BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
August 17, 2007

Directions: There are three parts. Please follow the instructions for each part carefully. Write the answer to each part in separate bluebooks and on the answer sheet for Part II.

Write your alias, question number(s) on the front of each blue book and on the Part II answer sheet.

Please read the entire exam before writing anything.

Part I. This part consists of a single question that you are required to answer.

An individual has a utility function $U(x_1, x_2) = x_1x_2 + 5x_2$, where x_1 is the number of times per week he plays tennis at a neighborhood tennis club, and x_2, which sells for a dollar per unit, is his weekly consumption of a composite commodity. To play tennis at the club, you have to be a member. There is also a charge each time that you play. For week 0, the club sets a membership fee of $12 and a charge of $2 each time that a person plays. The individual's income in week 0 (and in every other week) is $40.

(a) Determine whether the individual will choose to join the club in week 0.

(b) Given the club's charge of $2 per visit, at what level would its membership fee have to be set to make the individual indifferent between belonging and not belonging to the club?

(c) For week 1, the club sets the membership fee at only $6 but raises the charge for each time that a person plays to $2.50. Calculate the compensating variation for the individual for the change in the tennis club's fee structure from week 0 to week 1.

Part II

Allocate about two hours for this part. Please answer all questions in this section. Mark your answers on the special answer sheet provided.

Each of the following questions has several possible answers. Mark all the correct answers, and only the correct answers. Important: There may be no correct answers, several correct answers, and it may even happen that all answers are correct!

All 34 questions have the same value of 3 points.

1. For each of social rankings (1)–(4), find which of the axioms (i)–(iv) is violated by it. In all cases, there are at least three individuals and three social options.

   1. For every x and y, $x \succ y$ iff for all i and for all $z \neq x$, $x \succ_i z$. If $x \not\succ y$ and $y \not\succ x$, then $x \sim y$.
   2. For every x and y, $x \succ y$ iff for all $i > 1$, $x \succ_i y$. If $x \not\succ y$ and $y \not\succ x$, then $x \sim y$.
   3. For every x and y, $x \succ y$ iff $y \succ_1 x$. If $x \not\succ y$ and $y \not\succ x$, then $x \sim y$.
   4. For every x and y, $x \succ y$ iff $x \succ_1 y$ and $x \succeq_2 y$, or if $x \succeq_1 y$ and $x \succ_2 y$. If $x \not\succ y$ and $y \not\succ x$, then $x \sim y$.

   (i) Unanimity.

   (ii) Transitivity.

   (iii) Independence of Irrelevant Alternatives.

   (iv) No Dictatorship.

2. Mark the correct statements. In all cases, x is an allocation.

   (a) If x is a WE allocation, then it must be efficient and it must be in the core.

   (b) If x is in the core, then it must be WE.

   (c) If there are two individuals and x is efficient and individually rational then it must be in the core.

   (d) The allocation that yields everything to one person and nothing to everyone else is always in the core.

3. Which of the following statements is correct?

   (a) If for all x, $R_A^u(x) = R_A^v(x)$ and $R_R^u(x) = R_R^v(x)$, then for all x, $u(x) = v(x)$.

   (b) The function $u(x) = x^\alpha$ represents constant relative risk aversion.

   (c) The assumption that the absolute measure of risk aversion is decreasing with wealth involves assumptions about the third order derivative of the utility function.

   (d) $R_u^A(xy) = R_u^A(x) + R_u^A(y)$.

4. $u_1(x, y) = \min\{x, y\}$, $u_2(x, y) = x + y$, $\omega = (10, 20)$. Which of the following statements is correct?

   (a) $(3, 3), (17, 7)$ is efficient.

   (b) $(10, 5), (10, 5)$ is efficient.

   (c) $(0, 0), (20, 10)$ can be WE.

   (d) $\max u_1 + u_2$ is obtained at the allocation $(10, 10), (10, 0)$ (but there may be other allocations where this maximum is achieved).

   (e) In WE, $p_x = p_y$.

   (f) Depending on the initial allocation, the core of this economy may be empty.

5. Which of the following is correct?

   (a) If not all individuals in the economy have the same probability of loss, then efficiency in an insurance market cannot be obtained.

   (b) Suppose that all individuals are risk averse. If person 1 is facing the risk $(100, H; 200, \neg H)$ and person 2 is facing the risk $(300, H; 200, \neg H)$, then there is no way to make both of them better off. (As always, $\neg H$ is "not H.")

