi > 0$ is a fixed cost. Firms are imperfectly competitive, with each firm charging a price that is μ_t times its marginal cost of production. There are no adjustment costs of changing K and L. Use the normalization that steady state utilization is 1 ($U^* = 1$).

A. For this part only, assume that the cost of utilizing capital more intensively is that it wears out faster. Thus, the rental rate of capital is $R_t = r_t + \delta(U_t)$, where r is the real interest rate and δ is the rate of depreciation. Assume $\delta' > 0$, $\delta'' > 0$. The real wage is W_t. The firm takes both r and W as given.

   1. Solve for the optimal choice of utilization using the firm's first-order conditions for optimization. Your result should express $\hat{u}$ only in terms of the other production-function variables (Y, K, L, Z), parameters (for example, but not limited to, α, Φ, δ', δ'', U^*) and prices exogenous to the firm (r, W). Eliminate terms that depend on μ or $\hat{\mu}$.

   2. Suppose a positive, persistent technology shock raises the real interest rate and the real wage above their steady-state levels. Will capital utilization rise with this positive technology shock? Is your result ambiguous? Explain the intuition.

B. For this part only, assume that the cost of utilizing capital more intensively is that production must take place outside normal business hours, which requires firms to pay their workers higher wages. (Unlike graduate students, workers expect to be compensated for working nights and weekends.) Thus, the labor cost is $W_t V(U_t)$, where V is a shift premium. Assume that $V' > 0$ and $V'' > 0$. There is no depreciation in use. The rental rate of capital is $R_t = r_t + \delta$, where r is the real interest rate and δ is the constant rate of depreciation. The firm takes the baseline real wage, W_t, as well as r_t, as given.

   1. Solve for the optimal choice of utilization using the firm's first-order conditions for optimization. Your result should express $\hat{u}$ only in terms of the other production-function variables (Y, K, L, Z), parameters (for example, but not limited to, α, Φ, V', V'', δ, U^*) and prices exogenous to the firm (r, W). Eliminate terms that depend on μ or $\hat{\mu}$.

   2. Suppose a positive, persistent technology shock raises the real interest rate and the real wage above their steady-state levels. Will capital utilization rise with this positive technology shock? Is your result ambiguous? Explain the intuition.

QUESTION #4

II. The Comovement Problem in an Open Economy

Suppose a standard business-cycle model, where a representative consumer does all consumption, supplies all labor, and maximizes discounted utility over an infinite horizon. The utility function takes the form:

$$U = \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) + V(\bar{L} - L_t) \right]$$

where L is labor and V is strictly concave. Consumers take all goods and factor prices as given.

Output is produced by many profit-maximizing firms. Firms take factor prices as given, but may or may not have market power in the product market (i.e., firms might sell their output at a markup, μ, over its marginal cost of production).

Assume that the economy in question is small and open. Thus, it can borrow and lend as much as it likes at the exogenous and constant world interest rate r^*. Also assume that there are no capital adjustment costs, so capital is freely mobile within and across countries.

In this setting, analyze the issue of consumption-labor comovement in response to an increase in government purchases financed by lump-sum taxes.

A. Suppose the model is fully neoclassical, so that firms have standard constant-returns production functions with diminishing marginal products of capital and labor and with $\mu = 1$. In this setting, is it possible for an increase in G to raise both C and L on impact (when the shock occurs)? How is your analysis affected by the openness of the economy?

B. Now sketch a model of firms with some non-neoclassical features. (You do not have to give all the details; just list the main features that you would add to the neoclassical RBC model.) Argue that if certain conditions are met (which ones?) your model predicts that both C and L will rise in response to an increase in G. Does the fact that the economy is open mean that it is easier to get positive consumption-labor comovement from G shocks? Define what you mean by "easier."

EC751 May 2008 - Macro Comp: Professor Matteo Iacoviello

Question 5 (A macro model with money and durables)

Consider the following macroeconomic model. A representative household chooses consumption, durables, capital, labor, real money balances $\{C_{t+i}, D_{t+i}, K_{t+i}, L_{t+i}, M_{t+i}/P_{t+i}\}_{i=0}^{\infty}$ to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, \frac{M_t}{P_t}, D_t, L_t\right)$$

with $u_C, u_m, u_D > 0$, $u_L < 0$, $u_{CC}, u_{DD}, u_{mm}, u_{LL} < 0$

subject to:

$$C_t + D_t + \frac{M_t}{P_t} + B_t + K_t =$$

$$w_t L_t + (z_t + 1 - \delta_K) K_{t-1} + (1 - \delta_H) D_{t-1} + \frac{M_{t-1}}{P_t} + R_{t-1} B_{t-1} + T_t$$

where w_t is real wage, z_t is the rental rate of capital, δ_K and δ_H denote depreciation rates for K and D, B is a real bond, T are lump-sum transfers from the central bank.

A firm maximizes profits by running a production function $Y_t = f(K_{t-1}, L_t)$ renting capital from households K_{t-1} and labor L_t. (Output Y_t can be then transformed into either C_t, K_t or D_t)

1. Derive and comment the first-order conditions for the household problem
2. Derive and comment the first-order conditions for the firm problem
3. On a diagram, plot the household labor supply and the firm's labor demand as a function of the real wage w_t (put w on the vertical axis). Explain clearly what shifts either labor demand or labor supply.
4. Assume the period t utility function takes the form

$$u = \log C_t + j \log D_t + \tau \log\left(\overline{L} - L_t\right) + b \log\left(\frac{M_t}{P_t}\right)$$

   and that the production function is

$$Y_t = K_{t-1}^{\alpha} L_t^{1-\alpha}$$

   Assume money grows over time at a constant rate

$$\frac{M_t}{M_{t-1}} = \theta > 1$$

   Calculate the steady state capital-output ratio, consumption-output ratio, durables-consumption ratio and steady state hours worked as a fraction of the total time endowment $\overline{L}$. Do they depend on the growth rate of money? Why?

5. Suppose you want to introduce nominal rigidity in the consumption good C_t in the model above. Sketch the steps required to do so.

Question 6 (An endowment economy)

Suppose there are n agents, indexed by i. Agent i utility is given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u_{it}$$
$$u_{it} = \log c_{it}$$

Each agent has an endowment of a non-storable commodity. The endowment for agent i is given by:

$$y_{it} = z_{it}$$
$$\log z_{it} \sim N\left(0, \sigma^2\right)$$
$$corr\left(z_{it}, z_{jt}\right) = 0 \ \forall \ i, j$$

1. Solve for the consumption allocation chosen by a social planner that maximizes social welfare assigning equal weights on all agents. At any point in time, is cross-sectional consumption inequality smaller or larger than income inequality?

2. Suppose now that there is no social planner, but agents can trade a long-term bond that costs p_t and pays one unit of consumption every period from periods $t+1$ on. Write down an agent's flow-of-funds constraint and first order conditions. Describe a competitive equilibrium for this economy as best as you can. Show that in a steady state with constant consumption for every agent, the bond price is positively related to β, the discount factor (explain the intuition).

BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
August 31, 2007

Directions: There are six questions. Please follow the instructions for each question carefully. Write the answer to each Professor's (Basu, Ghironi, Iacoviello) part in separate bluebooks.

Write your alias, question number(s) and Professor's Part on the front of each blue book.

Please read the entire exam before writing anything.

Susanto Basu's questions.

Question 1 (Long)

Consider a Ramsey economy with government purchases. Dynasties maximize the present value of utility from consumption:

$$U = \int_0^\infty e^{-\rho t} N(t) \frac{c(t)^{1-\sigma}}{1-\sigma} dt .$$

N is the number of people alive at time t, c is per-capita consumption and $\rho > 0$ is the discount rate. The population growth rate is n. Normalize $N(0) = 1$.

Household assets evolve as:

$$\dot{A} = rA + wN - C - G,$$

where A is assets, C is total consumption and G is total government purchases, financed by lump-sum taxes.

Output is produced by a standard neoclassical production function, F. There is no technological progress. Workers and firms behave competitively.

The government maintains a constant *per capita* level of spending, g:

$$G(t) = N(t)g .$$

A) Assume that the country is a small open economy and can borrow and lend freely at a fixed world interest rate r^*.

1. Suppose that the government unexpectedly raises g for a known length of time T, after which time g will revert to its original level. What will be the effect on c, k, and net exports over time?

2. Suppose that the government announces at time t_1 that it will follow the policy in part A.1 starting at some later time $t_2 > t_1$. What will be the effect on c, k, and net exports, starting from the announcement time t_1?

B) Now assume that the economy is closed, and there is no foreign borrowing or lending.

1. Repeat part A.1 for a closed economy. Draw the relevant phase diagram.

2. Repeat part A.2 for a closed economy. Draw the relevant phase diagram.

Question 2 (Short)

Consider the standard q model of investment presented in class. As usual, all firms maximize the expected present discounted value of cash flows:

$$V_0 = E_0\left[\int_0^{\infty} e^{-rt}\left\{\left[AF(K_t, L_t) - W_t L_t\right] - I_t\left[1+\phi(I_t/K_t)\right]\right\}dt\right]$$

subject to the constraints:

$$\dot{K}_t = I_t - \delta K_t$$
$$K_0 = \overline{K}.$$

The adjustment cost function, ϕ, is convex and has the standard properties.

