BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
September 3, 2010

Directions: There are two parts to this exam. Please follow the instructions for each part and question carefully. Write the answer to PART I, Question 1 and 2 in one bluebook and then PART II, Questions 3 and 4 in a separate bluebook.

**Write your Alias, Part Number, Question Number(s) on the front of each blue book.**

Please read the entire exam before writing anything.

PART I

F. Schiantarelli

Question1

(The 3 parts that make up Question 1 are unrelated. All figures are in the back, after Question 2)

Part 1

a) Consider the plot in Figure 1 (Kremer, QJE, 1993). The world population growth rate is on the vertical axis and populations levels on the horizontal axis. Can you rationalize this figure (or more specifically the upward sloping portion of the figure) with a model with the following characteristics:

Output is produced with labor, $L$, and land, $T$, according to the production function:

$Y = AT^{1-\alpha}L^{\alpha}$ , with $0<a<1$

Land is fixed and normalized to 1.

Output per capita ($Y/L$) is set to the level $y^M$ (Malthusian assumption).

Technological progress depends upon the stock of labor in the following way:

$$\frac{\dot{A}}{A} = gL$$

where g is a positive constant.

b) Assume now the production of knowledge equation is:

$$\dot{A} = \delta L A^{\phi}.$$

with $0<\phi<1$. What is the relationship between population levels and population growth rate in this case? Is this model "better" in explaining the data in figure one?

c) Continue to assume that the production of knowledge equation is as in b). Assume that the population growth rate is constant and it equals $n$. What is the steady state growth rate of knowledge? Assume that there is a country in which the growth rate of TFP is either constant or increasing, while the population growth rate is decreasing. Is this consistent with the steady state of the model?

Part 2

Consider the data in Table III (Jones, QJE, 1995) and focus on the figures for producer durables investment as a share of GDP for the US. Consider also the growth rate of per capita GDP in the US in the post War II period in Figure II (solid line). Discuss whether these data lend support (or not) to an AK model with production function:

$$Y = AK$$

And capital accumulation equation:

$$\dot{K} = sY - \delta K$$

With $s$ constant.

Part 3

Assume log income per capita in country $i$ in period $t$ ($log\, y_{it}$) depends upon the quality of institutions ($R_{it}$). In turn institutions depends upon the level of development summarized by log $y_{it}$ and a set of other exogenous variables $Z_{it}$ so that:

$$\log y_{it} = a + b * R_{it} + \varepsilon_{it}$$
$$R_{it} = c + d * \log y_{it} + f * Z_{it} + \eta_{it}$$

a) Describe why using Ordinary Least Squares would not provide you with the right answer about the causal effect of institutions on economic performance and suggest a way to address this issue

F. Schiantarelli

Question 2

a) Consider a Ramsey-Cass-Koopmans model with zero technical progress and population growth rate equal to $n$. Infinitely lived households maximize intertemporal utility:

$$U = \int_0^{\infty} \exp[-(\rho - n)t] \frac{c^{1-\vartheta}}{1-\vartheta} dt$$

subject to the constraint:

$$\dot{a} = (r - n)a + w - c$$

c denotes consumption per person, $\rho$ the subjective rate of discount, a assets per person, w the wage rate, r the real rate of interest . The standard condition to rule out chain-letter debt finance also holds.

a) Obtain the Euler equation for consumption and use it in conjunction with the optimality conditions for the firm and the resource constraint for this economy to obtain the steady state for this model. Illustrate its dynamics, using the appropriate phase diagram.

b) Assume that, while the economy is on the balanced growth path, the government at time $t_0$ starts taxing investment income at a rate $\tau$ (Hint: what is now the relationship between the interest rate and the marginal product of capital?). Assume this change is permanent and that this change in policy is un-anticipated. Assume also that the government returns the revenue it collects from this tax through lump sum transfers. How does the tax affect the $c = 0$ and the $k = 0$ locus? How does the economy respond to the adoption of the tax (describe the changes at time $t_0$ and the path to the new steady state. Use the phase diagram).

c) How would your answer to b) change if at time $t_0$, the introduction of the (permanent) tax on investment income is anticipated to take place at a future time $t_1$?

d) Analyze the following two further variations to problems b) and c).

(i) Assume that at time $t_0$ the government announces that it will tax investment income at rate $\tau$ from time $t_0$ to time $t_1$. After $t_1$ _investment income will not be taxed.
(ii) Assume that at time $t_0$ the government announces that it will tax investment income at rate $\tau$ from time some future time $t_1$ to time $t_2$ . Before $t_1$ and after $t_2$ investment income will not be taxed.

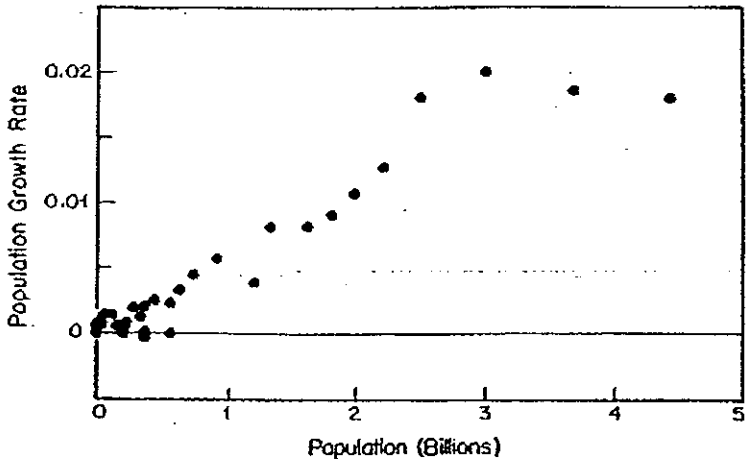


FIGURE I
Population Growth Versus Population

TABLE III
AVERAGE INVESTMENT SHARES OF GDP (PERCENT)