   (c) Selling a good to the highest bidder at the third-high offer price is truth revealing.

   (d) WE is truth revealing.

Part III: Game Theory and Asymmetric Information
NOTE: *Answer* ***three*** *problems out of five.*

1. Consider a voluntary contributions/public goods model where $n = 2$, each participant chooses a contribution $s_i \in [0, 1]$, and the utility to player $i = 1, 2$ from a vector of contributions $\mathbf{s} = (s_1, s_2)$ is equal to

$$u_i(\mathbf{s}) = f(\mathbf{s}) - s_i,$$

where $f(\mathbf{s}) = 1$ if $s_1 + s_2 \geq c$, and $f(\mathbf{s}) = 0$ otherwise ($c > 0$ is a public good provision cost). Answer the following question.

   (a) Suppose that $c = 0.5$. Calculate all pure-strategy Nash equilibria of this game.

   (b) Suppose that $c = 1.5$. Calculate all pure-strategy Nash equilibria of this game.

2. Consider a two person (private value) auction problem in which each bidder's willingness-to-pay is an i.i.d. random variable drawn from a uniform distribution over the interval $[0, 1]$. **Assume that there is an auctioneer's reservation value** $r \in [0, 1]$.

   (a) Find a Bayesian Nash equilibrium of the auction game under a first price sealed bid rule.

   (b) Find a Bayesian Nash equilibrium of the auction game under a second price sealed bid rule. (Recall that if $b_i > r > b_j$, then player i pays r.) What is special about this Bayesian equilibrium?

   (c) For a bidder with her willingness-to-pay being $v \in [0, 1]$, prove that the expected utilities are the same in the above two Bayesian Nash equilibria. You can use anything you have learned in class, but if you want to use a theorem (or something else), then you must make a precise statement of the theorem including conditions required for the validity of the statement of it.

3. True or False: **Explain the reasonings briefly and clearly.**

   (a) In two person two strategy strategic form games, if there are two Nash equilibria in pure strategies then there must be another Nash equilibrium in strictly mixed strategies.

   (b) If Nash equilibrium in pure strategies is unique, then the game is solvable by iterative eliminations of strictly dominated strategies.

4. Consider the Rothschild-Stiglitz insurance market problem (1976, QJE) discussed in class. There are a continuum of high- and low-risk type consumers. The fraction of high-risk type consumers is denoted by $\alpha \in (0,1)$. Take the law of large numbers as given.

   (a) Explain clearly how the two-state diagram explained in class works.

   (b) In a diagram, show the competitive equilibrium allocation under perfect information (insurance companies can tell which type each consumer belongs to.

   (c) Explain the Rothschild-Stiglitz equilibrium concept for the insurance market under asymmetric information.

   (d) In a diagram, show an example of nonexistence of the Rothschild-Stiglitz equilibrium.

   (e) What if this is a signalling game? With the intuitive criterion (R6 in Gibbons, 1991), what is the perfect Bayesian equilibrium in the example for the above subproblem (d)? equilibrium.

5. Consider the following strategic form game. Answer the following questions:

	L	R
T	$3,3$	$0,5$
B	$5,0$	$1,1$

   (a) (finitely repeated game): Suppose that players maximize simple sum of their payoffs (discount factor $\delta = 1$). How many times at least do we need to repeat this stage game in order to support (T, L) as the period 0 action pair of a subgame perfect Nash equilibrium of this repeated game (if you can)? Explain the reasoning.

(b) (infinitely repeated game with discounting): Suppose that this stage game is repeated infinite times, and that players maximize their discounted payoffs. What is the minimum value of common discount factor $\delta \in (0,1)$ in order to support a path $\{(B,L),(T,R),(B,L),(T,R),...\}$ as the equilibrium path of a subgame perfect equilibrium of the game? Describe your subgame perfect equilibrium strategy as well.

Part II

Alias________________________________

Mark your answers below.

1.

	(i)	(ii)	(iii)	(iv)
1				
2				
3				
4				

2. a b c d

3. a b c d

4. a b c d e f

5. a b c d

BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
June 8, 2007

Directions: There are three parts. Please follow the instructions for each part carefully. Write the answer to each part in separate bluebooks and on the answer sheet for Part II.

Write your alias, question number(s) on the front of each blue book and on the Part II answer sheet.

Please read the entire exam before writing anything.

Part I. This part consists of a single question that you are required to answer.