At time t_0 it is announced that there is uncertainty about the future path of productivity, A. Specifically, at time $t_1 > t_0$, A will either increase to some new, higher value, and stay at that value permanently, or we will find out that A will remain unchanged at its current value forever.

Assume that at the time of the arrival of the new information, t_0, the firm is in steady state.

A. What will happen to q and K between t_0 and t_1?

B. Conditional on A increasing, what will happen to q and K after t_1?

C. Conditional on A staying constant, what will happen to q and K after t_1?

D. Suppose we think of this as a stylized model of the Internet investment and stock market boom of the 1990s and the subsequent bust in the early 2000s. Many people interpreted this period as a stock market "bubble" (i.e., stock prices not driven by fundamentals). Based on your previous answers, is it easy to conclude that the Internet boom and bust was indeed caused by a bubble? What additional data would allow you to distinguish between the "fundamental" and "irrational exuberance" explanations for this period?

Fabio Ghironi, Short Question 3

This question asks you to use results from the Lucas Trees Model to prove Modigliani and Miller's (AER, 1958) result that the total value of a firm is independent of its financial structure (*i.e.*, of the particular structure of indebtedness or ownership that it issues).

Suppose that an agent starts a firm at time t with a Lucas tree as its sole asset, and then she/he immediately sells the firm to the public by issuing N number of shares and B number of bonds as follows. Each bond promises to pay off r per period, and r is chosen so that rB is less than all possible realizations of future crops from the tree y_{t+j}. After payments to bondholders, the owners of issued shares are entitled to the residual crop. Thus, the dividend of an issued share is equal to $(y_{t+j} - rB)/N$ in period $t+j$. Assume that the agent has an infinite horizon and period utility function $u(c_t)$, with the standard properties, and discount factor β such that $0 < \beta < 1$.

(a) Write the expressions for the equilibrium prices of issued bonds and shares, p_t^B and p_t^N, respectively. (You should be able to do this without having to do any math.) Briefly explain the intuition for these asset pricing formulas.

(b) Use the expressions in (a) to show that the total value of issued bonds and shares is just equal to the tree's initial value, p_t (*i.e.*, the total value of the firm does not depend on its financing structure).

(c) Briefly discuss the intuition for the result in (b). (If you want to introduce math details in the discussion as we did in class (not required), feel free to make the special assumptions that will simplify matters: $u(c_t) = ln(c_t)$, y_{t+j} i.i.d. over time so that $E(y_{t+j}) = y$, and y_{t+j}^{-1} also i.i.d. for all $j = 1$.)

Fabio Ghironi, Long Question 4

Consider the following model.

The economy consists of two regions, North and South. Each region is populated by a representative household that receives exogenous endowment income. Income is identical across regions in steady state, but it is subject to shocks that can differ across regions outside the steady state.

The representative household in the North maximizes:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \log C_{N,s} \, ,$$

where $C_{N,t}$ denotes period-t consumption of a common consumption good by the representative household in the North, and $0 < \beta < 1$.

Asset markets are incomplete. The representative household in the North can trade a risk-free bond denominated in units of consumption with the representative household in the South to smooth the consequences of unexpected income shocks. The budget constraint of the household is:

$$B_{N,t+1} + \frac{\mu}{2} B_{N,t+1}^2 = (1 + r_t) B_{N,t} + y_{N,t} + T_{N,t} - C_{N,t} \, ,$$

where $B_{N,t+1}$ denotes the household's holdings of the bond entering period $t + 1$, r_t is the economy-wide, risk-free interest rate on the bond, and $y_{N,t}$ is time-t endowment income in the North. The term $\frac{\mu}{2} B_{N,t+1}^2$, $\mu = 0$, represents a cost of adjusting bond holdings relative to a constant level equal to zero: In order to change its bond holdings relative to this level, the household must pay a fee equal to $\frac{\mu}{2} B_{N,t+1}^2$ to a financial intermediary. The fee is then rebated to the household through a lump-sum transfer $T_{N,t}$, so that $T_{N,t} = \frac{\mu}{2} B_{N,t+1}^2$ in equilibrium. However, the household takes $T_{N,t}$ as given when solving its maximization problem.

The representative household in the South maximizes a similar utility function, with identical discount factor, subject to a similar budget constraint:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \log C_{S,s} \, ,$$

$$B_{S,t+1} + \frac{\mu}{2} B_{S,t+1}^2 = (1 + r_t) B_{S,t} + y_{S,t} + T_{S,t} - C_{S,t} \, .$$

For the bond market to be in equilibrium, it must be:

$$B_{N,t} + B_{S,t} = 0 \;\; \forall t.$$

(a) Use dynamic programming to obtain the Euler equation for bond holdings for the representative household in the North.

(b) Use the Euler equation to argue that, when $\mu > 0$, if the household borrows ($B_{N,t+1} < 0$), it pays a premium on the gross interest rate $1 + r_{t+1}$ that increases with the size of debt. (Assume $1 + \mu B_{N,t+1} > 0$.)

(c) Focus on the non-stochastic case. Assume that endowment income is constant at the same level $\bar{y}$ in the North and in the South in steady state. Consumption is also constant in steady state. (Use overbars to denote steady-state levels of variables.) Prove that, if $\mu = 0$ (*i.e.*, if households pay no fee to adjust their bond holdings), the steady-state level of bond holdings is indeterminate (*i.e.*, any initial level of $B_N = -B_S$ will be the steady state). (Hint: To prove this, focus on the North household. Use the Euler equation and the equilibrium *intertemporal* budget constraint to obtain the consumption function for the representative household. Substitute the consumption function in the equilibrium *period* budget constraint. Set bond holdings and income to be constant in the resulting equation, as well as the interest rate at its steady-state level implied by the Euler equation. Does the equation you obtain allow you to solve for steady-state bond holdings as a function of steady-state income?)

(d) Prove that, if $\mu > 0$, $\overline{B}_N = -\overline{B}_S = 0$ is the *unique* steady state with $\beta(1+\overline{r}) = 1$. What is the intuition for the difference in results relative to part (c)?

(e) Set $\mu = 0$ and suppose you *picked* $\overline{B}_N = -\overline{B}_S = 0$ as initial position. Take the ratio of Euler equations in the North and in the South for the stochastic case and log-linearize the resulting equation under standard assumptions of log-normality and homoskedasticity. What is the process for the North-South consumption differential in log-linear terms? Does the economy return to the initial position after a temporary, unexpected, asymmetric income shock? Why? Briefly, discuss the consequences of your result for solution of DSGE models through log-linearization around the initial steady state.

(f) How do your conclusions for part (e) change if $\mu > 0$?

(g) Intuitively: Assume $\mu = 0$. What would happen to the marginal utility ratio if agents in the North and in the South could trade in a complete set of contingent securities? Assuming zero initial financial wealth, would the two regions return to the initial position after an unexpected, transitory, asymmetric income shock? Why?

EC751 August 2007 - Macro Comp: Professor Matteo Iacoviello

Question 5 (long)

Assume the momentary utility of the representative agent is given by:

$$\frac{C_t^{1-\sigma}\left(\frac{M_t}{P_t}\right)^{1-b}}{1-\sigma}-\frac{N_t^{1+\eta}}{1+\eta}$$

A competitive final good firm produces the final good: $Y_t = \left[\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$.

Each individual variety Y_{jt} is produced by monopolistic firms according to $Y_{jt} = A_t N_{jt}$. Demand for each variety is given by $Y_{jt} = (P_{jt}/P_t)^{-\varepsilon} Y_t$.

1. Derive the linearized household's first order condition for labor supply.

2. Each monopolistic firm freely sets her own relative price $r_{jt} = P_{jt}/P_t$ in order to maximize profits which are given by:

   $$r_{jt}Y_{jt} - Z_t Y_{jt}$$

   where Z_t is the firm's real marginal cost (Lagrange multiplier on the production constraint).

   Express each firm's optimal relative price as a function of the real marginal cost.

3. From the cost minimization problem of the firm, derive the labor demand condition.

4. Combining the results in parts (1) to (3), derive an expression for equilibrium output in terms of percentage deviations around the steady-state as a function of technology and real balances.

5. Calculate the impulse response of output for the first three periods to an unexpected 1% technology shock with an autocorrelation coefficient of ρ_z.

6. Do real money balances affect the equilibrium? Explain.

EC751 August 2007 - Macro Comp: Professor Matteo Iacoviello

Question 6 (short)

The Calvo model of partial adjustment of prices shows that (in a neighborhood of the steady state) inflation can be approximated as:

$$\pi_t = \kappa d_t + \beta E_t \pi_{t+1}$$

where β is the discount rate and κ is a constant, and $d_t = y_t - y_t^*$ is the difference between actual output y_t and its natural level y_t^*.

