BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
August 17, 2012

Directions:

There are four parts to this exam. Allow one hour for each part.

Please use a separate bluebook for each part.

Please follow the instructions for each part carefully and **write your alias, part number and question number(s) on the front of each blue book.**

Please read the entire exam before writing anything.

PART I —Kraus

Part I. Answer both questions in this part.

1. Consider the utility maximization problem in the case of a two good utility function of the form $U(x_1, x_2) = f(x_1) + g(x_2)$. Assume that f and g have positive first derivatives and negative second derivatives.

(a) Prove that U is strictly quasiconcave.

(b) Assuming an interior solution to the utility maximization problem, derive an expression for the comparative statics derivative $\frac{\partial x_1}{\partial p_1}$.

(b) What does the expression you derived imply about the sign of $\frac{\partial x_1}{\partial p_1}$?

2. Consider the standard utility maximization problem in the case of three commodities. Suppose that the utility function takes the special form $U(x_1, x_2, x_3) = x_1 g(x_2, x_3)$, where g is increasing in its arguments and is strictly quasiconcave. Prove that the ordinary demands for commodities 2 and 3 are independent of p_1. For simplicity, you can ignore corner solutions (Hint: This question is related to a question that appeared on the spring comp).

PART II --Konishi

PART II (General Equilibrium)

Allocate about 60 minutes for this part.

Answer all three questions.

1. Explain why the first welfare theorem can fail in an overlapping-generations model (say by Peter Diamond). (20 points)

2. A Robinson-Crusoe economy: Robinson Crusoe has utility function $u(x_1, x_2) = x_1^{\frac{1}{3}} \times x_2^{\frac{2}{3}}$ where x_1 is leisure consumption and x_2 is fish consumption. He has $\bar{L} = 18$ hours everyday to work on fishing and relaxing (leisure endowment). If he spends z hours in fishing, he can catch $z^{\frac{1}{2}}$ fish: i.e., his production function $f(z) = z^{\frac{1}{2}}$. Answer the following questions: (40 points)

   (a) Find a Pareto efficient allocation.

   (b) Find a market equilibrium that supports the Pareto efficient allocation.

   (c) Draw a picture and explain.

   (d) Now suppose $f(z) = z^{\frac{3}{2}}$ instead. Find a Pareto efficient allocation.

   (e) Find a market equilibrium that supports the Pareto efficient allocation in (d) if any. Explain the second welfare theorem.

3. Consider the following Edgeworth economy: $\omega^1 = (2, 0)$, $\omega^2 = (2, 2)$, $u^1(x_1^1, x_2^1) = \max\{x_1^1, x_2^1\}$, and $u^2(x_1^2, x_2^2) = \max\{x_1^2, x_2^2\}$. (40 points)

   (a) Draw each consumer's offer curves (demand curves).

   (b) Draw Edgeworth box and find a market equilibrium if any. Is it unique?

   (c) Find the Pareto set and the core if any.

PART III --Segal

Allocate about one hour for this part. Please answer all questions in this section.

1. There are $n \geqslant 2$ individuals and $k \geqslant 3$ options, $x_1, \ldots, x_k$. Define the following social ranking based on individual rankings of these options. For two options x_ℓ and x_m,
   - If for all i, $x_\ell \sim_i x_m$, then $x_\ell \sim x_m$.
   - If for all i, $x_\ell \succeq_i x_m$, and there is j such that $x_\ell \succ_j x_m$, then $x_\ell \succ x_m$.
   - If the number of people who strictly prefer x_ℓ to x_m equals the number of people who strictly prefer x_m to x_ℓ, then $x_\ell \sim x_m$.

   Which of Arrow's axioms does the social ranking $\succeq$ violate and which does it satisfy? Explain.

2. In each of the following cases, all lotteries have a finite number of outcomes. In each case, prove or find a counter example. As always, FOSD is first order stochastic diminance and SOSD is second order stochastic dominance.
   (a) If X dominates Y by SOSD *and* by FOSD, then $X = Y$.
   (b) If X dominates Y by SOSD and Y dominates Z by SOSD, then X dominates Z by SOSD. In other words, SOSD is a transitive relation.
   (c) On the set of lotteries $\mathcal{X} = \{X : \mathrm{E}[X] = 10\}$, the SOSD relation is complete.

3. For two lotteries X and Y define:

   $$X \succeq Y \text{ iff } \mathrm{E}[u(X)] + \mathrm{E}[v(X)] \geqslant \mathrm{E}[u(Y)] + \mathrm{E}[v(Y)]$$

   Where u and v are strictly increasing and continuous. Are these preferences consistent with expected utility theory? Explain.

   Answer the question again for the case

   $$X \succeq Y \text{ iff } \mathrm{E}[u(X)] + (\mathrm{E}[v(X)])^2 \geqslant \mathrm{E}[u(Y)] + (\mathrm{E}[v(Y)])^2$$

   (Warning: This last part is somewhat harder than the rest of the questions. Do other questions first).

PART IV --Unver

Utku Ünver's Part. Summer, 2012 Micro Comp.

1. 60 pt. Consider the model in which agents are voting for a candidate. Suppose that they have strict single-peaked preferences over $c > 2$ candidates. There is an odd number, n, voters. A Condorcet winner is defined as a candidate who would win the majority of votes against any other candidate if only those two candidates competed in an election.

   (a) Prove that there is always a Condorcet winner for any strict single-peaked preference profile.

   (b) Prove that the social choice function that chooses the Condorcet winner is implementable in dominant strategies for the strict single-peaked domain.

   (c) Is the Condorcet (winner) social choice function in the strict-single peaked domain anonymous? Unanimous? Pareto efficient? Group strategy-proof? Explain briefly. No formal proof is needed. Also write the definition of each property.

   (d) Can you think of ways to implement the Condorcet social choice function through an indirect mechanism with as simple message space as possible for the strict single-peaked domain? Can you give examples from real-life elections where (approximations of) the Condorcet social choice function are implemented?

   Hint: Recall that a preference profile domain is strict and single peaked if there exists a linear order of candidates as $x_1, x_2, ..., x_c$ such that for any preference relation $\succ_i$ in the domain for any voter i, there is a "peak" candidate x_k such that for any candidate x_ℓ to the right of x_k (i.e., $c \geq \ell > k$), $x_{\ell-1} \succ_i x_\ell$, and for any candidate x_ℓ to the left of x_k (i.e., $k > \ell \geq 1$), $x_{\ell+1} \succ_i x_\ell$.

2. 40 pt Plot the set of average-stage-game subgame-perfect-Nash-equilibrium payoffs of the following repeated game as $p \to 0$ (see below for definition of p).

   The following stage game is repeated infinitely many times with actions being made public after each repetition. Also there is a sufficiently small probability of game being terminated, p, after each repetition (otherwise there is no discounting). If the game ends after a repetition, then in each of the following periods the payoff of zero will be earnt by each player. Suppose that the total utility of each agent is the expected value of the sum of the payoffs he will earn during the whole repeated game.

		P2	
		L	R
P1	U	2,1	1,4
	D	4,-1	0,0