| | France | Germany | Japan | United Kingdom | United States |
|---|---|---|---|---|---|
| Total investment | | | | | |
| 1950—1954 | 18.4 | 26.1 | 16.1 | 12.1 | 16.5 |
| 1955—1959 | 20.8 | 29.2 | 19.0 | 14.3 | 16.0 |
| 1960—1964 | 24.0 | 30.3 | 26.8 | 16.7 | 15.7 |
| 1965—1969 | 26.9 | 29.5 | 30.7 | 18.9 | 16.9 |
| 1970—1974 | 29.5 | 28.7 | 36.5 | 19.6 | 17.2 |
| 1975—1979 | 26.4 | 24.7 | 32.5 | 18.7 | 17.4 |
| 1980—1984 | 24.2 | 23.9 | 29.4 | 16.2 | 17.3 |
| 1985—1988 | 23.7 | 23.6 | 29.6 | 18.8 | 18.1 |
| Producer durables investment | | | | | |
| 1950—1954 | 4.3 | 4.8 | 3.4 | 4.8 | 4.4 |
| 1955—1959 | 5.1 | 5.5 | 3.8 | 5.5 | 4.3 |
| 1960—1964 | 6.3 | 6.8 | 5.6 | 6.0 | 4.2 |
| 1965—1969 | 6.9 | 6.9 | 6.0 | 6.6 | 5.2 |
| 1970—1974 | 8.1 | 7.8 | 7.4 | 6.9 | 5.4 |
| 1975—1979 | 8.0 | 7.3 | 6.4 | 6.9 | 5.9 |
| 1980—1984 | 7.9 | 7.6 | 7.5 | 6.6 | 6.2 |
| 1985—1988 | 8.0 | 8.1 | 9.8 | 7.5 | 7.2 |

*Source.* Summers and Heston [1991] and unpublished data courtesy of Robert Summers.

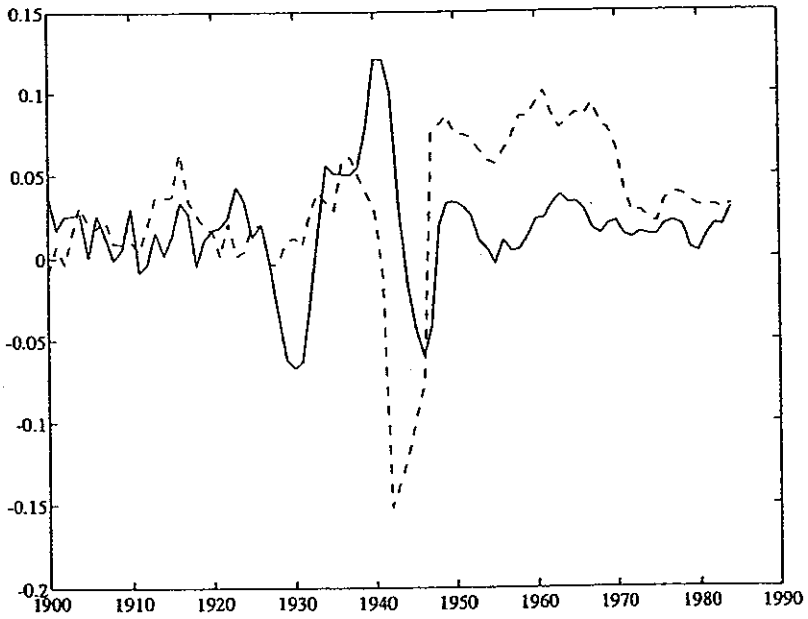


FIGURE II
Annual Growth Rates for the United States (solid) and Japan (dashed), 1900–1987

*Source.* Five-year moving averages are plotted. The data are from Maddison [1982, 1989] as compiled by Bernard [1991].

PART II

**Question 3:**
In much popular writing about the stock market, stock prices are said to follow a random walk (or martingale). The intuition is that changes in stock prices must be unpredictable, because any expected change in prices will be arbitraged away. This question asks you to investigate whether this intuition is consistent with formal models of asset pricing.

Suppose there is a fixed amount of capital in the economy, $\bar{K}$. There is no investment or depreciation. The price of one unit of capital (the stock price) is $P_t$. Output depends on stochastic technology and the fixed capital input:

$$Y_t = Z_t \bar{K}.$$

Output is distributed to consumers depending on their ownership of the capital stock (think of this distribution as "dividends"). A person who owns $K_t$ units of capital receives

$$D_t = Y_t \frac{K_t}{\bar{K}}$$

units of output as dividends. People must choose the amount of capital they hold one period before dividends are paid, so holdings of $K_{t+1}$ are decided at time $t$. (Any sale of capital takes effect after the dividend for that period is received by the original owner.)

Output is used only for consumption: $Y_t = C_t$.

A continuum of identical consumers maximize the present value of utility, given by

$$V = E_t \sum_{s=0}^{\infty} \frac{U(C_{t+s})}{(1+\rho)^s},$$

subject to a standard budget constraint.

(a) Suppose a consumer buys one tiny unit of capital at time $t$, receives the dividend at time $t+1$, sells the capital, and consumes both the dividend and the proceeds from the sale at time $t+1$. Along the optimal path, what is the expected benefit to the consumer from taking this action? Use your answer to derive an expression for the price of capital today, $P_t$.

(b) Suppose that $U(C) = \ln(C)$. Under this assumption, what is the price of capital today as a function of current and expected future dividends only? (Eliminate any future prices from your solution. You may assume a standard no-bubble condition if necessary.)

(c) Does your answer in part (b) prove that stock prices always follow a random walk or martingale? Explain why or why not. Give economic intuition for your answer.

(d) Suppose technology $Z_t$ does NOT follow a random walk. Are there any conditions under which the stock price will still follow a random walk/martingale? Either state conditions that you believe are sufficient, and show that they lead to the desired result. Or show that such conditions do not exist—that is, in this model, stock prices cannot follow a random walk unless $Z_t$ follows a random walk. (For this part of the question, you do not have to assume that $U(C) = \ln(C)$.)

