BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
June 10, 2011

Directions: There are three parts to this exam. Part 2 has an answer sheet and for Parts 1 and 3, you will need a bluebook. Please follow the instructions for each part carefully. Write your **Alias** on the answer sheet for Part 2 and on Parts 1 and 3 write your **Alias, part number, question number(s)** on the front of each blue book.

Please read the entire exam before writing anything.

Part I. Answer both questions in this part.

1. In Alphatown, commodity one is more expensive on the north side of town than on the south side, while the reverse is true for commodity two. For commodity one, $p_{1n} = 2p_{1s}$, where p_{1n} and p_{1s} are the north and south-side prices, respectively, while for commodity two, $p_{2n} = (2/3)p_{2s}$. Each of Alphatown's inhabitants has to choose which side of town to live on, knowing, for example, that a north-side choice means making all purchases at north-side prices. All inhabitants have a utility function of the form $U(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$, but have different values of α (between zero and one). Determine which individuals (as characterized by their values of α) will become north-side residents. (Note: An individual's income is not affected by her location choice.)

2. A consumer has the quasilinear utility function $U(x) = x_n + f(x_1, \ldots, x_{n-1})$, where $f()$ is strictly concave and increasing in its arguments. Assume that $p_n = 1$ and that the consumer's utility maximization problem always has an interior solution. Consider a change in the price of commodity 1 from p_1^0 to p_1^1. Show that the change in utility that occurs is exactly equal to the change in (Marshallian) consumer's surplus. Your answer should be in the form of a formal proof, with a justification written out for each step. A valuable hint is that the change in utility can be expressed as

$$\int_{p_1^0}^{p_1^1} \frac{\partial V(p,I)}{\partial p_1} dp_1 .$$

Allocate about two hours for this part. Please answer all questions in this section. Mark your answers on the special answer sheet provided.

Each of the following questions has several possible answers. Mark all the correct answers or statements, and only the correct ones. Important: There may be no correct answers, several correct answers, and it may even happen that all answers are correct!

Your grade for this part (out of 100) will be

$$2x + 6y - 4$$

where x is the number of correct answers you'll give in question 1 and y is the number of correct answers you'll give in questions 2–4.

1. Assume $n \geqslant 2$ agents and $k \geqslant 3$ social policies. Each of the following aggregation rules violates at least one of Arrow's axioms (Transitivity, Unanimity,[1] IIA, No dictatorship). Mark all assumptions that are violated.

   (a) $x \succ y$ iff for all i, $x \succ_i y$. Otherwise, $x \sim y$.

   (b) Suppose $k = 3$ and the three options are x, y, z. If $x \succeq_1 y \succeq_1 z$, $y \succeq_2 z \succeq_2 x$, and $z \succeq_3 x \succeq_3 y$, or if $x \succeq_1 z \succeq_1 y$, $y \succeq_2 x \succeq_2 z$, and $z \succeq_3 y \succeq_3 x$, then $x \sim y \sim z$. Otherwise, we use majority rule.

   (c) Each person receives one unit of voting power, which he allocates over the k options. For each option we then sum the weights it received, and rank options accordingly. Formally, person i gives option j weight α_{ij} such that $\sum_j \alpha_{ij} = 1$. Option j is socially weakly preferred to option ℓ iff $\sum_i \alpha_{ij} \geqslant \sum_i \alpha_{i\ell}$.

   (d) As in (c), only that each person has to put all the weight on his best option.

2. There are two consumers and two goods ($n = 2$, $k = 2$).

   - $u_1(x^1, x^2) = \max\{x^1, x^2\}$, $\omega_1 = (3, 7)$
   - $u_2(x^1, x^2) = (x^1)^2 + (x^2)^2$, $\omega_2 = (7, 3)$

   Which of the following statements is correct?

[1] If for all i, $x \succ_i y$, then $x \succ y$. If for all i, $x \succeq_i y$, then $x \succeq y$. If for all i, $x \sim_i y$, then $x \sim y$.

(a) The set of WE (Walrasian equilibrium) allocations is empty.

(b) The allocation $(10, 0)$ to person 1 and $(0, 10)$ to person 2 is in the core.

(c) The allocation $(10 - \sqrt{58}, 10 - \sqrt{58})$ to person 1 and $(\sqrt{58}, \sqrt{58})$ to person 2 is efficient.

(d) There are no allocations that make *both* consumers strictly better off than their initial endowments.