1. Briefly explain how this "Phillips curve" can be derived from explicit microfoundations.

2. What factors can affect the natural level of output y_t^*?

3. Suppose d_t follows an exogenous $AR(1)$ process, e.g.

$$d_t = \rho d_{t-1} + e_t$$

where e is white noise. Calculate the implied autocorrelation coefficient of inflation given the process for d.

BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
May 25, 2007

<u>Directions:</u> There are six questions. Please follow the instructions for each question carefully. Write the answer to each part in separate bluebooks.

<u>Write your alias, question number(s) on the front of each blue book.</u>

Please read the entire exam before writing anything.

Susanto Basu's questions. The two questions will be weighted equally. Thus, you should spend about the same length of time on each one.

Question 1

Consider the following OLG model. People live for two periods. They are born and die with zero assets. They work only in the first period of their lives. The capital stock in any period is composed of the savings of the older generation.

People have the following lifetime utility functions:

$$U = \ln(c_1) + \ln(c_2)$$

The production function is

$$y = k^{\alpha},$$

where y is output per worker and k is capital per worker. Workers are paid their marginal products.

Population grows at rate n, so that the number of young in period $t+1$ is $(1+n)$ times the number of young people in period t. There is no technological progress.

Capital depreciates fully after it is used in production. This means that the elderly consume the earnings from the capital they own, but not the capital itself.

The economy is in steady state.

A) Solve for the steady-state level of capital per worker.

B) Solve for the steady-state levels of first and second period consumption.

C) Solve for the steady-state level of lifetime utility.

D) Assume that $\alpha = ½$. Is steady-state lifetime utility higher in a country where population growth is positive (i.e., $n > 0$) or negative (i.e., $n < 0$)?
Indicate how you know this.

E) <u>Completely optional, no credit, for fun only</u>
Based on your answer to part D, should you move to Italy or stay in the U. S.?

Question 2

Consider a country with production function:

$$Y = K^{0.5}H^{0.5}$$

where K is the stock of physical capital and H is the stock of human capital. There is no depreciation, population growth, or technological change. Physical capital is accumulated according to:

$$\dot{K} = s_K Y,$$

and human capital is accumulated according to:

$$\dot{H} = s_H Y.$$

The saving rates are exogenously given, not chosen optimally.

A country is growing at a constant, positive rate along its balanced growth path. At a point in time, the country's saving rate in physical capital (s_K) doubles, while its saving rate in human capital (s_H) remains constant.

[Note: You do not have to answer part A in order to answer part B]

A. What will be the effect of this change on the growth rate of output along the new balanced growth path? That is, by what factor will the growth rate of output along the new balanced growth path differ from that along the initial balanced growth path?

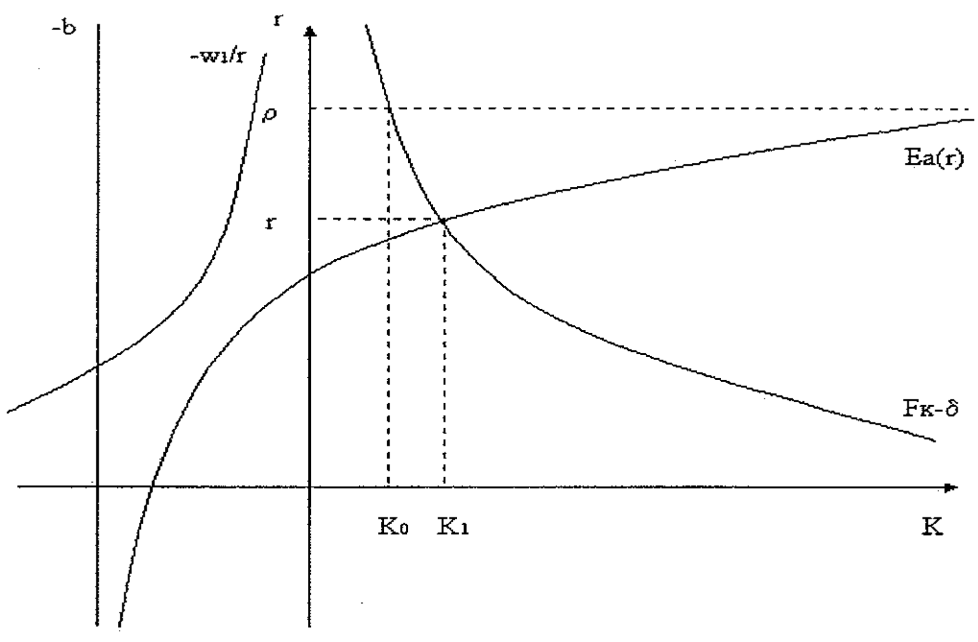
B. What will be the instantaneous effect of this change on the growth rate of output? That is, by what factor will the growth rate of output change at the moment the saving rate changes?

Question 3

Fabio Ghironi, Short Question

A result of the partial equilibrium savings problem with uncertainty in which a household is constrained to accumulating non-negative amounts of a single risk-free asset, and we assume that the household's discount factor and the gross return on the asset are such that $\beta(1 + r) = 1$, is that the household's holdings of the asset and consumption will diverge to infinity in the long run. Explain the intuition for this result, assuming that the household's period utility function $u(c)$ is such that $u' > 0$, $u'' < 0$, $u''' > 0$.

Suppose next that the economy is populated by many households who are subject to idiosyncratic employment uncertainty. There is no aggregate uncertainty. The single asset that the households can accumulate is physical capital, which they rent to firms in a competitive capital market. Capital depreciates at rate δ, $1 > \delta > 0$. Firms combine capital and labor to produce output with technology $F(K,N) = AK^{\alpha}N^{1-\alpha}$, $1 > \alpha > 0$, where K and N are the average levels of capital and employment, respectively, and $A > 0$. Assume that there exists a stationary distribution of households over capital-employment state pairs. Average asset holdings as a function of r are denoted by $Ea(r)$. The interest rate r is now determined endogenously in general equilibrium. Explain intuitively the equilibrium determination of capital and the interest rate with the aid of the following diagram. Make sure to explain the difference relative to the partial equilibrium case above, and compare the equilibrium with uncertainty to what would happen with certainty or complete asset markets. (In the diagram below, $\rho > 0$ is the household's discount rate, such that $\beta = 1/(1 + \rho)$, $b > 0$ would be an *ad hoc* debt limit, and w_l/r would be the natural debt limit, where w_l is labor income in the "worst case scenario.")



Question 4

Fabio Ghironi, Long Question

Consider the following version of the stochastic growth model. The representative household maximizes:

$$E_t \sum_{i=0}^{+\infty} \beta^i \left[\log C_{t+i} + \theta \frac{(1-N_{t+i})^{1-\gamma_n}}{1-\gamma_n} \right], \quad 0 < \beta < 1, \theta > 0, \text{ and } \gamma_n > 0,$$

subject to the budget constraint:

$$B_{t+1} + P_t X_{t+1} + C_t + I_t = R^f_{t-1,t} B_t + (P_t + D_t) X_t + \tilde{r}_t K_t + w_t N_t .$$

The household enters period t with holdings B_t of a risk-free one-period bond and receives the gross return $R^f_{t-1,t}$ on this bond position. It also starts the period with holdings X_t of a risky asset, and receives the dividend $D_t X_t$ from this position and the value of selling it, $P_t X_t$. (There is no government. Assume that the bond and the risky asset are issued by the households.) The household owns the capital stock and rents it to firms for the rental rate $\tilde{r}_t$. It supplies labor for the wage w_t. The household uses its income to finance consumption, investment, and purchases of the risk-free bond and risky asset to be carried into next period.

Capital accumulation obeys the standard law of motion:

$$K_{t+1} = (1-\delta) K_t + I_t , \quad 0 < \delta < 1.$$

Output is produced according to the Cobb-Douglas production function:

$$Y_t = (A_t N_t)^\alpha K_t^{1-\alpha}, \quad 0 < \alpha < 1.$$

Output is used for consumption and investment, so that the resource constraint of the economy is:

$$Y_t = C_t + I_t .$$

(Recall that $B_{t+1} = B_t = X_{t+1} = X_t = 0$ in equilibrium.)

(a) Write the Euler equations for the pricing of the risk-free one-period bond B, paying the certain gross return $R^f_{t,t+1}$ between period t and $t+1$, and of the asset X, paying the uncertain gross return $R_{t,t+1} = (P_{t+1} + D_{t+1}) / P_t$ between period t and $t+1$.

(b) Assume log-normality and homoskedasticity. Show that

$$\log E_t R_{t,t+1} - \log R^f_{t,t+1} = \text{cov}_t (c_{t+1} - c_t , r_{t,t+1}),$$

where $r_{t,t+1}$ is the percent deviation of $R_{t,t+1}$ from the steady state and c_t is the percent deviation of consumption from the steady state.