**Question 4:**

In one version of the New Keynesian model with Calvo price rigidity, the time paths of output and the price level can be determined from the following four equations:

$$y_t = m_t - p_t \tag{1}$$

$$\mu_t = -\phi y_t \tag{2}$$

$$p_t - p_{t-1} \equiv \pi_t = \beta E_t \pi_{t+1} - \gamma \mu_t \tag{3}$$

$$m_t = \rho m_{t-1} + \varepsilon_t \tag{4}$$

$y$ is output, $m$ is the money supply, $p$ is the price level, $\pi$ is the inflation rate, and $\mu$ is the markup of price over marginal cost. All variables are in log deviations from steady state. The steady-state inflation rate is zero. $\phi$, $\beta$, $\gamma$ and $\rho$ are all positive parameters. In addition, assume that $\beta \in (0,1)$ and $\rho \in (0,1)$. $\varepsilon$ is a mean-zero, iid error term.

a) Briefly explain the assumptions underlying each equation. What are the assumptions that allow us to derive equations (2) and (3), and why are they thought to be central to New Keynesian economics?

b) Consumption is one major component of output, and consumption growth depends on the real interest rate (via the consumer's Euler equation). So how can this model solve for output without any explicit solution for the real interest rate? Explain the economics at work.

c) Suppose there is a 1 percent shock to $\varepsilon$ at time $t = 0$. What are the predicted time paths of output, the price level, and the inflation rate?

If you cannot solve the model formally, make your verbal explanation as tight and clear as possible. Explain why your proposed time paths are consistent with the model equations.

d) Nekarda and Ramey (2010) argue that in US data one often cannot reject the hypothesis that $\phi = 0$. Suppose it is indeed true that $\phi = 0$. How does this change your solution to part (c)?

e) Nekarda and Ramey (2010) suggest that the lack of evidence proving $\phi > 0$ is a major challenge to New Keynesian Economics (NKE). Suppose they had instead found that $\phi$ is definitely much bigger than zero. Which result would be worse for NKE? Why?

BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
May 28, 2010

Directions: There are two parts. Please follow the instructions for each part carefully. Write the answer to Part 1 and 2 in a separate bluebooks.

**Write your alias, Part number, question number(s) on the front of each blue book.**

Please read the entire exam before writing anything.

NO QUESTION 1 & 2 (Schiantarelli)
Made up his questions night before & added to exam on his own

Question 3:

In a classic paper in the 1999 *American Economic Review*, Jordi Galì proposed a method for estimating the effects of "technology" and "non-technology" shocks without writing down a specific economic model. He argued that this method would work regardless of the model describing the economy, as long as the model satisfied two properties: (1) that technology shocks *would* have a permanent effect on **labor productivity** (output per hour worked), and (2) that non-technology shocks *would not* have a permanent effect on labor productivity.

Assess Galì's method of identifying supply and demand shocks using a benchmark RBC model.

Note: This is <u>not</u> the question from last year's comp. Read and answer the question as written.

(a) Begin by writing down a benchmark RBC model. Your model must have the following properties:

(i) A representative consumer who maximizes discounted utility from consumption and leisure over an infinite horizon. Assume that each period the utility function is $\ln(C_t)+\ln(\bar{H}-H_t)$

(ii) A representative firm that is perfectly competitive and maximizes profits, with the production function $Y_t = K_t^{\alpha}(Z_t H_t)^{1-\alpha}$

(iii) Government purchases, financed by lump-sum taxes.

(iv) A closed economy with capital accumulation, so $Y_t = C_t + I_t + G_t$.

(v) Variable technology (i.e., $Z$ changing over time).

Write down the consumer and firm optimization problems and the equilibrium conditions for the economy. Derive the key first-order conditions.

(b) Solve for the steady-state levels of $Y$ and $H$ in terms of steady-state $Z^*$, $G^*$ and exogenous parameters.

(c) Suppose there is a permanent shock to technology. Will this shock have a permanent effect on the level of labor productivity ($Y/H$)? (Interpret "permanent effect" as a change in the steady-state level.) Explain your answer and the economics at work thoroughly and rigorously.

(d) Suppose there is a permanent shock to government purchases. Will this shock have a permanent effect on labor productivity? Explain your answer and the economics at work thoroughly and rigorously.

(e) Call your model of part (a), driven by permanent technology and government purchase shocks, the *baseline model*.

If the baseline model does satisfy Galì's identifying assumptions, then suggest some modifications to the model or the assumptions about the shock processes that would cause the model to fail to identify technology shocks properly. (When proposing modifications, feel free to introduce new variables and shocks to those variables.)

If the baseline model does not satisfy Galì's assumptions, then suggest some modifications to the model or assumptions about the shock processes that would make the model predictions *consistent* with his identifying assumptions.

Question 4:

Suppose a small, open economy where many identical consumers maximize an intertemporal utility function that depends only on consumption, $C$:

$$E_t \sum_{i=0}^{\infty} \beta^i \ln\left(C_{t+i}\right).$$

Consumers can invest in two assets. The first is internationally traded bonds, $B$, which have a fixed rate of return $r$. The other asset is domestic capital, $K$. The consumer's budget constraint is:

$$B_{t+1} + K_{t+1} + \frac{\kappa}{2}\left(K_{t+1} - K_t\right)^2 + C_t = W_t\bar{H} + \left(R_t + (1-\delta)\right)K_t + \left(1+r\right)B_t + \Pi_t.$$

$R$ is the rental rate of capital, $\delta$ is the depreciation rate, $W$ is the real wage, and $\Pi$ is economic profit (if any). The consumer inelastically supplies $\bar{H}$ units of labor. $(\kappa/2)\Delta K_{t+1}^2$ is an adjustment cost to changing the capital stock. Assume that $\beta(1+r)=1$. The consumer must satisfy a no-Ponzi-game condition:

$$\lim_{t\to\infty} \frac{B_t}{\left(1+r\right)^t} = 0.$$

Firms have constant-returns Cobb-Douglas production functions:

$$Y_t = K_t^{\alpha}(Z_t\bar{H})^{1-\alpha}.$$

a) Derive the two Euler equations for investing in bonds, $B$, and capital, $K$.

b) Assume that future consumption is uncertain, because of shocks to $Z$. Will consumption follow a "random walk" (technically, a martingale) in this economy? Up to a first-order, log-linear approximation, will consumption follow a random walk (martingale)? Explain.