3. In this question you should assume expected utility theory and you should also assume risk aversion. Mark the correct statements. **Important:** There is no connection between the different parts of this question.

   (a) Fig. (1) depicts a case where separating contracts equilibrium is possible.

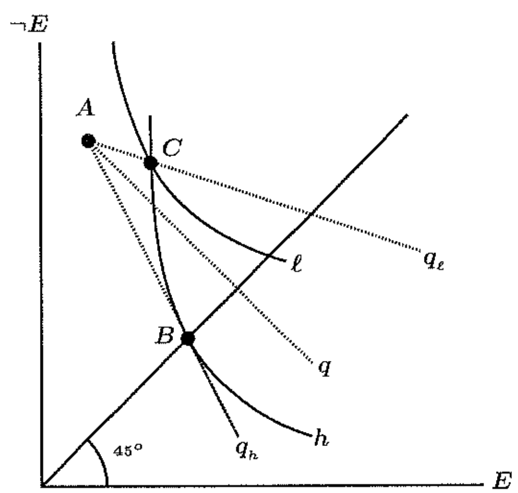

Figure 1:

   (b) Assume symmetric information. If a consumer wants to over-insure his asset then the market is not in equilibrium.

(c) The relation $\trianglerighteq$ over lotteries, given by $X \trianglerighteq Y$ iff X (weakly) dominates Y by FOSD (first order stochastic dominance), is transitive.

(d) The relation $\gtrapprox$ over lotteries, given by $X \gtrapprox Y$ iff X (weakly) dominates Y by SOSD (second order stochastic dominance), is incomplete.

4. Which of the following statements is correct?

   (a) The rank-dependent model with non-linear probability transformation function violates the independence axiom.

   (b) The proof we had that WE exists needed the assumption that the aggregate excess demand function is continuous.

   (c) If WE is unique, then the gross substitution assumption is satisfied.

   (d) Behaving according to the Allais paradox[2] violates not only expected utility theory, but also the assumption that decision makers are risk averse.

[2] $(5, 0.1; 0, 0.9) \succ (1, 0.11; 0.0.89)$ together with $(1, 1) \succ (5.0.9; 1, 0.89; 0, 0.01)$.

Mark your answers below.

1.

	(a)	(b)	(c)	(d)
Transitivity				
Unanimity				
IIA				
ND				

2. a b c d

3. a b c d

4. a b c d

Utku Ünver's Part:

1. 60pt Consider the following public goods game: Building a bridge...

		P2 Contribute	Don't
P1	Contribute	$1-c_1, 1-c_2$	$1-c_1, 1$
	Don't	$1, 1-c_2$	$0, 0$

The benefits of the bridge (1 to each player if at least one player contributes) are common knowledge, but the cost of contributing c_i is private information for player i.

   (a) 20pt Suppose that it is common knowledge that $c_1 = 0.25$ but that Player 1 does not know c_2. Player 1 believes that $c_2 = 0.25$ with probability 1/2 and $c_2 = 2$ with probability 1/2.

      i. If $c_2 = 2$, does Player 2 have a dominant action? If so, what is it? Does Player 2 have a dominant action if $c_2 = 0.25$?

      ii. Let z denote the probability that Player 2 contributes if her cost is $c_2 = 0.25$. What is Player 1's expected payoff from "Don't", given her beliefs, in terms of z?

      iii. What is the Bayesian Nash equilibrium? What is the probability that the bridge will be built at the equilibrium?

   (b) 20pt Suppose that Player i believes that $\Pr[c_j = 0.25] = \Pr[c_j = 2\ ] = 0.5$ for $i = 1, 2$ and $j = 1, 2,\ j \neq i$. What is the Bayesian Nash equilibrium of the game? What is the probability that the bridge will be built at the equilibrium?

   (c) 20pt Suppose that both players believe that the c_i are drawn independently from a uniform distribution on the interval $[0, 2]$. What is the Bayesian Nash Equilibrium now? What is the probability that the bridge will be built at the equilibrium?

2. 40pt Briefly (in 4 lines or less) answer each of the following questions:

   (a) 10pt. "Ordinary people are not familiar with the Nash concept. There is no reason to expect them to play such equilibrium strategies." Do you agree? Why or why not?

   (b) 10pt. When should the Nash concept be applied?

   (c) 10pt. Is there any alternative?

   (d) 10pt. Are there good arguments for the use of mixed strategy equilibria as solutions to games?