(c) Given solutions for consumption, capital, and the asset return:

$$c_t = \eta_{ck} k_t + \eta_{ca} a_t,$$
$$k_{t+1} = \eta_{kk} k_t + \eta_{ka} a_t,$$
$$r_{t-1,t} = \eta_{rk} k_t + \eta_{ra} a_t,$$

and the technology process $a_t = \phi a_{t-1} + \varepsilon_t,\ \ E_{t-1}\varepsilon_t = 0$, show that

$$\log E_t R_{t,t+1} - \log R^f_{t,t+1} = \eta_{ca}\eta_{ra}\sigma^2_\varepsilon,$$

where σ^2_ε is the variance of the normally-distributed technology innovation ε_t. Comment on this expression.

(d) Use the solution for consumption to write the solution for the percent deviation of the risk-free rate of return from the steady state as:

$$r^f_{t,t+1} = \frac{1}{1-\phi L}\left[-(1-\phi)\eta_{ca} + \frac{\eta_{ck}\eta_{ka}(1-L)}{1-\eta_{kk}L}\right]\varepsilon_t \ .$$

Explain the effect of a technology shock on the risk-free rate of return, clarifying the role of the two channels through which technology affects $r^f_{t,t+1}$.

Question 5: Money Demand

Assume the momentary utility of the representative agent is given by

$$u(C_t, 1-N_t) = \frac{C_t^{1-\eta}(1-N_t)^{\theta(1-\eta)}}{1-\eta}$$

Total utility is $U = \sum_{t=0}^{\infty} \beta^t u_t$, $0 < \beta < 1$. N_t denotes total working hours, and η and θ are positive and satisfy $\eta > \theta/(1+\theta)$. The household receives wages W_t, rental income from capital $(r_t K_{t-1})$, dividends D_t and a lump-sum transfer T_t from the government. The household allocates its income net of transaction costs TC_t to consumption C_t, capital K_t and real money balances M_t/P_t. Hence its budget constraint is

$$\frac{W_t}{P_t}N_t + r_t K_{t-1} + D_t + T_t - TC_t - C_t \geq \frac{M_t - M_{t-1}}{P_t} + K_t - (1-\delta)K_{t-1}$$

Transaction costs are given by:

$$TC_t = \gamma\left(\frac{C_t}{M_t/P_t}\right)^{\kappa} C_t,\ \gamma, \kappa > 0$$

The idea is that more money facilitates transaction, thus increasing the real stock of resources available for consumption to the household.

1. Provide the first order conditions and the transversality conditions for the household problem. Interpret them.

2. Assume there is a (price taking) final good firm that purchases and assembles the output of a large number J of intermediary producers to the single good Y_t according to

$$Y_t = \left[J^{-1/\varepsilon}\sum_{j=1}^{J} Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},\ \varepsilon > 1$$

   Let the money price of each intermediate good j be P_{jt} and let P_t the money price of the final good.

   Write down the optimization problem of the final good firm and its optimality condition. Does the final good firm make profits in equilibrium? What is the aggregate price index in this economy?

Question 6: Output and Inflation

Consider the following New-Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

where y_t is log deviation of output from its flexible price value, and π_t is inflation (in deviation from its mean).

1. Assume y_t follows an exogenous stochastic process of the form

   $$y_t = \rho y_{t-1} + \varepsilon_t$$

   where ε_t is zero mean, iid, with variance σ^2.

   Find the solution for inflation. Show that in equilibrium the persistence (as measured by the autocorrelation) of inflation is equal to the persistence of output, that is

   $$corr(\pi_t, \pi_{t-1}) = corr(y_t, y_{t-1}) = \rho$$

2. Suppose now y_t is the sum of two autoregressive components

   $$y_t = y_t^P + y_t^C$$

   where:

   $$y_t^i = \rho_i y_{t-1}^i + \varepsilon_t^i,\ i = P, C.$$

   Calculate the solution for inflation.

3. Suppose that ρ_C is low (say, 0) and ρ_P is high (say, 0.95). Let the variances of ε_t^P and ε_t^C be equal. Will the persistence of inflation (as measured by its autocorrelation) be closer to 0 or closer to 0.95?

BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
September 1, 2006

Directions: There are three parts to the exam. Please follow the instructions for each part carefully. Write the answer to each part in separate bluebooks.

Write your alias, part number and question number(s) on the front of each blue book.

Please read the entire exam before writing anything.

Part I. Economic Growth (Basu)

You should plan to spend about 60 minutes on the long question and 20 minutes on the shorter one. The grading weights will reflect this suggested allocation of time.

Question 1 (Long Question)

Recent tax reforms and tax reform proposals have emphasized the importance of reducing distortions in the rate of return to saving. Consider the merits of these proposals by working through the two benchmark models below.

In both models, the aggregate production function is $Y = F(K,L)$. F has constant returns to scale and satisfies standard neoclassical assumptions.

The rate of depreciation is δ. With competition, the interest rate paid by firms is

$$r_t = f'(k_t) - \delta$$

where $k_t \equiv K_t / L_t$ and $f(k) \equiv F(k,1)$.

A. Consider a standard Ramsey representative household. It takes prices as given and inelastically supplies one unit of labor. There is no population growth. Assets are denoted a, and w is the real wage. $\rho > 0$. At time t, the household solves:

$$\underset{c_s}{Max} \int_t^{\infty} e^{-\rho s} \ln(c_s) ds$$

$$\text{subject to: } \dot{a}_s = r_s(1-\tau)a_s + w_s - c_s$$

$$a_t \text{ given}$$

The government taxes interest income at $\tau \in (0,1)$ and wastes the revenue ("throws the taxes in the ocean").

i) Write down the first-order conditions for household optimization.

ii) The steady state of this economy has constant levels of consumption and capital (denoted c^* and k^*). Derive the sign of $dk^*/d\tau$.

B. Switch to a 2-period OLG model in discrete time. Each household lives two periods; there are L households born each year t; each household inelastically supplies one unit of labor in youth and zero in old age; there is no population growth. The production function, depreciation and equation for r_t are as before.

Each household solves

$$\underset{c_t^1, c_t^2}{\text{Max}} \left\{ \ln\left(c_t^1\right) + \ln\left(c_t^2\right) \right\}$$

$$\text{subject to: } c_t^1 + \frac{c_t^2}{1 + r_t(1-\tau)} \le w_t$$

For simplicity, assume $F(K_t, L) = [K_t]^{\alpha} [L]^{1-\alpha}, \quad \alpha \in (0,1)$.

i) Showing all your steps, solve the household maximization problem for c_t^1 and c_t^2.

ii) Derive an equation of motion for $k_t \equiv K_t / L$ (i.e., how does k evolve over time?) Show all your steps.

iii) If $k_t = k^* > 0$ is a steady-state equilibrium, derive the sign of $dk^*/d\tau$.

C. Many people assume that a lower tax rate on interest income will promote domestic accumulation of physical capital. What do you think? (Be brief!)

Question 2 (Short Question)

Explain why each of the following statements is True, False or Uncertain.
<u>Explanation determines grade</u>.

A. The smoothness of consumption relative to income is a key implication of the permanent income hypothesis.

B. In neoclassical growth models (e.g., the Solow and Ramsey models), neither Tobin's average q nor Hayashi's marginal q is of much interest.

Fabio Ghironi, Short Question

Suppose bonds and stocks yield gross returns $1+r_{t+1}^{b}$ and $1+r_{t+1}^{s}$ at time $t+1$, respectively. These returns are uncertain as of time t. The representative consumer maximizes a standard intertemporal utility function that depends on consumption only. The discount factor is $0<\beta<1$.

(a) Write the Euler equations for bonds and stocks.

Assume that the period utility function is $u(C)=\frac{C^{1-\gamma}}{1-\gamma}$, $\gamma>0$. Suppose also that consumption growth and returns obey:

$$\frac{C_{t+1}}{C_t}=\overline{C}_\Delta \exp\left(\varepsilon_{C,t+1}-\frac{\sigma_C^2}{2}\right),$$

$$1+r_{t+1}^{i}=\left(1+\overline{r}^{i}\right)\exp\left(\varepsilon_{i,t+1}-\frac{\sigma_i^2}{2}\right),\quad i=s,b;$$

where $\{\varepsilon_{C,t+1},\varepsilon_{s,t+1},\varepsilon_{b,t+1}\}$ are jointly normally distributed with zero means and variances $\{\sigma_C^2,\sigma_s^2,\sigma_b^2\}$. Thus, the log of consumption growth and the logs of returns are jointly normally distributed. Recall that, if $\log x$ is normal with mean μ and variance σ^2, it is

$$E(x)=\exp\left(\mu+\frac{\sigma^2}{2}\right).$$

(b) Use these assumptions and the unconditional expectation of the Euler equation for bonds in part (a) to obtain:

$$\log\left(1+\overline{r}^{b}\right)=-\log\beta+\gamma\log\overline{C}_\Delta-(1+\gamma)\gamma\frac{\sigma_C^2}{2}+\gamma\operatorname{cov}(\varepsilon_b,\varepsilon_C).$$

Interpret this equation.