For the parts (c) and (d), assume that the log deviation of $Z$ from the steady state, $\hat{Z}_t$, follows the AR(1) process $\hat{Z}_t = \rho \hat{Z}_{t-1} + \varepsilon_t$, where $1 > \rho > 0$.

c) Assume that $\kappa = 0$. Suppose there is a 1 percent positive shock to $\hat{Z}_t$. What will be the time paths of $Y$, $K$, $B$ and $C$ following the shock? (A qualitative answer is sufficient.) What is the economic reasoning behind your answer?

d) Assume that $\kappa > 0$. Suppose there is a 1 percent positive shock to $\hat{Z}_t$. What will be the time paths of $Y$, $K$, $B$ and $C$ following the shock? (A qualitative answer is sufficient.) How do these time paths compare to your answers in part (c)?

BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
September 4, 2009

Directions: There are three parts to this exam and each part has two questions. Please follow the instructions for each question carefully. Please use a separate blue book for each part.

**Write your alias, part number, question number(s) on the front of each blue book.**

Please read the entire exam before writing anything.

PART 1

Fabio Schiantarelli

**Question 1**

Consider a standard Diamond OG model with production. Assume the utility function is logarithmic and the production function Cobb-Douglas. The utility function is, therefore:

$$U_t = \log C_{1t} + \frac{1}{1+\rho} \log C_{2t+1}$$

Output per unit of effective labor, $f(k_t)$ is:

$$f(k_t) = Bk_t^{\alpha}$$

where $k_t = K_t / A_t L_t$. $A_t L_t$ is labor in efficiency units and $K_t$ is capital. Remember that the budget constraint for the individual is:

$$A_t w_t = C_{1t} + \frac{1}{1+r_{t+1}} C_{2t+1}$$

where $w$ is the real wage per unit of effective labor and $r_{t+1}$ the interest rate.

Since the capital stock in period t+1 equals the saving of the young, the accumulation equation for capital is:

$$K_{t+1} = s_t L_t A_t w_t$$

where $s_t$ denotes the saving rate. $L_t$ and $A_t$ evolve according to:

$$L_t = (1+n) L_{t-1}$$
$$A_t = (1+g) A_{t-1}$$

a) Derive $C_{1t}$ as a function of the parameters of the utility function, of the rate of interest, $r_{t+1}$, and of $A_t w_t$.

b) Use the result in a) and the FOC's for capital (assume zero depreciation rate, $\delta = 0$) and labor to derive the equation of motion for $k_{t+1} = K_{t+1} / L_{t+1} A_{t+1}$ and plot it in a graph with $k_{t+1}$ on the vertical axis and $k_t$ on the horizontal axis. Use the graph to illustrate that the equilibrium is unique and globally stable.

c) Derive the steady state level of k, k*, and show what effect a fall in the discount rate, $\rho$, has on k* (use also the graph to illustrate the results).

d) Assume now that g=o and recall that the Golden Rule level of capital, $k_{GR}$ (given that g=0 and $\delta = 0$) satisfies f'($k_{GR}$) = n. Can this economy be dynamically inefficient (k*> $k_{GR}$)?

**Question 2**

Suppose that output, $Y$, is produced according to the following production function:

$$Y = K^{\alpha} A^{1-\alpha}$$

where $K$ is capital and $A$ is knowledge. The capital accumulation equation is $\dot{K} = sY$, where $s$ is the saving rate. Knowledge accumulation is a byproduct of production, i.e. $\dot{A} = BY$, where $B$ is a positive constant.

a) Find expressions for the growth rate of A, $\gamma_A$, and for the growth rate of $K$, $\gamma_K$, as a function of A/K.

b) (After taking logs) Show how the growth rate of $\gamma_A$ and $\gamma_K$ ($\dot{\gamma}_A/\gamma_A, \dot{\gamma}_K/\gamma_K$) depend upon the growth rate of K and A ($\gamma_A$ and $\gamma_K$).

c) Sketch the $\dot{\gamma}_K = 0$ and the $\dot{\gamma}_A = 0$ loci in a graph that has $\gamma_K$ on the vertical axis and $\gamma_A$ on the horizontal axis.

d) Show that the economy converges to a balanced growth path in which capital knowledge and output grow at a common growth rate $\gamma$. Calculate this growth rate and discuss how it depends upon the saving rate, s, and B (hint: use the equations derived under a) to obtain a relationship between $\gamma_A$ and $\gamma_K$. This provides another locus in the graph to pin down the steady state).

Question 3 (Long):

In a classic paper in the 1989 *American Economic Review*, Blanchard and Quah proposed a method for estimating the effects of "supply" and "demand" shocks without writing down a specific economic model. They argued that their method would work regardless of the model describing the economy, as long as the model satisfied two properties: (1) that supply shocks *would* have a permanent effect on output, and (2) that demand shocks *would not* have a permanent effect on output.

Assess Blanchard and Quah's method of identifying supply and demand shocks using a benchmark RBC model. For the purposes of this question, interpret "supply" shocks as technology shocks, and "demand" shocks as shocks to government purchases.

(a) Begin by writing down a benchmark RBC model. Your model must have the following properties:
(i) A representative consumer who maximizes discounted utility from consumption and leisure over an infinite horizon. Assume that each period the utility function is $\ln(C_t)+\ln(\bar{L}-L_t)$
(ii) A representative firm that is perfectly competitive, produces with constant returns to scale in capital and labor, and maximizes profits.
(iii) Capital accumulation.
(iv) Government purchases, financed by lump-sum taxes.
(v) Variable technology.