A consumer has a strictly quasiconcave utility function $U(x_1, x_2)$. Assume that $U(\cdot)$ is twice continuously differentiable and has positive first derivatives. Consuming a unit of commodity 1 entails a monetary outlay of p_1 dollars (money price *per unit*) and a time outlay of t_1 hours (time requirement *per unit*). Consuming a unit of commodity 2 entails a monetary outlay of p_2 dollars but no time outlay. The consumer has T hours per period to devote to work and meeting the time requirements of consuming commodity 1. Work pays an hourly wage of w, and the consumer has nonwage income of M.

(a) Set up the consumer's utility maximization problem and derive the first-order conditions for an interior solution.

(b) Assuming an interior solution to the utility maximization problem (and that the strong form of the second-order condition holds), use the perturbation method to derive an expression for the comparative statics derivative $\dfrac{\partial x_1}{\partial w}$.

(c) Suppose that the consumer is slowed by an injury, causing t_1 to double. For the case of the Cobb-Douglas utility function $U = x_1 x_2$, derive an expression for the compensating variation (CV) in terms of the model's parameters. For this same Cobb-Douglas case, also derive an expression for the equivalent variation (EV).

PART II

Allocate about two hours for this part. Please answer all questions in this section. Mark your answers on the special answer sheet provided.

Each of the following questions has several possible answers. Mark all the correct answers, and only the correct answers. Important: There may be no correct answers, several correct answers, and it may even happen that all answers are correct!

All 34 questions have the same value of 3 points.

1. For each of the following preferences or properties 1–4, mark the properties (a)–(d) with which it is consistent. In all cases, all outcomes are non-negative.

   1. $(200, \frac{1}{4}; 300, \frac{1}{2}; -100, \frac{1}{4}) \succ (300, \frac{2}{3}; -100, \frac{1}{3}) \sim (200, 1)$
   2. $(x, p; 0, 1-p) \succeq (y, q; 0, 1-q)$ iff for all $\lambda \in (0, 1]$, $(x, \lambda p; 0, 1-\lambda p) \succeq (y, \lambda q; 0, 1-\lambda q)$.
   3. The preferences are represented by $V(X) = \left(\sum_i p_i \sqrt{x_i}\right)^3$.
   4. The preferences are represented by $V(X) = \sum_i p_i x_i \sqrt{x_i}$.

   (a) Expected utility theory.

   (b) Risk aversion.

   (c) First order stochastic dominance.

   (d) Quasi concavity: For $X \neq Y$, $X \sim Y$ implies for all $p \in (0, 1)$, $(X, p; Y, 1-p) \succ X$.

2. Mark the correct statements.

   (a) If all consumers have the same strictly quasi concave preferences, then in WE they will all receive the same outcome.

   (b) If all consumers have the same strictly quasi concave preferences, then in the core they will all receive the same outcome.

   (c) If all consumers have the same strictly quasi concave preferences, then there are PE allocation where all consumers receive the same outcome.

   (d) If all consumers have the same strictly quasi convex preferences, then there cannot be PE allocation where all consumers receive the same outcome.

3. Which of the following statements is correct?

   (a) Universal non-dictatorial transitive social rankings must violate IIA.

   (b) Universal Pareto liberal social rankings must violate transitivity.

   (c) With single-peaked preferences, majority rule is transitive (assume odd n).

   (d) When there are only two options, all preferences can be viewed as if they are single peaked.

4. Society needs to allocate \$100 between n individuals with the utility functions $u_1, \ldots, u_n$. The social welfare function is $W(u_1, \ldots, u_n)$. For which of the following cases is the allocation $(\frac{100}{n}, \ldots, \frac{100}{n})$ the only social optimum?

   (a) $u_i(x) = \sqrt{x}$, $i = 1, \ldots, n$, $W(u_1, \ldots, u_n) = \sum u_i$.
   (b) $u_i(x) = i\sqrt{x}$, $i = 1, \ldots, n$, $W(u_1, \ldots, u_n) = \sum u_i$.
   (c) $u_i(x) = \sqrt{x}$, $i = 1, \ldots, n$, $W(u_1, \ldots, u_n) = \sum i u_i$.
   (d) $u_i(x) = \sqrt{x}$, $i = 1, \ldots, n$, $W(u_1, \ldots, u_n) = \min\{u_i\}$.
   (e) $u_i(x) = \sqrt{x}$, $i = 1, \ldots, n$, $W(u_1, \ldots, u_n) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} u_i u_j$.
   (f) $u_i(x) = \sqrt{x}$, $i = 1, \ldots, n$, $W(u_1, \ldots, u_n) = \sum u_i^2$.