(c) Show that

$$\log\left(1+\overline{r}^{s}\right)-\log\left(1+\overline{r}^{b}\right)=\gamma\left[\operatorname{cov}(\varepsilon_s,\varepsilon_C)-\operatorname{cov}(\varepsilon_b,\varepsilon_C)\right].$$

What is the "equity premium puzzle"?

Fabio Ghironi, Long Question

Consider the following model.

The economy consists of two regions, North and South. Each region is populated by a representative household that receives exogenous endowment income. Income is identical across regions in steady state, but it is subject to shocks that can differ across regions outside the steady state.

The representative household in the North maximizes:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \log C_{N,s} ,$$

where $C_{N,t}$ denotes period-t consumption by the representative household in the North of a common consumption good, and $0 < \beta < 1$.

Asset markets are incomplete. The representative household in the North can trade a risk-free bond denominated in units of consumption with the representative household in the South to smooth the consequences of unexpected income shocks. The budget constraint of the household is:

$$B_{N,t+1} + \frac{\mu}{2} B_{N,t+1}^2 = (1 + r_t) B_{N,t} + y_{N,t} + T_{N,t} - C_{N,t} ,$$

where $B_{N,t+1}$ denotes the household's holdings of the bond entering period $t + 1$, r_t is the economy-wide, risk-free interest rate on the bond, and $y_{N,t}$ is time-t endowment income in the North. The term $\frac{\mu}{2} B_{N,t+1}^2$, $\mu \geq 0$, represents a cost of adjusting bond holdings relative to a constant level equal to zero: In order to change its bond holdings relative to this level, the household must pay a fee equal to $\frac{\mu}{2} B_{N,t+1}^2$ to a financial intermediary. The fee is then rebated to the household through a lump-sum transfer $T_{N,t}$, so that $T_{N,t} = \frac{\mu}{2} B_{N,t+1}^2$ in equilibrium. However, the household takes $T_{N,t}$ as given when solving its maximization problem.

The representative household in the South maximizes a similar utility function, with identical discount factor, subject to a similar budget constraint:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \log C_{S,s} ,$$

$$B_{S,t+1} + \frac{\mu}{2} B_{S,t+1}^2 = (1 + r_t) B_{S,t} + y_{S,t} + T_{S,t} - C_{S,t} .$$

For the bond market to be in equilibrium, it must be:

$$B_{N,t} + B_{S,t} = 0 \ \forall t.$$

(a) Use dynamic programming to obtain the Euler equation for bond holdings for the representative household in the North.

(b) Focus on the non-stochastic case. Assume that endowment income is constant at the same level $\bar{y}$ in the North and in the South in steady state. Consumption is also constant in steady state. (Use overbars to denote steady-state levels of variables.) Prove that, if $\mu = 0$ (*i.e.*, if households pay no fee to adjust their bond holdings), the steady-state level of bond holdings is indeterminate (*i.e.*, any initial level of $B_N = -B_S$ will be the steady state). (Hint: To prove this, focus on the North household. Use the Euler equation and the equilibrium *intertemporal* budget constraint to obtain the consumption function for the representative household. Substitute the consumption function in the equilibrium *period* budget constraint. Set bond holdings and income to be constant in the resulting equation, as well as the interest rate at its steady-state level implied by the Euler equation. Does the equation you obtain allow you to solve for steady-state bond holdings as a function of steady-state income?)

(c) Prove that, if $\mu > 0$, $\overline{B}_N = -\overline{B}_S = 0$ is the *unique* steady state with $\beta(1+\overline{r}) = 1$. What is the intuition for the difference in results relative to part (b)?

(d) Take the ratio of Euler equations in the North and in the South for the stochastic case. Assume $\mu = 0$ and suppose you *pick* $\overline{B}_N = -\overline{B}_S = 0$ as initial steady-state position (so that $\overline{C}_N = \overline{C}_S$). Log-linearize the ratio of Euler equations in this case under standard assumptions of log-normality and homoskedasticity. Does the economy return to the initial position $\overline{B}_N = -\overline{B}_S = 0$, $\overline{C}_N = \overline{C}_S$ after a temporary, unexpected, asymmetric income shock? Why? Briefly, discuss the consequences of your result for solution of DSGE models through log-linearization around the initial steady state.

(e) Intuitively: How do your conclusions for part (d) change if $\mu > 0$?

(f) Intuitively: Assume $\mu = 0$. What would happen to the marginal utility ratio if agents in the North and in the South could trade a complete set of contingent securities? Assuming zero initial financial wealth, would the two regions return to the initial position after an unexpected, transitory, asymmetric income shock? Why?

Question 5 (long): Two agents, and a long-term bond

Suppose there are two agents, indexed by i. Agent i utility is given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u_{it}$$
$$u_{it} = \log c_{it}$$

Each agent has an endowment of a non-storable commodity. The endowment for agent i is given by:

$$y_{it} = z_{it}$$
$$\log z_{it} \sim N\left(0, \sigma^2\right)$$
$$corr\left(z_{1t}, z_{2t}\right) = 0$$

1. Solve for the consumption allocation in a competitive equilibrium with complete markets, by solving a social welfare maximization problem with equal weights on the two agents.

   What is the correlation between c_{1t} and c_{2t}?

2. Suppose now that there is no social planner, but agents can trade with each other a long-term bond b_t that costs p_t and pays one unit of consumption every period from periods $t+1$ on. Write down agents' flow-of-funds constraint and agents' first order conditions for the choice of b_t. Describe a competitive equilibrium for this economy as best as you can. Show that, in a neighborhood of the steady state with constant consumption for each agent, the bond price is positively related to β, the discount factor.

3. Suppose now that:

   $$y_{it} = z_{it} n_{it}^{\alpha}$$

   where n_{it} is labor supply of agent i. The exogenous component of output, z, is still *iid* and has zero correlation across agents. Assume

   $$u_{it} = \log\left(c_{it} - n_{it}\right)$$

   Solving the social planner problem like in part 1, find the optimal n_{it}.

   Does this modification in affect the prediction for the consumption correlation across the two agents? Why?

Question 6 (short): Monetary shocks and output

Consider the following economy. On the demand side, output is determined by the following equation

$$y_t = E_t y_{t+1} - (r_t - E_t \pi_{t+1})$$

where y_t is output, r_t is the nominal interest rate, and π_t denotes the rate of inflation between $t-1$ and t. All variables are expressed in log deviations from their steady state mean. Inflation evolves according to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

where a_t is a variable that describes the evolution of technology and follows a mean zero iid process.

Suppose the central bank follows an interest rate rule of the form:

$$r_t = \phi \pi_t + e_t$$

where e_t is a zero-mean, iid monetary shock.

1. Derive a closed form solution for equilibrium output and inflation as a function of the state variables of the model.
2. Discuss how the response of output and inflation to monetary shocks depends on the coefficient ϕ in the interest rate rule.

BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
May 26, 2006

Directions: There are six questions. Please follow the instructions for each question carefully. Write the answer to each part in separate bluebooks.

Write your alias, question number(s) on the front of each blue book.

Please read the entire exam before writing anything.

Question 1

In a certain country, consumption and investment decisions are made by a social planner who maximizes

$$\int_0^{\infty} e^{-\rho t} \ln(c)\,dt$$

where c is consumption per capita, and $\rho > 0$ is the time discount rate.

The population (which is equal to the labor force) grows exogenously at rate $n > 0$.

There are two kinds of capital used in production. The quantities per worker are denoted k_1 and k_2. Once capital has been created, it cannot be turned back into output. Similarly, one type of capital cannot be turned into the other. Call i_1 the per-worker quantity of investment in Type-1 capital and i_2 the per-worker quantity of investment in Type-2 capital.

At each instant, the social planner's budget constraint is

$$y = c + i_1 + i_2$$

The per-worker production function is

$$y = k_1^{\alpha} k_2^{\alpha} \quad \alpha < 1/2$$

The depreciation rate is zero.

A. Solve for the steady state levels of c, k_1 and k_2.
   [Note, there are some slightly ugly algebraic expressions that crop up here and in Part C. You do not have to simplify too much, as long as what you are doing is clear.]

   Now, suppose that the economy is in steady state when suddenly an asteroid collides with the earth, destroying half of the Type-2 capital. None of the Type-1 capital is destroyed.

B. Draw a picture with the quantity of k_1 on the horizontal axis and the quantity of k_2 on the vertical axis. Draw a point indicating the steady state. Now sketch on the figure the time path of (k_1, k_2) following the collision with the asteroid. Also describe (in words) how investment behaves during this transition.

C. Solve for the growth rate of consumption immediately following the collision with the asteroid.

Question 2

Suppose production in each country takes place at many identical small firms. There is a continuum of such firms, indexed by $i \varepsilon [0,1]$. The production function for each firm is:

$$Y_i = AK_i^{\alpha} L_i^{1-\alpha} \qquad 0 < \alpha < 1.$$

Note that A is the same for all firms within a country (but not necessarily across countries). Each small firm takes the economy-wide level of A as given, but for the economy as a whole A depends on the aggregate capital/labor ratio:

$$A = B\left(\frac{K}{L}\right)^{\beta} \qquad \beta > 0,$$

where $B > 0$ is a constant. α, β, B and δ are the same for all countries.

