BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
June 8, 2012

Directions:

There are four parts to this exam. Allow one hour for each part.

Please use a separate bluebook for each part.

Please follow the instructions for each part carefully and **write your alias, part number and question number(s) on the front of each blue book.**

Please read the entire exam before writing anything.

Part I. Answer both questions in this part.

1. Consider the standard utility maximization problem in the case of three commodities. Suppose that the utility function takes the special form $U(x_1, g(x_2, x_3))$, where g is increasing in its arguments and is homothetic. Determine what restriction this special structure places on the cross-price elasticities $E_{x_2:p_1}$ and $E_{x_3:p_1}$. Your answer must be carefully backed up. For simplicity, you can ignore corner solutions.

2. Consider the standard expenditure minimization problem in the case of three commodities. Corner solutions can again be ignored. The compensated demand functions for commodities 1 and 2 can respectively be written

$$x_1 = h_1(p_1, p_2)$$
$$x_2 = h_2(p_1, p_2),$$

where the price of commodity 3 has been normalized to one and the level of utility is fixed. Assuming that commodities 1 and 2 are substitutes, consider the inverse compensated demand functions

$$p_1 = \phi_1(x_1, x_2)$$
$$p_2 = \phi_2(x_1, x_2).$$

Determine the signs of $\partial\phi_1 / \partial x_1$ and $\partial\phi_1 / \partial x_2$.

PART II (General Equilibrium)

Allocate about 60 minutes for this part.

Answer all three questions.

1. Consider an economy with two types of goods: a composite good (money) and car. Each consumer can consume only an interger numer of cars, while the composite good is perfectly divisible (can take any real number). Composite good consumption is denoted by x, and car consumption is denoted by y (thus, $y \in \{0, 1, 2, ...\}$). We take the composite good as the numeraire, and the price of car is denoted by $p > 0$. Answer the following questions:

   (a) Draw a consumption space by taking the composite good on the horizontal axis.

   (b) Suppose that a consumer's endowment is given by the composite good $I > 0$. Describe the consumer's budget correspondence. Is the budget correspondence continuous for $p > 0$?

   (c) Suppose that the consumer's utility function is $U(x, y) = x + y$, and $I = 2.5$. Calculate demand correspondence. Is the demand corrspondence upper hemicontinuous for $p > 0$?

   (d) What is the relevant mathematical theorem for upper hemicontinuity of demand correspondence? State the theorem. Explain the result in (c) in this context.

2. Suppose that there is a continuum of atomless consumer-workers in an economy with two goods — labor/leisure and a consumption good. Consumers only care about the amount of consumption good (no disutility fron labor). Each consumer-worker is endowed with one unit of labor. There are two locations 1 and 2, with different production technologies: $y_1 = (x_1)^{\frac{1}{3}}$ in location 1, and $y_2 = (x_2)^{\frac{2}{3}}$ in location 2, where y_i and x_i denote consumption good output and labor input in location $i = 1, 2$, respectively. Workers in the same location receive the same amount of output by dividing the total output in the location equally. Consumer-workers are freely mobile across locations: thus, in

equilibrium, consumer-workers' utilities in the two locations need to be equated. The population measure of consumer-workers is 9. Answer the following questions:

(a) Find an equilibrium labor allocation.

(b) Find a Pareto efficient labor allocation. Explain.

(c) Compare them. Why aren't they the same? Explain the reason clearly.

(d) In order to support a Pareto-efficient allocation, what modification is needed to this economy?

3. Consider the following Edgeworth economy: $\omega^1 = (2, 0)$, $\omega^2 = (2, 2)$, $u^1(x_1^1, x_2^1) = x_1^1 \times x_2^1$, and $u^2(x_1^2, x_2^2) = (x_1^2)^{\frac{1}{2}}(x_2^2)^{\frac{1}{2}}$.

   (a) Draw an Edgeworth box with offer curves (demand curves).

   (b) Find a market equilibrium if any. Is it unique?

   (c) Draw the Pareto set and the core.

Part III

Allocate about one hour for this part. Please answer all questions in this section.

1. For two lotteries X and Y define:

   - $X \succ Y$ iff $\mathrm{E}[X] - \mathrm{E}[Y] > 1$
   - $X \sim Y$ iff $-1 \leqslant \mathrm{E}[X] - \mathrm{E}[Y] \leqslant 1$
   - $Y \succ X$ iff $\mathrm{E}[X] - \mathrm{E}[Y] < -1$

   Are these preferences consistent with expected utility theory? Explain.

2. There are three individuals and $n \geqslant 3$ options, $x_1, \ldots, x_n$. For two options x_i and x_j, define:

   - $x_i \succeq x_j$ iff $x_i \succeq_1 x_j$ whenever $i + j = 1 \bmod 3$ (that is, $i + j = 4, 7, 10, \ldots$).
   - $x_i \succeq x_j$ iff $x_i \succeq_2 x_j$ whenever $i + j = 2 \bmod 3$ (that is, $i + j = 2, 5, 8, \ldots$).
   - $x_i \succeq x_j$ iff $x_i \succeq_3 x_j$ whenever $i + j = 0 \bmod 3$ (that is, $i + j = 3, 6, 9, \ldots$).

   Which of Arrow's axioms does the social ranking $\succeq$ violate? Explain.

3. In each of the following cases, u and v are vNM utility functions (that is, of expected utility preferences). For each of the following, prove or find a counter example. In all cases, both functions u and v are strictly increasing and non-negative.

   (a) u and v represent CRRA (constant relative risk aversion). Then so does uv.

   (b) u and v represent risk aversion. Then so does uv.

   (c) u and v represent risk loving. Then so does $uv + u + v$.

Part IV

1. 50 pt Army A has a single plane with which it can strike one of three possible targets. Army B has one anti-aircraft gun that can be assigned to one of the targets. The value of target k is v_k and satisfies $v_1 > v_2 > v_3 > 0$. Army A can destroy a target only if the target is undefended and A attacks it. Army A wishes to maximize the expected value of the damage while army B wishes to minimize it. Formulate the situation as a strategic-form game and find its mixed-strategy Nash equilibria. (Observe that this is a strictly competitive game, i.e., the sum of the payoffs of the agents is a constant for all strategy profiles.)

2. 50 pt Player 1 is involved in an accident with Player 2. Player 1 knows whether she is negligent or not, but Player 2 does not know it. Suppose Player 2's prior on Player 1 being not negligent is $p_1(Not) \in (0, 1)$. If the case comes to the court the judge learns the truth. Player 1 sends a "take-it-or-leave-it" pre-trial offer of compensation either 3 or 5, which player 2 accepts or rejects it. If he accepts the offer the parties do not go to the court. If he rejects it the parties goes to the court and Player 1 has to pay 5 if he is negligent and 0 otherwise; in either case Player 1 has to pay court expense of 6. The payoffs are summarized below in two tables. Formulate this situation as a signaling game, draw its game tree, and find its perfect Bayesian equilibria. Can you suggest a criterion for ruling out unreasonable equilibria?

If Player 1 is not negligent:

	Y	N
3	$-3, 3$	$-6, 0$
5	$-5, 5$	$-6, 0$

If Player 1 is negligent:

	Y	N
3	$-3, 3$	$-11, 5$
5	$-5, 5$	$-11, 5$