5. Which of the following are correct?

   (a) Economies with continuum of agents are competitive.

   (b) If u_i is replaced with $h(u_i)$ where h is convex, then the set of WE allocations may change.

   (c) If all consumers have quasi-linear utilities, then an allocation is efficient if and only if it maximizes the sum of utilities.

   (d) Suppose that all preferences are strictly quasi concave and that the initial allocation is in the core. All agents but i are price taker. Then the allocations obtained when i is *(i)* a price taker; or *(ii)* a fully discriminating monopolist are the same.

PART III

Allocate about 60 minutes for this part (Game Theory).
Answer the following three questions.

1. Consider the following strategic form game. Answer the following questions:

	L	R
T	$3,3$	$1,5$
B	$5,1$	$0,0$

   (a) (finitely repeated game): Suppose that players maximize simple sum of their payoffs (discount factor $\delta = 1$). How many times at least do we need to repeat this stage game in order to support (T, L) as the period 0 action pair of a subgame perfect Nash equilibrium of this repeated game?

   (b) (infinitely repeated game with discounting): Suppose that this stage game is repeated infinite times, and that players maximize their discounted payoffs. What is the minimum value of common discount factor $\delta \in (0,1)$ in order to support a path $\{(T,L),(T,L),(T,L),...\}$ as the equilibrium path of a subgame perfect equilibrium of the game? Describe your subgame perfect equilibrium strategy as well.

2. Consider a two person Rubinstein-type bargaining game with common discount factor $\delta \in (0,1)$. A possible variation of the original Rubinstein's alternating-move game is a bargaining game in which in each round one of the players is randomly selected as the proposer of the round. Each player has *equal chance* of being selected in each round (probability 1/2 each), and this stochastic process is i.i.d. Thus, if a player i rejects player j's offer in a round, in the next round both players have equal chances to become the proposer of the round (the same player may become a proposer in consecutive rounds). Each round t has three stages: (1) Nature chooses a proposer. (2) Proposer makes an offer. (3) Proposee accepts or rejects the offer. After each rejection, discount factor δ applies to both players' payoffs, and game ends whenever a proposee accepts an offer as in the original Rubinstein game.

We consider a symmetric equilibrium of this game, in which both players choose the same strategy whenever they become a proposer (and they choose the same strategy whenever they become a proposee).[1] This modified Rubinstein bargaining game is called a *random-proposer* game.

(a) Derive a SPNE in the **original** (alternating-move) Rubinstein bargaining game.

(b) Find a symmetric subgame perfect equilibrium payoffs of the **random proposer game**. (Ex post payoffs)

(c) Compare proposers' payoffs in the above two games. Which one is higher? Any insight on your result?

3. Consider an n-person third-price sealed-bid auction problem (the highest bidder gets the object by paying the third highest price). Each bidder's willingness-to-pay is uniformly distributed over $[0, 1]$, and is i.i.d. There is no reservation price.

   (a) Is truth-telling strategy profile a Bayesian Nash equilibrium in this auction problem? Explain clearly.

   (b) Compare the bid-functions of the first-price and the third-price sealed-bid auction problems. (You do not have to solve the equilibrium if you can compare the two without explicit solution of bid-functions. You can use any proposition proved in class.)

[1] *Although it is not needed to understand the game formally, I provide a little more formal description of the game just in case.* Player i's strategy as a proposer (PR) in round t is $\sigma_i^{PR}[t] : H_t \to [0, 1]$, and her strategy as a proposee (PE) in round t is $\sigma_i^{PE}[t] : H_t \times [0, 1] \to \{accept, reject\}$, where H_t is the set of histories up to round $t-1$ (a typical history $h_t \in H_t$ is a list of proposer's name and her offer in each round up to round $t-1$. A *symmetric* subgame perfect Nash equilibrium is such that both players choose the same strategy in a strong sense (with *anonymity*: proposers' names in the history do not affect players' action choices).

PART II -- ANSWER SHEET

ALIAS:________________________________

Mark your answers below.

1.

	(a)	(b)	(c)	(d)
1				
2				
3				
4				

2. a b c d

3. a b c d

4. a b c d e f

5. a b c d

BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
August 18, 2006

Directions: There are four parts to the exam. Please follow the instructions for each part carefully. Write the answer to each part in separate bluebooks except for Part III. There is an answer sheet for Part III.

Write your alias, part number and question number(s) on the front of each blue book.

Please read the entire exam before writing anything.

Part I.

