BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
August 19, 2011

Directions: There are three parts to this exam. For Parts 1 and 3, you will need a bluebook for your answers. Part 2 has a separate answer sheet to mark your answers and you will need to use a bluebook to explain your answers.

Please follow the instructions for each part carefully. Write your **Alias** on the answer sheet for Part 2 and on Parts 1, 2 (explanations) and 3 write your **Alias, part number, question number(s)** on the front of each blue book.

Please read the entire exam before writing anything.

Part I. This part consists of a single question that you are required to answer.

1. An individual has a monthly utility function $U(s,x)$, where s is the number of square feet of living space he decides to rent, and x is his consumption of a general-purpose commodity. The monthly rent for a square foot of living space is r; the general-purpose commodity has a price of unity. There are two possible locations at which the individual can reside. With location choice 0, there are no commuting costs and $r = r_0$. With location choice 1, there are monthly commuting costs of c and $r = r_1$. The individual has a given monthly income of I with which he has to cover his commuting costs, rent, and consumption of the general-purpose commodity.

(a) Given c, r_0 and I, suppose that r_1 adjusts so that the individual's utility would be the same regardless of location choice. Prove formally that $\partial r_1 / \partial c < 0$.

(b) Assume again that r_1 adjusts to equalize utility across locations. For the Cobb-Douglas utility function $U(s,x) = s^{\alpha} x^{1-\alpha}$, derive an explicit expression for r_1 in terms of the other parameters of the model.

(c) For the more general case considered in part (a), determine the sign of $\partial^2 r_1 / \partial c^2$.

PART II Instructions

Please read the instructions carefully as they have been recently changed.

Allocate about two hours for this part. Please answer all questions in this section. Mark your answers on the special answer sheet provided.

Each of the following 24 questions has two possible answers. To receive full credit for a question all you have to do is to mark the correct answer. However, you may feel uncertain about the meaning of a question, and therefore uncertain about the correct answer. If you so wish you can add an explanation regarding your understanding of the question. The following is an example:

- WE always exists.

The correct answer is "False" (we have seen examples for that in the problem sets), but you may be worried that the question may implicitly assume some properties of preferences. So the first two of the following answers will get you full credit, the third will get you half credit, but the forth and fifth will get you no credit (even though the statement in the forth answer is correct and the fifth states correctly that the statement is false):

1. Mark "False," no explanations.
2. Mark "False" and add the explanation: If $u_1(x^1, x^2) = u_2(x^1, x^2) = (x^1)^2 + (x^2)^2$, $\omega_1 = \omega_2 = (5, 5)$, then WE doesn't exist.
3. Mark "True" and add the explanation: If preferences are continuous and quasi concave, then WE always exists.
4. Mark "True" and add the explanation: If preferences are Cobb-Douglas, then WE exists.
5. Mark "False" and add the explanation: If preferences are Cobb-Douglas, then WE does not exist.

Please don't waste your time on unnecessary explanations. Use this option only if you are genuinely uncertain about the meaning of a question.

PART II

1. Assume $n \geqslant 2$ agents labeled $1, \ldots, n$ and two goods: one private good m (say, money) and one public good x which must be between zero and 1. Each person i will announce quasi linear utility of the form $m + u_i(x)$, $i = 1, \ldots, n$ where u_i must be strictly concave (but not necessarily increasing). A planner will then decide how to allocate a given amount of money $\bar{m}$ and how many units of the public good x to produce (the production cost of x is zero). For each of the following allocation rules, check all the properties they satisfy. The properties are: Efficiency, Symmetry (in the sense that two individuals with the same preferences should receive the same amount of money), and Truth revealing (in the sense that no one can ever benefit by misrepresenting his preferences)

   (a) Give everyone $\bar{m}/n$ units of the private good and produce the quantity of x that maximizes $\sum_i u_i(x)$.

   (b) Give $\bar{m}/(n-1)$ units of the private good to each of the first $n-1$ individuals and produce the quantity of x person n considers optimal.

   (c) Produce zero units of x, and give person i $2i\bar{m}/n(n+1)$ units of the private good.