Write down the consumer and firm optimization problems and the equilibrium condition(s) for the economy. Derive the key first-order conditions.

(b) Suppose there is a permanent shock to technology. Will this shock have a permanent effect on output? Explain your answer and the economics at work as thoroughly and rigorously as you can.

(c) Suppose there is a permanent shock to government purchases. Will this shock have a permanent effect on output? Explain your answer and the economics at work as thoroughly and rigorously as you can.

(d) Does the model you wrote down in part (a), if driven by permanent technology and government purchase shocks, satisfy the two identifying assumptions made by Blanchard and Quah? (Try this, even if you could not do some of (b) or (c).) Explain your answer.

(e) If the model satisfies the Blanchard-Quah assumptions, then suggest some modifications to the model or assumptions about the shock processes that would cause it to *violate* their identifying assumptions. If it does not satisfy their assumptions, then suggest some modifications to the model or assumptions about the shock processes that would make the model predictions *consistent* with the identifying assumptions.

Question 4 (short):

Suppose a government wants to raise a specified amount of revenue while causing the least economic distortion possible. The government does not have access to lump-sum taxes, so it must use distortionary taxes. The government's problem is:

$$\underset{\{T_{t+s}\}}{Min}\ E_t \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} Y_{t+s} f\left(\frac{T_{t+s}}{Y_{t+s}}\right)$$

subject to

$$\sum_{s=0}^{\infty} \frac{1}{(1+r)^s} T_{t+s} = D_t + \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} G_{t+s}$$

where output, $Y$, government purchases, $G$, and the initial value of government debt, $D_t$, are exogenously given. The function $f$ represents the distortions from raising tax revenues, and satisfies $f(0)=0, f'>0, f''>0$. The expectations operator is needed because future output and government purchases are not known with certainty. $r>0$ is a constant.

a) Derive the dynamic first-order condition (Euler equation) for this problem. (Your equation should relate some function of $T_t$ to some function of $T_{t+1}$, with any other variables or expectations that are necessary.)

b) Now assume that the function $f$ is quadratic: $f(x)=x^2$. Insert this assumption into your Euler equation from part (a). What is the expected time path for the tax rate, *T/Y*? Why does the time path for the tax rate not depend on anything that is known about the time path of government purchases?

c) Suppose that $G_t$ unexpectedly increases to $\tilde{G}_t > G_t$, but it is known that this change will last for only one period, after which we go back to the previous time path for $G$. What is the predicted effect of this change in $G$ on the time path of *T/Y*? What is the predicted effect on the time path of government debt?

PART 3

EC751 September 2009 - Macro Comp: Professor Matteo Iacoviello

# Question 5: Money in Utillity.

Assume the household maximizes following utility function:

$$U = \sum_{s=t}^{\infty} \beta^{s-t} \left( \theta \ln C_s + (1-\theta) \ln \frac{M_s}{P_s} \right)$$

subject to the following constraints in each period

$$P_t C_t + P_t T_t + M_t + B_t = (1 + I_{t-1}) B_{t-1} + M_{t-1} + P_t Y_t$$

where $Y_t$ is an endowment of goods, $T_t$ are taxes in real terms, $C_t$ is consumption, $B_t$ are government bonds, $M_t$ is money, and $G_t$ is government spending. The government constraint is given by

$$P_t G_t + (1 + I_{t-1}) B_{t-1} + M_{t-1} = P_t T_t + M_t + B_t.$$

1. Assume $Y_t$ is exogenous and fixed, that is $Y_t = Y$ for all $t$. Assume government spending is $g\%$ of total output, that is $G_t = gY_t$.

2. Find the household's first order conditions for optimization. Interpret them.

3. Suppose that the government chooses fiscal and monetary policies to guarantee that:

$$\begin{aligned} M_t &= \overline{M} \\ B_t &= 0 \end{aligned}$$

   at all times. Show that the model exhibits a unique solution for the price level (as a function of the model exogenous parameters or variables). Interpret this equation.

4. Suppose that government spending exogenously rises as a fraction of total output. Assume no change in government debt policy or in money supply. Derive and interpret the implications for consumption, the price level and inflation of such a policy change.

# Question 6: Central Bank and Inflation Volatility

It is argued that a central bank that responds strongly to current inflation can reduce the volatility of inflation itself, but at the cost (possibly) of higher volatility of output. Use the simple three equation new-keynesian model (where the variables are output $y_t$, the nominal interest rate $r_t$ and inflation $\pi_t$) to discuss this claim. Assume the economy is hit by two types of shocks, an i.i.d. demand shock and an i.i.d. cost-push shock.

Note: Assume that a strong response to inflation is a statement about the response coefficient $\phi$ to inflation in the interest rate rule.

BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
May 22, 2009

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# PART 1

Question 1

Assume infinitely lived households maximize intertemporal utility:

$$U = \int_0^{\infty} \exp[-(\rho - n)t] \frac{c^{1-\vartheta}}{1-\vartheta} dt$$

subject to the constraint:

$$\dot{a} = (r - n)a + w - c$$

c denotes consumption per person, $\rho$ the subjective rate of discount, $a$ assets per person, $w$ the real wage rate, $r$ the real rate of interest and $n$ population growth. The standard condition to rule out chain-letter debt finance also holds.