This part of the exam consists of a single question, which you are required to answer.

A consumer having the Cobb-Douglas utility function $U = x_1^{\alpha} x_2^{1-\alpha}$ has an income of I and initially faces prices of p_1 and p_2. The government then introduces a sales tax of t on commodity one.

(a) Derive an expression for the compensating variation as a function of t.

(b) Derive an expression for the equivalent variation as a function of t.

(c) Derive an expression for the Marshallian consumer surplus measure of welfare change as a function of t.

(d) Assume that the sales tax is set to yield tax revenue of R. Derive an expression for the equivalent variation as a function of R.

Part II

Question for August Microeconomics Comprehensive Examination

Answer no more than a question asks. When the question says "calculate", show your calculations.

1. Consider a representative individual, one-good, two-factor general equilibrium production economy. The good is produced according to the production function

$$C = L^{1/2}K^{1/2}.$$

The economy is endowed with one unit of labor and one unit of capital. Take the wage rate as the numeraire, and let r be the price of capital, p the producer price of the good, and q the consumer price of the good.

a) Calculate the unit cost function and the unit factor demand functions.

b) Assume that all markets operate under perfect competition. Calculate the equilibrium price of capital, producer price of the good, and consumer price of the good.

c) State Walras' Law, indicate whether it applies to the economy in part b), and explain why.

d) Suppose that the goverment imposes a 100% tax on the good, redistributing the tax revenue as a lump sum to the representative individual. Using elementary reasoning (including symmetry of the two factors) and no algebra, derive the equilibrium allocation, price of capital, producer price of the good, and consumer price of the good. Check that the individual's budget constraint is satisfied.

e) Suppose that the government imposes a price ceiling of $r = 0.5$ on the price of capital. How *qualitatively* (no calculation expected) does the price ceiling alter the equilibrium allocation, producer price of the good, and consumer price of the good? Does the individual's budget constraint continue to be satisfied?

f) Suppose that the production function faced by each firm is instead

$$C = L^{1/4}K^{1/4}.$$

By symmetry of the two factors, $r = 1$. What is the equilibrium price of the good? Check that the individual's budget constraint is satisfied.

Part III

Allocate about one hour for this part. Please answer all questions in this section. Mark your answers on the special answer sheet provided.

Mark each of the following statements that is true. All 8 questions have the same value of 12.5 points.

1. It follows from Arrow's Impossibility Theorem that if a social ranking is dictatorial and satisfies unanimity and IIA, then it cannot be transitive.

2. If people are willing to consider other people's welfare, then social preferences must violate transitivity.

3. The social ranking $x \succ y$ iff either $x \succ_1 y$ (that is, person 1 prefers x to y) or $x \succ_2 y$ (that is, person 2 prefers x to y), but not both, violates either unanimity or transitivity, but not both.

4. If the preferences $\succeq$ over lotteries satisfy for all lotteries X and Y and $\alpha \in [0, 1]$,

$$X \sim Y \Longrightarrow (X, \alpha; Y, 1-\alpha) \sim (X, 1-\alpha; Y, \alpha)$$

then they cannot be expected utility.

5. If $R_A^u \equiv R_A^v$,[1] and $u'(3) = v'(3)$, then $u \equiv v$.

6. $(-6000, \frac{1}{4}; 6000, \frac{3}{4}) \succ (3500, 1)$ is inconsistent with expected utility theory.

7. An expected utility maximizer will never buy full insurance.

8. John and Mary will have together \$1000 if it rains and \$1000 if it doesn't rain. Then giving John \$500 if rains and \$500 if it doesn't rain, and giving Mary \$500 if rains and \$500 if it doesn't rain is always efficient, but is not always the *only* efficient allocation.

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[1] $\equiv$ between two functions means "they are always the same."

Allocate about 60 minutes for this part.
!Answer two of the following three problems!

1. Consider the following n player game. Each player report a real number between 0 and 100. Take the average of reported numbers. The player who reported a number closest to the average is the winner, and she receives her reported number as her payoff. The tie-breaking rule is equal division: i.e., if there are k players whose reports are closest to the average, then each player gets $\frac{1}{k}$ of the average number. Answer the following question.

   (a) Find all pure strategy equilibria.

   (b) Is there a mixed strategy equilibrium in this game? Discuss.

   (c) Let's modify the game a bit. Now the winner is not the one who reported a number that is closest to the average. Instead, the one who reported a number that is closest to a half of the average of the reported numbers. (For example, if the average is 50, the one who reported a number closest to 25 is the winner.) Find all Nash equilibria. Explain clearly.