Aggregate quantities are the sums of firm-level quantities. For example, $K = \int_0^1 K_i di$.

There is a fixed saving rate s and the population growth rate is constant:

$$\dot{K} = sY - \delta K$$

$$\frac{\dot{L}}{L} = n$$

A. Write the firm-level production function for the firm's output per worker (y_i) in terms of the firm's capital per worker (k_i) and A.

B. Assume that firms are competitive and take factor prices as given. Show that the production function for per-capita output for the whole economy is $y = Bk^{\alpha+\beta}$.

C. Suppose $\beta < 1 - \alpha$, but still quite large relative to α. Write the equation for per-capita capital accumulation, and draw the picture that shows the steady state of the Solow model. How does the picture change because of the existence of β, which is not in the standard model? [If you could not show the result in Part B, just assume it here and in Part D.]

D. Suppose you do "development accounting" in this world, as in Hall and Jones (1999). Would you conclude that differences in productivity explain much of per-capita income differences across countries? Is that conclusion correct in this model? What kinds of evidence would let you decide whether the story implied by this model or the story of cross-country TFP differences is more important? Could both stories be true? Discuss.

Question 3

Fabio Ghironi, Long Question

A pure endowment economy consists of two households with identical preferences but different endowments. Household i has preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_{it}),\ 0 < \beta < 1,$$

where $u(C_{it}) = u_1 C_{it} - \frac{1}{2} u_2 C_{it}^2$, with u_1, $u_2 > 0$.

Household 1 has a stochastic endowment of the consumption good governed by:

$Y_{1t+1} = Y_{1t} + \sigma \varepsilon_{t+1}$,

where $\sigma > 0$ and ε_{t+1} is an i.i.d. Gaussian process with mean zero and variance 1. Household 2 has endowment:

$Y_{2t+1} = Y_{2t} - \sigma \varepsilon_{t+1}$,

where ε_{t+1} is the same random process as for household 1.

At time t, Y_{it} is realized before consumption at t is chosen. Assume that at time 0, $Y_{10} = Y_{20}$ and that Y_{10} is strictly smaller than the bliss point u_1/u_2. Assume that there is no disposal of resources.

Part A

Suppose that asset markets are incomplete. There is only one traded asset: a one-period risk-free bond that both households can either purchase or issue. The gross rate of return on the asset between date t and date $t + 1$ is R_t. Household i's budget constraint at time t is:

$C_{it} + R_t^{-1} B_{it+1} = Y_{it} + B_{it}$,

where B_{it} is the value in terms of time t consumption of household i's holdings of one-period risk-free bonds.

Assume that $B_{10} = B_{20} = 0$.

Prove *analytically* that the competitive equilibrium of the economy is such that $R_t = 1/\beta$ and $B_{1t+1} = B_{2t+1} = 0$ for all t, so that each household simply consumes its endowment ($C_{it} = Y_{it}$) in each period. (Hint: Among other things, you will want to use the equilibrium condition for the bond market and the consumption function for each household. In deriving this function, impose the condition $\lim_{T\to\infty} \beta^T E_t B_{it+T} = 0$.) What is the intuition for these results?

Part B

Assume now that there are complete markets in history- and date-contingent claims to consumption. What is the competitive equilibrium allocation of consumption in this case? What is the intuition for this result and for the difference relative to Part A?

Question 4

Fabio Ghironi, Short Question

Consider an economy consisting of a constant population of infinitely-lived individuals. The representative individual maximizes the expected intertemporal utility function $E_0 \sum_{t=0}^{+\infty} u(C_t)/(1+\rho)^t$, $\rho > 0$. The period utility function is $u(C_t) = C_t - \theta\, C_t^2$, $\theta > 0$. Assume that C is always in the range where $u'(C)$ is positive.

Output is linear in capital, pus an additive disturbance: $Y_t = AK_t + e_t$. There is no depreciation. Thus, $K_{t+1} = K_t + Y_t - C_t$, and the interest rate is A. Assume $A = \rho$. Finally, the disturbance follows a first-order autoregressive process: $e_t = \phi\, e_{t-1} + \varepsilon_t$, $-1 < \phi < 1$, and ε_t is a zero-mean i.i.d. shock.

(a) Find the Euler equation for consumption.

(b) Guess that consumption takes the form $C_t = \alpha_1 + \alpha_2 K_t + \alpha_3 e_t$. Given this guess, what is K_{t+1} as a function of K_t and e_t?

(c) What values must the parameters α_1, α_2, α_3 have for the first-order condition in part (a) to be satisfied for all values of K_t and e_t? (If you are running out of time, just state the conditions that the parameters α_1, α_2, α_3 should solve, without solving them.)

(d) What are the effects of a one-time shock to ε on the paths of Y, K, and C?

EC751 - May 2006 - Macro Comp: Professor Matteo Iacoviello

Question 5 (long): Money, Technology and Output

Assume the momentary utility of the representative agent is given by

$$u_t = \frac{c_t^{1-\theta}\left(\frac{M_t}{P_t}\right)^{1-b}}{1-\theta} - L_t$$

Total utility is $U = \sum_{t=0}^{\infty} \beta^t u_t$, $0 < \beta < 1$.

A representative firm owned by the household produces the final good according to $Y_t = A_t L_t^{\alpha}$ (where A_t is the level of technology and $0 < \alpha < 1$), pays labor at the real wage rate w_t, and transfers profits (if any) to the household. A central bank issues money and makes lump-sum transfers of seignorage revenues to the households. There are no bonds and prices are fully flexible. Money is the only asset that can transferred intertemporally.

1. Given the above setup, write down the household flow of funds.
2. Derive the linearized first order condition for household's labor supply.
3. Derive the linearized first order condition for firm's labor demand.
4. Express equilibrium output as a loglinear function of real money balances $m_t \equiv M_t/P_t$ and of the technology parameter A_t according to:

   $$\widehat{Y}_t = e_m \widehat{m}_t + e_A \widehat{A}_t$$

   where e_m and e_A are functions of the model parameters (α, b, θ).
5. Using the expression derived in part 4 above, discuss under which conditions and why money can affect output in this model.
6. Suppose there is a constant money growth supply rule, so that $M_t/M_{t-1} = \gamma$ for all t. Assume there is a steady state in which consumption and real balances are constant and positive. Calculate the steady state inflation rate and the steady state ratio of real balances to consumption.

Question 6 (short): An economy with two agents and durables

Suppose there are two agents, indexed by i. Agent i lifetime utility is given by:

$$u_i = \max E_0 \sum_{t=0}^{\infty} \beta_i^t \left(\log c_{it} + j \log h_{it}\right)$$

Each agent has an endowment of a good y_{it}, which can be either converted into durables h_{it} (which depreciate at rate δ) or into non-durable consumption c_{it}. Agents may have different discount factors.

Absent markets, the individual constraint is

$$y_{it} = c_{it} + h_{it} - (1-\delta) h_{it-1}$$

The process for endowment for agent i is given by:

$$y_{it} \sim iid, \; corr\left(y_{1t}, y_{2t}\right) = 0$$

Solve for the allocation that would be chosen by a social planner who pools together all the resources of the economy and maximizes a welfare criterion with equal weights on the two agents.

1. Will c_{1t} and c_{2t} be equal? Why?

2. Will h_{1t} and h_{2t} be equal? Why?

BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
September 2, 2005

Directions: There are six questions. Please follow the instructions for each question carefully. Write the answer to each part in separate bluebooks.

Write your alias, question number(s) on the front of each blue book.

Please read the entire exam before writing anything.

Question 1

Assume that the production function is (Mankiw, Romer and Weill (1992)):

$$Y = K^{\alpha} H^{\lambda} (AL)^{1-\alpha-\lambda}$$

with:

$$\alpha > 0, \lambda > 0, \alpha + \lambda < 1$$

where Y is output, K physical capital, H human capital, and A the level of technology. L and A grow at the constant rate n and x respectively. Both types of capital depreciate at the rate δ. Assume that gross investment in physical capital is a fraction s_K of output and gross investment in human capital a fraction s_H of output.

a) Derive the laws of motion for physical and human capital (each per unit of effective labor).

b) What are the steady state values of physical capital, human capital and output (all per units of effective labor)?

c) Can the introduction of human capital help in explaining large cross country differences in income, compared to the Solow model?

Question 2

a) Assume infinitely lived households maximize intertemporal utility:

$$U = \int_0^{\infty} \exp[-\rho t] \frac{c^{1-\vartheta}}{1-\vartheta} dt$$

subject to the constraint:

$$\dot{a} = ra + w - c$$

c denotes consumption per person, ρ the subjective rate of discount, a assets per person, w the wage rate, and r the real rate of interest. Population is assumed constant. The standard condition to rule out chain-letter debt finance also holds.