Assume that the production function is Cobb Douglas with constant return to scale in physical and human capital:

$$Y = K^{\alpha}(H)^{1-\alpha}$$

with: $0 < \alpha < 1$

where $Y$ is output, $K$ physical capital, $H$ human capital that depreciate at rate $\delta_K$ and $\delta_H$ respectively.

a) Show that using the first order conditions for profit maximization (denote with $r + \delta_K$ and $r + \delta_H$ the rental rates of $K$ and $H$), one can show that the production function is of the $AK$ type, where $A$ is a function of the (constant) $H/K$ ratio.

b) Using the Euler equation for consumption (please derive) show that the model displays endogenous growth (derive the steady state growth rate for consumption).

c) Show that this growth rate coincides with the one chosen by the central planner (Hint: write the resource constraint as: $\dot{k} = (A - \delta - n)k - c$)

d) Consider now the learning by doing (investing) model, where the productivity of labor, $L_i$, depends upon the aggregate capital stock, $K$, in the following way:

$$Y_i = K_i^{\alpha}(KL_i)^{1-\alpha}$$

_where the index $i$ denotes individual firms'. Derive the steady state growth rate in this model, assuming now that total employment, $L$ is fixed. (Hint: Note that all firms are identical and that in equilibrium they will have the same capital labor ratio $k_i=k$. Note also that $k=K/L$.. Use this in the expression for the marginal product of capital, from the point of view of the individual firm, to show that it depends only upon $L$)

e) Explain why the social planner would choose a higher growth rate (At a minimum, use a verbal argument. If you can, it is better to demonstrate this formally. Hint: what is the average product of capital, as a function of $L$? What is the resource constraint from the point of view of the planner?) How can the planner get the market to replicate the optimal solution?

Question 2

Assume the firm value is given by:

$$V_t(K_t,\xi_t) = \underset{I}{Max}\{D_t + E_t\left[\sum_{s=1}^{\infty}\beta^s D_{t+s}\right]\}$$

Subject to:

$$D_t = \Pi(K_t,\xi_t) - \frac{b}{2}\left(\frac{I_t}{K_t}\right)^2 K_t - I_t$$

$$K_{t+1} = (1-\delta)K_t + I_t$$

$$D_t \geq 0$$

Where $D_t$ are dividends, $K_t$ is the beginning of period capital stock, $I_t$ investment, $\xi_t$ a stochastic shock, $\delta$ the depreciation rate, and $\beta$ the discount rate. $\Pi(K_t,\xi_t)$ is the profit function, while $\frac{b}{2}\left(\frac{I_t}{K_t}\right)^2 K_t$ are adjustment costs.

a) From the FOC's for this problem, derive the Euler equation for the capital stock. Do you have any suggestion on how one should estimate it?

b) Assume that $\Pi(K_t, \xi_t) = \Pi(\xi_t)K_t$. Show how you can derive a Q model of investment in which the investment rate, $\frac{I_t}{K_t}$, is related to the market value of the firm relative to the replacement cost of capital, $\frac{V_{t+1}}{K_{t+1}}$.

Question 3:

Suppose an economy where many identical consumers maximize an intertemporal utility function of consumption, $C$, and labor supply, $N$:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \ln\left(C_{t+i}\right) + \ln\left(\bar{N} - N_{t+i}\right) \right].$$

Consumers are subject to a series of budget constraints:

$$K_{t+1} + A_{t+1} = W_t N_t + \left(1 + r_t^f\right) A_t + R_t K_t + \Pi_t - C_t .$$

$K$ is physical capital; $A$ is a private, risk-free bond (issued by households to other households); $r^f$ is the risk-free interest rate; $R$ is the rental rate of capital; $W$ is the marginal product of labor; and $\Pi$ is economic profit (if any). Capital does not depreciate:

$$K_{t+1} = K_t + I_t .$$

All production is done by a representative, competitive firm with the technology:

$$Y_t = K_t^{\alpha} (Z_t N_t)^{1-\alpha},$$

where $Z$ is a stochastic technology parameter:

$$Z_t = \rho Z_{t-1} + \varepsilon_t \quad 0 < \rho < 1,$$

and $\varepsilon$ is distributed $N(0, \sigma^2)$.

Output is consumed or invested (there are no government purchases):

$$Y_t = C_t + I_t .$$

(a) What is the Euler equation (dynamic first-order condition) for holding capital?

(b) What is the Euler equation for holding risk-free bonds, $A$?

(c) Solve for an equation expressing $r^f$ in terms of $R$ and any other relevant variables.

(d) Define an equity premium as a situation where $E(R) > E(r^f)$ (note these are unconditional expections). Argue that there is an equity premium in this model. Why is this the case? (You should not try to solve explicitly for the size of the premium. Just argue clearly why there will be one.)

(e) Why must $A = 0$ in equilibrium? Given this fact, what is the interpretation of $r^f$?

Now suppose the government notices that there is an equity premium, and comes up with the following plan. The government will make a one-time issue of risk-free real government bonds, $B$, and use the proceeds to buy some capital, $K^G$, which it will then rent out to firms on the capital market. The government will take the difference between what it earns in capital income and what it spends paying interest on its bonds, and will rebate that amount lump-sum to households. Call the lump-sum rebate $T$.

The government will keep its debt $B$ and capital holdings $K^G$ constant over time.

Given an equity premium, on average the government will earn more from renting out capital to firms than it has to pay in interest to bond-holders. Thus, on average, the lump-sum transfers will be positive. The government is happy, since it believes it has just made its citizens better off by giving them subsidies rather than taxes.

(f) Write down the law of motion for government debt, relating $B_{t+1}$ to $B_t$, government capital income, interest payments on the debt, and lump-sum transfers. (Government purchases are still zero at all dates.)

(g) Use the fact that debt and capital holdings are constant over time to solve for the size of the lump-sum transfer in each period.

(h) Is this scheme solvent in an intertemporal sense (and therefore something feasible for the government to do)? That is, under this scheme, is the present value of government assets equal to or greater than the present value of its liabilities? Write down the government balance sheet as part of your answer.

(i) Now write down the representative household's budget constraint under this scheme. Use the notation that private capital is called $K^P$. (You can assume from the beginning that private bonds $A = 0$.) Let total capital in the economy be $K$, where $K = K^G + K^P$.

(j) Can this government scheme make consumers better off? Why or why not? Make as rigorous an argument as you can.

(k) Can this government scheme make consumers worse off? Why or why not?