   (d) Denote the optimal quantity of x for person i by x_i and let $\bar{x}$ be the average of these values. Produce $\bar{x}$ units of the public good, and give each person $\bar{m}/n$ units of the private good.

For each of the following 12 statements decide whether they are true or false. Answer them according to the explanations in the introduction.

2. For every price vector p, $p \cdot z(p) = 0$ ($z(p)$ is the aggregate excess demand function).

3. If all preferences are quasi concave, then the set of efficient allocations is convex. (That is, if x and y are efficient allocations that so is $\alpha x + (1-\alpha)y$ for all $\alpha \in [0, 1]$).

4. If there are only two types of individuals in the economy, then the core equals the set of efficient, individually rational allocations.

5. Two individuals who have the same utility functions and who receive the same utility from their initial endowments will receive the same utility in all WE allocations.

6. Harsanyi's utilitarian theorem may imply that giving all the resources of the economy to one person is optimal even if all individuals have the same weight in the social welfare function.

7. Suppose we accept Diamond's criticism and use a non linear social welfare function. If society prefers policy x to policy y, then it impossible that it will be optimal to use policy x with probability p and policy y with probability y with $p < \frac{1}{2}$.

8. In Arrow's theorem, a dictatorial rule satisfies IIA (independence of irrelevant alternatives axiom).

9. Arrow's impossibility disappears if all preferences must be "single bottom." (Preferences $\succeq$ on the real line are single bottom if there is x^* such that $z < y < x^*$ implies $z \succ y \succ x^*$ and $z' > y' > x^*$ implies $z' \succ y' \succ x^*$).

10. The preferences

$$(500, \tfrac{1}{2}; 100, \tfrac{1}{2}) \succ (500, \tfrac{1}{3}; 300, \tfrac{1}{3}; 100, \tfrac{1}{3}) \succ (300, 1)$$

    are consistent with the axioms of expected utility theory.

11. If the vNM utility function of a decision maker is $u(x) = e^x$ and the price q of a dollar insurance is $q = p$ (p is probability of loss), then he will buy full insurance.

12. The vNM utility function represents CRRA and $u(4) = 2$. Then it must be the case that $u(9) = 3$.

13. No pair of the lotteries of question 10 above can be compared by FOSD.

PART III

Utku Ünver's Part:

1. 60 pt Two firms, A and B, are in a market that is declining in size. The game starts in period 0, and the firms can compete in periods $0, 1, 2, 3, \ldots$ (i.e., indefinitely) if they so choose. Duopoly profits in period t for firm A are equal to $105 - 10t$, and they are $10.5 - t$ for firm B. Monopoly profits (those if a firm is the only one left in the market) are $510 - 25t$ for firm A and $51 - 2t$ for firm B.

   Suppose that at the start of each period, each firm must decide either to "stay in" or "exit" if it is still active (they do so simultaneously if both are still active). Once firm exits, it is out of the market forever and earns zero profit in each period thereafter. Firms maximize their (undiscounted) sum of profits.

   What is this game's subgame perfect Nash equilibrium outcome (and what are the firms' strategies in the equilibrium)?

2. 40 pt A buyer and seller are bargaining over the sale of an indivisible good. The buyer's valuation is $\theta_b = 10$. The seller's valuation takes one of the two values: $\theta_s \in \{0, 9\}$. Let t be the period in which trade occurs ($t = 1, 2, \ldots$). Let p be the price agreed. Both the buyer and seller have discount factor $\delta < 1$. Only seller knows his value, i.e., it is private information.

   (a) 15 pt What would be the set of alternatives if this were a social choice problem and a social decision was being taken centrally about the trade?

   (b) 25 pt Suppose that in a Bayesian Nash equilibrium of this bargaining process, trade occurs immediately when seller's valuation is 0 and the price agreed to when the seller has announced his valuation as $\theta'_s \in \{0, 9\}$ is $(10 + \theta'_s)/2$. What is the earliest possible time the trade can occur when the seller's valuation is 9?