What are the first order conditions for an optimum? Use them to obtain the Euler equation for consumption.

b) (Congestion Model of Government Services) Assume that for each firm i the production function depends upon public goods in the following way:

$$Y_i = AK_i \, h(G/Y)$$

where Y_i denotes output, K_i capital and G public goods. Y is total output. Assume $h' > 0, h'' < 0$. This captures the idea that government services are in input to production, but, for a given G, the quantity available to individual firms decline as others use the congested facility. The government runs a balanced budget and finances public goods with a proportional tax on aggregate output, i.e.:

$$G = \tau Y$$

The firm maximizes after tax profits that can be written as:

$$(1-\tau)AK_i \, h(G/Y) - (r+\delta)K_i$$

(i) Obtain the first order conditions for K_i.

(ii) Use the first order condition for K_i, the production function, the budget constraint and the Euler equation for consumption to obtain an expression for consumption growth as a function of τ, A, δ, ϑ, and ρ.

(iii) Describe the effect of public goods, as a share of output, on growth, analytically and graphically. Given the decisions of individual agents, what is the value of G/Y $(=\tau)$ that maximizes growth? What does this imply for $\partial Y/\partial G$?

(iv) What would be the growth rate chosen by the social planner? (remember that the resource constraint is $Y = C + G + \dot{K} + \delta K$). Would a shift to a lump sum tax be Pareto improving? (explain the intuition).?

(v) Assume the production function is now $Y_i = AK_i$, but the probability, *p*, of maintining ownership in one's output is a function of *G/Y*, so that *p=p(G/Y),* with *p'>0 and p"<0.* What is the steady state growth rate? What is the relationship of this model with the congestion model of government services?

Question #3

Fabio Ghironi, Short Question

Consider the stochastic growth model with inelastic labor supply in Campbell (JME, 1994). The representative consumer maximizes:

$$E_t \sum_{i=0}^{\infty} \beta^i u(C_{t+i,}), \; 0 < \beta < 1.$$

The law of motion for capital is:

$$K_{t+1} = (1-\delta)K_t + I_t,$$

where I_t is investment.

Output is:

$$Y_t = A_t^{\alpha} K_t^{1-\alpha}, \; 0 < \alpha < 1,$$

where A_t is exogenous technology.

The resource constraint of the economy is $Y_t = C_t + I_t$.

Finally, the gross, one-period rate of return on investment in capital is:

$$R_{t+1} \equiv (1-\alpha)\left(\frac{A_{t+1}N_{t+1}}{K_{t+1}}\right)^{\alpha} + (1-\delta).$$

Assume $u(C) = \log C$ and $\delta = 1$, so that capital depreciates fully in one period. Let $s_t \equiv I_t / Y_t$ denote the saving rate (so that $C_t = (1-s_t)Y_t$).

Show that the model can be solved analytically, without taking any approximation, and $s_t = \hat{s} = \beta(1-\alpha)$ in all periods. In particular, obtain the equilibrium process for $\log Y_t$ and show that this makes it possible to reconstruct the paths of $\log K_{t+1}$ and $\log C_t$, given assumptions on the exogenous process for $\log A_t$.

Question #4

Fabio Ghironi, Long Question

A pure endowment economy consists of two households with identical preferences but different endowments. Household i has preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_{it}),\ 0 < \beta < 1,$$

where $u(C_{it}) = u_1 C_{it} - \frac{1}{2} u_2 C_{it}^2$, with u_1, $u_2 > 0$.

Household 1 has a stochastic endowment of the consumption good governed by:

$Y_{1t+1} = Y_{1t} + \sigma \varepsilon_{t+1}$,

where $\sigma > 0$ and ε_{t+1} is an i.i.d. Gaussian process with mean zero and variance 1. Household 2 has endowment:

$Y_{2t+1} = Y_{2t} - \sigma \varepsilon_{t+1}$,

where ε_{t+1} is the same random process as for household 1.

At time t, Y_{it} is realized before consumption at t is chosen. Assume that at time 0, $Y_{10} = Y_{20}$ and that Y_{10} is strictly smaller than the bliss point u_1/u_2. Assume that there is no disposal of resources.

Part A

Suppose that asset markets are incomplete. There is only one traded asset: a one-period risk-free bond that both households can either purchase or issue. The gross rate of return on the asset between date t and date $t+1$ is R_t. Household i's budget constraint at time t is:

$C_{it} + R_t^{-1} B_{it+1} = Y_{it} + B_{it}$,

where B_{it} is the value in terms of time t consumption of household i's holdings of one-period risk-free bonds.

Assume that $B_{10} = B_{20} = 0$.

Prove *analytically* that the competitive equilibrium of the economy is such that $R_t = 1/\beta$ and $B_{1t+1} = B_{2t+1} = 0$ for all t, so that each household simply consumes its endowment ($C_{it} = Y_{it}$) in each period. (Hint: Among other things, you will want to use the equilibrium condition for the bond market and the consumption function for each household. In deriving this function, impose the condition $\lim_{T \to \infty} \beta^T E_t B_{it+T} = 0$.) What is the intuition for these results?

Part B

Assume now that there are complete markets in history- and date-contingent claims to consumption. What is the competitive equilibrium allocation of consumption in this case? What is the intuition for this result and for the difference relative to Part A?

Question 5 (long): An economy with two agents

Suppose there are two agents, indexed by i. Agent i utility is given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u_{it}$$
$$u_{it} = \log c_{it}$$

Each agent has an endowment of a non-storable commodity. The endowment for agent i is given by:

$$y_{it} = z_{it}$$
$$z_{it} \sim iid$$
$$corr\left(z_{1t}, z_{2t}\right) = 0$$

1. Solve for the consumption allocation in a competitive equilibrium with complete markets, by solving a social welfare maximization problem with equal weights on the two agents.

   What is the correlation between c_{1t} and c_{2t}?

2. Suppose now that there is no social planner, but agents can trade with each other a riskless one-period bond b_t that costs p_t and pays one unit of consumption next period. Write down agents' flow-of-funds constraint and agents' first order conditions for the choice of b_t. Describe a competitive equilibrium for this economy as best as you can.

   Intuitively, will the consumption correlation between agents be larger or smaller than the endowment correlation? Explain.

3. Suppose now that:

   $$y_{it} = z_{it} n_{it}^{\alpha}$$

   where n_{it} is labor supply of agent i. The exogenous component of output, z, is still iid and has zero correlation across agents. Assume

   $$u_{it} = \log\left(c_{it} - n_{it}\right)$$

   Solving the social planner problem like in part 1, find the optimal n_{it}.

   Does this modification in affect the prediction for the consumption correlation across the two agents? Why?

Question 6 (short): Technology shocks and output

Consider the following economy. On the demand side, output is determined by the following equation

$$y_t = E_t y_{t+1} - (r_t - E_t \pi_{t+1})$$

where y_t is output, r_t is the nominal interest rate, and π_t denotes the rate of inflation between $t-1$ and t. All variables are expressed in log deviations from their steady state mean. Inflation evolves according to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - \eta a_t)$$

where a_t is a variable that describes the evolution of technology and follows a mean zero iid process.

Suppose the central bank follows an interest rate rule of the form:

$$r_t = \phi_\pi \pi_t$$

1. Derive a closed form solution for equilibrium output and inflation as a function of the state variables of the model.

2. Show that, the larger the response of interest rates to inflation, the larger the effect of technology shocks on output and the smaller the effect of technology shocks on inflation.

3. Provide an intuition for the result in (2)

BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
May 27, 2005

Directions: There are six questions to the exam. Please follow the instructions for each part of the exam carefully. Write the answer to each part in separate bluebooks. **Write your alias, part number and question number(s) on the front of each book.** Read the entire exam before writing anything.

Question 1

a) Assume infinitely lived households maximize intertemporal utility:

$$U = \int_0^\infty \exp[-\rho t] \frac{c^{1-\vartheta}}{1-\vartheta} dt$$

subject to the constraint:

$$\dot{a} = ra + w - c$$

c denotes consumption per person, ρ the subjective rate of discount, a assets per person, w the wage rate, and r the real rate of interest. Population is assumed constant. The standard condition to rule out chain-letter debt finance also holds.

What are the first order conditions for an optimum? Use them to obtain the Euler equation for consumption.

b) Assume that for each firm i the production function is Cobb-Douglas, and depends upon public goods in the following way:

$$Y_i = AL_i^{1-\alpha} K_i^{\alpha} G^{1-\alpha}$$

where Y_i denotes output, L_i labor, K_i capital and G public goods. The government runs a balanced budget and finances public goods with a proportional tax on aggregate output, i.e.:

$$G = \tau Y$$

The firm maximizes after tax profits, that can be written as:

$$L_i \left[(1-\tau) A k_i^{\alpha} G^{1-\alpha} - w - (r+\delta) k_i \right]$$

where k_i is the capital labor ratio.

(i) Obtain the first order conditions for k_i.

(ii) Use the first order condition for k_i, the production function, the budget constraint and the Euler equation for consumption to obtain an expression for consumption growth as a function of τ and L and the parameters of the production function.

(iii) Describe the effect of public goods, as a share of output, on growth. Given the decisions of individual agents, what is the value of G/Y $(=\tau)$ that maximizes growth?