Question 4:

In the current crisis, there is a lot of interest in figuring out whether increases in government purchases will also raise private consumption (thus implying a larger impact of government spending on output). This question asks you to think about this issue using simple models.

a) Take a standard, dynamic RBC model without investment; the capital stock is fixed for all time. Assume that government purchases are pure waste ("goods thrown in the ocean"), and are financed by lump-sum taxes. Show that in this setting, under normal assumptions, private consumption will decline if government purchases increase.

b) Now suggest a modification to this model that would predict the opposite result: an increase in government purchases will increase private consumption. You don't have to write down all the details of the model, but be precise about the parts of the model you are changing. State the economic intuition explaining why your changes will have the desired effect.

PART 3

EC751 May 2009 - Macro Comp: Professor Matteo Iacoviello

# Question 5: An Agent Living for two Periods.

Consider an agent living for two periods, with preferences over consumption and leisure given by the following utility function:

$$u = \ln c_1 + \gamma \ln c_2$$

and subject to the following constraints in each period

$$\begin{aligned} c_1 &= w_1 l_1 + a + b_1 \\ c_2 &= w_2 l_2 - R b_1 \end{aligned}$$

where $\gamma$ is the discount factor, $a$ is the agent's initial wealth, $b_1$ is first period's borrowing (saving if negative), $R$ is the gross interest rate, $l$ are hours worked, $w_1$ and $w_2$ is the wage in each period. Assume $l_1 = l_2 = 1$.

1. Assume that the agent can borrow in time 1 at the interest rate $R$, subject to the constraint that $b_1 < \bar{b}$. Taking the other parameters as given, show that if $\gamma$ is "low enough", the borrowing constraint will be binding in the first period.

2. Suppose $\gamma$ is low enough so that the borrowing constraint holds in period 1. Show what happens to consumption in period 1 and 2 following a "small" relaxation of the borrowing constraint that increases $\bar{b}$.

3. Suppose now that labor supply is endogenous, that is, utility takes the form

$$u = \ln c_1 + \gamma \ln c_2 + \tau \ln \left(\bar{l} - l_1\right) + \tau\gamma \ln \left(\bar{l} - l_2\right).$$

   where $\bar{l}$ is the endowment of time. Solve for consumption and labor supply in each period under the assumption that the borrowing constraint does not bind. Show that, in response to a wage increase in period 1 ($w_1$ rises), labor supply will respond less the lower initial wealth $a$.

# Question 6: Money Growth and Inflation

Consider the following macroeconomic model. A representative agent chooses consumption and money balances to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t + b \log \frac{M_t}{P_t} \right)$$

subject to:

$$C_t + \frac{Z_t}{P_t} + B_t + \frac{M_t}{P_t} = I_{t-1} \frac{Z_{t-1}}{P_t} + R_{t-1} B_{t-1} + Y_t + \frac{M_{t-1}}{P_t} + T_t$$

where $Z$ is a nominal bond paying gross nominal rate $I$, $T$ are lump-sum transfers of seigniorage revenues from the government, and $Y_t$ is a random endowment of final goods. Let $m_t \equiv \frac{M_t}{P_t}$ and $\pi_t \equiv \frac{P_t}{P_{t-1}}$.

1. Derive the optimality conditions for the household problem. Show how to derive an equation relating the nominal rate to the real rate and to expected inflation in this context.

2. Consider the case where money supply follows the following exogenous stochastic process

$$\log \frac{M_t}{M_{t-1}} = \theta (1 - \rho) + \rho \log \frac{M_{t-1}}{M_{t-2}} + e_t$$

where $\varepsilon_t$ is a zero mean, iid shock, $\rho$ is positive but less than one, and $\theta$ is the steady state growth rate of money supply. Let $\theta_t \equiv \frac{M_t}{M_{t-1}}$

Assume output is constant at all times. Write down the equilibrium conditions involving the nominal variables of the model as a three equation dynamic system in the variables $\theta_t, \pi_t$ and $m_t$. Linearize this system around the steady state.

3. Solve the above system using the method of the undetermined coefficients. Guess that the solution for $\pi_t$ takes the following form

$$\pi_t = \varepsilon_1 m_{t-1} + \varepsilon_2 \theta_t$$

Find the solution for $\varepsilon_1$ and $\varepsilon_2$ as a function of the model parameters.

4. Suppose the economy is in steady state. Plot the impulse response over time of $\widehat{\pi}_t$, $\widehat{m}_t$ and $\widehat{\theta}_t$ following an innovation in $\varepsilon_t = 1\%$ (that is, a 1% increase in the rate of growth of money supply). Will inflation rise more or less than 1%? Will real balances ($m_t$) rise or fall? Discuss

BOSTON COLLEGE
Department of Economics

Macroeconomics Theory Comprehensive Exam
August 29, 2008

Directions: There are two parts. Please follow the instructions for each part carefully. Write the answer to Part A and Part B in a separate bluebook.

Part A contains 4 questions that are weighted equally.

**Write your alias, Part number, question number(s) on the front of each blue book.**

Please read the entire exam before writing anything.

PART A

Econ 750 August 2008 – Macro Comp: Professor Susanto Basu

# Question 1

Consumption in an economy is determined by a Ramsey-style social planner who derives log utility from consumption and has a time discount rate $\rho = 0.05$. The rates of depreciation and population growth are both zero. Time is continuous and the time horizon is infinite. The production function in per-capita form is

$$y = Ak$$

From time zero to time 1, the value of $A$ is 0.05. At time 1, $A$ will remain at 0.05 with probability ½ and will rise to 0.10 with probability ½. After time 1, $A$ will be constant forever.

What can you say about the growth rate of output between time zero and time 1? You don't have to give an exact initial value or time path. Just say whether output is rising, falling, or constant, and explain why.

Hint: Begin by analyzing the two possible outcomes from time 1 forward.

PART A

Econ 750 August 2008 – Macro Comp: Professor Susanto Basu

# Question 2: Explaining Cross-Country Growth Patterns

The following are three important facts about cross-country growth:

1. GDP and consumption per capita are higher by a factor of 20 in rich countries relative to poor countries.

2. Interest rates appear roughly similar across countries, and there is no marked tendency for capital to flow from rich to very poor countries.

3. There is evidence of "convergence" within rich and middle-income countries (countries with lower GDP per capita tend to grow faster and catch up to richer countries). However, there is no evidence of such convergence for the world as a whole.

This question asks you to explain how different models of growth and development either match or fail to match these three facts. List your assumptions clearly—for each model, what are you holding constant across countries and what are you allowing to vary?

A. How would the Solow model explain the facts?

B. How would a simple endogenous-growth model, the "*AK*" model, explain these facts?

C. If neither the Solow nor the *AK* model gives fully satisfactory explanations of the three facts, pick ONE of the two models and explain how you would modify it to do a better job. (If you think that at least one of the models does a good job of explaining all three facts, then present a fourth stylized fact about cross-country growth and explain how that model either can or cannot explain this fact.)

# Question 3: Government Purchase Shocks

Suppose a standard neoclassical business-cycle model, where a representative consumer receives utility from consumption and leisure, and maximizes discounted utility over an infinite horizon. Consumers take all prices, wages and interest rates as given.

Output is produced by many profit-maximizing firms. Firms are competitive and produce with constant returns to scale.

The economy is closed. The law of motion for capital is:

$$K_{t+1} = (1-\delta)K_t + AK_t^{\alpha}L_t^{1-\alpha} - C_t - G_t$$

Government purchases, $G$, are financed by lump-sum taxes. The economy is at its steady state.

A. Assume that government purchases are wasted ("thrown in the ocean"). Now suppose that $G$ rises unexpectedly at time $t$ for one period, and then returns to its steady-state level. What are the effects of this shock on $C_t$, $L_t$ and $K_{t+1}$? Write down the model you are using for analysis, and relate your answers to the model as precisely as you can.

B. Under the same assumption of wasteful government expenditures, suppose that the increase in government expenditures is permanent. How do the levels of $C$, $L$ and $K$ in the new steady state compare to their values in the old steady state before this shock?

C. Now assume that government expenditures provide real services that consumers view as perfect substitutes for consumption in providing utility. How does this new assumption change your answer to part A?

D. Suppose that instead of being viewed as a perfect substitute for consumption, government purchases are a perfect substitute for $K_{t+1}$. How does this new assumption change your answer to part A?

# Question 4: The Labor Market in Macro Models

"Macroeconomists say that they 'calibrate' their models using micro data, but in one case this is simply untrue. All modern macro models need to assume very large labor supply elasticities—Frisch elasticities of 4 or larger. But microeconometric estimates of the response of hours worked to changes in the real wage suggest a labor supply elasticity close to zero. (It is true that these estimates are restricted to hours worked by continuously employed workers, but this is the right concept for most macro models, where everyone is continuously employed and all adjustments in labor supply come from hours worked.) If macroeconomists used realistic numbers in calibrating the labor market, their models would be completely unable to explain business cycle fluctuations."

Discuss this criticism of modern business-cycle models in a well-crafted essay. Your essay should address at least the following issues:

1. Is it true that macro models "need" large labor supply elasticities to explain basic business-cycle facts? Which facts in particular?

2. Is it correct that micro labor supply estimates are small relative to values usually assumed in macro?

3. How do macro models deal with the intensive versus extensive margins of labor supply (changes in hours per worker versus changes in the number of workers)? Is the quote correct in saying macro models assume that everyone is always employed, and it is only their hours that vary?

4. Are there any macro models of labor supply that are <u>not</u> subject to this critique? If so, discuss ONE such model, and say how it deals with the issues raised in the quote.

PART B EC751 August 2008 - Macro Comp: Professor Matteo Iacoviello

# Question 5 (A macro model with capital and bonds)

Consider the following macroeconomic model. A representative household chooses consumption, capital, labor and foreign assets $\{C_{t+i}, K_{t+i}, L_{t+i}, B_{t+i}\}_{i=0}^{\infty}$ to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t + \tau \left(1 - L_t\right)\right)$$

where $\tau > 0$. The budget constraint is:

$$C_t + B_t + K_t = w_t L_t + \left(z_t + 1 - \delta\right) K_{t-1} + \left(1 + r^f\right) B_{t-1}$$

where $w_t$ is real wage, $z_t$ is the rental rate of capital, $\delta$ is the depreciation rate for $K$, $B$ is a the amount of international bonds that the household is holding (a negative $B_t$ means borrowing from the rest of the world). It is assumed that the international interest rate is $r^f = 1/\beta - 1$. The household is prevented from borrowing an overgrowing amount by the transversality condition

$$\lim_{t \to \infty} \frac{B_t}{\left(1 + r^f\right)^t} = 0$$

A firm maximizes profits by running a production function

$$Y_t = A_t K_{t-1}^{\theta} L_t^{1-\theta}$$

renting from households capital $K_{t-1}$ and labor $L_t$. $A_t$ is a random technology variable that follows

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_t, \ \varepsilon_t \text{ zero mean and iid}$$

1. Derive and comment the first-order conditions for the household problem
2. Derive and comment the first-order conditions for the firm problem
3. Having derived (1) and (2), assume that the initial level of foreign bonds is $B = \overline{B}$, where $\overline{B}$ is a constant. Show as carefully as you can how to derive a competitive equilibrium and a steady state involving constant wages, rental rates of capital, consumption, capital and output.

PART B

EC751 August 2008 - Macro Comp: Professor Matteo Iacoviello

# Question 6

Define and discuss the significance of the following questions. The more formal the discussion is, the better.

1. Vector Autoregressions
2. Identification of Shocks in Vector Autoregressions
3. Neutrality and Superneutrality of Money
4. The Inflationary Bias under Discretionary Monetary Policy, and how to solve it.