(iv) What would be the growth rate chosen by the social planner? (remember that the resource constraint is $Y = C + G + \dot{K} + \delta K$. Define it in terms of per capita variables before doing the optimization exercise). Does it equal the one generated by the market?

(v) Would your answer to the very last question under (iv) change if public spending is financed through lump sum taxes?

Question 2

Summarize the Romer/Grossman-Helpman/Aghion-Howit model in the following two equations:

$$Y = K^{1-\alpha}(AL_y)^{\alpha}$$

$$\frac{A}{A} = \delta L_A$$

where Y is output, A knowledge, K capital, L_Y labor used in the production of final output, and L_A labor used in the production of knowledge. Assume that $L_A = sL$. Assume also that the capital output rate is constant in the steady state. (So that Y is proportional to L_Y. What is the constant of proportionality?)

a) Does this model have a scale effect? Is it supported by the data?

b) Assume now the production of knowledge equation is (as in Jones (95)):

$$A = \delta L_A A^{\phi}.$$

with $\phi < 1$. Assume also that the labor force increases at a rate equal to n. Consider the steady state of this model. Do we still have a scale effect? Does government policy in the form of tax incentives to investment or R&D affect the steady state?

c) Assume that aggregate consumption (or output) is the following CES composite:

$$C = \left(\int_0^B Y_i^{1/\theta} di\right)^{\theta}$$

where B measure the variety of goods and Y_i denotes variety i of the intermediate good. $\theta > 1$ also holds. Assume that B evolves according to:

$$B = L^{\beta}$$

Assume $\beta = 1$ and that each intermediate goods are used in the same amount so that per capita consumption (output) C/L equals:

$$c = B^{\theta} y$$

where $y = Y/L$. Continue to assume that the production technology for Y is the same as before. Now the growth rate of A, g_A, depends upon research effort per variety L_A/B in the following way:

$$g_A = \delta s L / B$$

Derive the steady state growth rate of consumption (output) per capita. Discuss the implications of this model and how they differ from those of the model derived under b).

QUESTION 3

Fabio Ghironi, Short Question

This question asks you to use results from the Lucas Trees Model to prove Modigliani and Miller's (AER, 1958) result that the total value of a firm is independent of its financial structure (*i.e.*, of the particular structure of indebtedness or ownership that it issues).

Suppose that an agent starts a firm at time t with a Lucas tree as its sole asset, and then she/he immediately sells the firm to the public by issuing N number of shares and B number of bonds as follows. Each bond promises to pay off r per period, and r is chosen so that rB is less than all possible realizations of future crops from the tree y_{t+j}. After payments to bondholders, the owners of issued shares are entitled to the residual crop. Thus, the dividend of an issued share is equal to $(y_{t+j} - rB)/N$ in period $t + j$. Assume that the agent has an infinite horizon and period utility function $u(c_t)$, with the standard properties, and discount factor β such that $0 < \beta < 1$.

(a) Write the expressions for the equilibrium prices of issued bonds and shares, p_t^B and p_t^N, respectively. (You should be able to do this without having to do any math.) Briefly explain the intuition for these asset pricing formulas.

(b) Use the expressions in (a) to show that the total value of issued bonds and shares is just equal to the tree's initial value, p_t (*i.e.*, the total value of the firm does not depend on its financing structure).

(c) Briefly discuss the intuition for the result in (b). (If you want to introduce math details in the discussion as we did in class (not required), feel free to make the special assumptions that will simplify matters: $u(c_t) = ln(c_t)$, y_{t+j} i.i.d. over time so that $E(y_{t+j}) = y$, and y_{t+j}^{-1} also i.i.d. for all $j \geq 1$.)

QUESTION 4

Fabio Ghironi, Long Question

Consider the stochastic growth model in Campbell, JME, 1994. Agents choose optimally how much to consume (C) and how much labor to supply in each period (N, denote the real wage with W). The representative consumer maximizes:

$$E_t \sum_{i=0}^{+\infty} \beta^i u(C_{t+i}, N_{t+i}),$$

where $0 < \beta < 1$. The period utility function takes the form:

$$u(C_t, N_t) = \frac{\left[C_t^{\rho}(1-N_t)^{1-\rho}\right]^{1-\gamma}}{1-\gamma}, \ \gamma > 0,\ 0 < \rho < 1.$$

The law of motion for capital is:

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t .$$

Output is:

$$Y_t = (A_t N_t)^{\alpha} K_t^{1-\alpha}, \ 0 < \alpha < 1,$$

where A is technology, which grows at the constant gross rate G in steady state. (Assume $a_t = \phi a_{t-1} + \varepsilon_t$, $0 < \phi < 1$, where a is the percent deviation of technology from its steady-state path and ε is a zero-mean, homoskedastic innovation.)

Finally, the gross, one-period rate of return on investment in capital is:

$$R_{t+1} \equiv (1-\alpha)\left(\frac{A_{t+1}N_{t+1}}{K_{t+1}}\right)^{\alpha} + (1-\delta).$$

(a) Write the Bellman equation for the planner's optimization problem. What is the intuition for this equation?

(b) Obtain the first-order conditions. What is the intuition for them?

(c) Solve for the balanced growth path and log-linearize the model around it. Assume log-normality, so that $\log(E_t X_{t+1}) \approx E_t(\log X_{t+1}) + \frac{1}{2}\mathrm{var}_t(\log X_{t+1})$.

(d) Conjecture the solution of the log-linearized model. (*Do not* solve for the elasticities η_{xy} as functions of the deep parameters of the model. *Just conjecture the solution.*) Motivate your conjecture.

(e) Suppose the economy was in steady state up to and including period $t = -1$ and that there is an innovation to technology $\varepsilon_0 > 0$ at time $t = 0$. There is no other innovation in periods 1, 2, 3… Use the solution you conjectured for capital and consumption to compute the responses of these variables to the shock in periods 0, 1, and 2.

Question 5 (long): An economy with credit constraints

Consider the following economy. There are entrepreneurs and workers. Entrepreneurs hire workers and borrow from them at the interest rate R_t, subject to a borrowing constraint. They solve:

$$\max E_0 \sum_{t=0}^{\infty} \gamma^t (\ln c_t)$$
$$\text{s.t. } c_t + k_t + R_{t-1}b_{t-1} = y_t - w_t l_t + b_t + (1-\delta) k_{t-1}$$
$$\text{and } b_t \leq m k_t$$
$$\text{and } y_t = A_t k_{t-1}^{\mu} l_t^{1-\mu}$$

where k is capital that can be accumulated by entrepreneurs only that depreciates at rate δ.

The workers do not face borrowing constraints. They lend b_t to entrepreneurs and solve the following problem, where l_t' are hours worked:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln c_t' + \tau \ln \left(1 - l_t'\right)\right)$$
$$\text{s.t } c_t' - R_{t-1}b_{t-1} = w_t l_t' - b_t$$

Assume $\gamma < \beta$.

Answer the following:

1. Calculate the optimality conditions (first-order condition and market clearing conditions) for this economy. (Check that in equilibrium there are as many equations and endogenous variables). Assume productivity follows a standard stationary Markov process around a constant mean. Obtain and describe a competitive equilibrium for this economy.

2. Show that, in a neighborhood of the steady state, entrepreneurs will be borrowing constrained if $\gamma < \beta$.

3. Calculate the equilibrium, steady state division of output between (1) consumption of workers, (2) consumption of entrepreneurs and (3) investment as a function of the structural parameters of the economy. (That is, find steady state expressions for c/y, c'/y and so on).

4. Describe intuitively the effect on the economy of a persistent productivity shock.

5. Consider the problem of a central planner who wants to maximize the sum of the utilities of farmers and borrowers, given the economywide resource constraint (total consumption must equal total output). Will the social planner choose an allocation where consumption is the same for both entrepreneurs and workers? Will he choose a higher or lower capital-output ratio than in the competitive economy?

Question 6 (short): Technology and business cycles

Consider the following economy. On the demand side, output is determined by the following equation

$$y_t = E_t y_{t+1} - (r_t - E_t \pi_{t+1})$$

where y_t is log output, r_t is the nominal interest rate, and π_t denotes the rate of inflation between $t-1$ and t. Inflation evolves according to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - \overline{y}_t)$$

where $\overline{y}_t$ is the natural level of output that would prevail in absence of nominal frictions. Let the solution for output in absence of nominal frictions be:

$$\overline{y}_t = \eta a_t$$

where $\eta > 1$, and a_t is a technology parameter that follows an AR(1) process

$$a_t = \rho_a a_{t-1} + \varepsilon_t$$

Suppose the central bank follows an interest rate rule of the form:

$$r_t = \phi_\pi \pi_t + \phi_y y_t$$

1. Derive a closed form solution for equilibrium output of the form

$$y_t = \theta a_t$$

2. Is θ bigger or smaller than η? Show that, the larger nominal rigidities, the smaller the effect of technology shocks on output.

3. Provide an intuition for the result in (2)