2. Recall the job market signalling game studied in class. In a diagram, describe a semi-pooling (hybrid) perfect Bayesian equilibrium in which high type randomizes its education level. *You also need to describe all the elements of perfect Bayesian equilibrium.* Does it satisfy Requirement 5? Does it satisfy Requirement 6? Note that your answer can depend on your diagram, so you need to draw it very clearly (if you are not good at drawing pictures clearly, then your diagram should be supplemented by a nicely written description).

3. Consider the following strategic form game. Answer the following questions:

	L	R
T	$5,5$	$3,10$
B	$10,3$	$1,1$

   (a) (finitely repeated game): Suppose that players maximize simple sum of their payoffs (discount factor $\delta = 1$). Can we support (T, L) as the period 0 action pair of a subgame perfect equilibrium? If so, how many times at least do we need to repeat this stage game in order to do so?

(b) (infinitely repeated game with discounting): Suppose that this stage game is repeated infinite times, and that players maximize their discounted payoffs. What is the minimum value of common discount factor $\delta \in (0,1)$ in order to support a path $\{(B,L),(T,R),(B,L),(T,R),...\}$ as the equilibrium path of a subgame perfect equilibrium of the game? Explain clearly.

Part III Answer Sheet

Alias:________________________________

Mark your answers below.

1. T F
2. T F
3. T F
4. T F
5. T F
6. T F
7. T F
8. T F

BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
June 9, 2006

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Write your alias, part number and question number(s) on the front of each blue book.

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Part I. This part consists of a single question which you are required to answer.

Mr. A has a utility function $U = x_1 x_2$, where x_1 and x_2 are his consumption levels of commodities one and two, respectively. Throughout the problem, assume that commodity two is available at $1 per unit. In period 0, commodity one is also available at $1 per unit, and Mr. A's resources consist of his $120 of earnings.

(a) Solve numerically for Mr. A's period 0 equilibrium.

In period 1, a rationing scheme is in effect for commodity one. In addition to a cash price, commodity one now has a coupon requirement. Assume the same $1 cash price as in period 0 and a coupon requirement of one coupon per unit of commodity one (For example, for 5 units of commodity one, a consumer must surrender 5 coupons in addition to $5 cash). Each consumer receives a coupon allotment from the government. The coupons are received gratis. The size of Mr. A's allotment is 40. Mr. A has access to a coupon market in which coupons can be bought or sold at a unit price of P_c. Mr. A can buy or sell in this market according to his desire to set his consumption of commodity one above or below 40. Mr. A takes P_c as given. His resources again include $120 of earnings.

(b) Derive Mr. A's period 1 budget constraint. Show diagrammatically how it compares with his period 0 budget constraint. Provide numerical labels.
(c) Derive Mr. A's period 1 consumption levels of commodities one and two in terms of P_c.
(d) With period 0 as the base period, derive an expression for CV in terms of P_c.
(e) For $P_c = 0.25$, solve numerically for Mr. A's period 1 equilibrium, and show diagrammatically how it compares with his period 0 equilibrium.
(f) What is the critical value of P_c that determines whether Mr. A will be a buyer or seller in the coupon market?
(g) With P_c as the only free parameter, determine whether Mr. A can be better off in period 1 than in period 0.

Part II. **GENERAL EQUILIBRIUM PART – ANSWER ALL THREE QUESTIONS**

Answer no more than a question asks; for example, if you are asked only to give the equilibrium allocation, you need not indicate how you derived it.

A. Consider a simple exchange economy. The economy comprises two individuals, Yolanda and Zacharias, and two goods, apples and bananas. Yolanda has a utility function of the form $U = \ln a + \ln b$ and is endowed with two apples and no bananas. Zacharias has a utility function of the form $U = \min(a,b)$, and is endowed with two bananas and no apples.
a) Show the set of Pareto optimal allocations in the Edgeworth exchange box.
b) Provide a definition of a competitive (Walrasian) equilibrium in an exchange economy.
c) Indicate a competitive equilibrium allocation for this economy, and demonstrate that it is indeed a competitive equilibrium.

B. Consider a simple exchange economy that is the same as in A. except that Yolanda has a utility function of the form $U = a^2 + b^2$.
a) Show the set of Pareto optimal allocations in the Edgeworth exchange box.
b) Demonstrate that the economy has no competitive equilibrium, and indicate the source of non-existence.
c) State Walras' Law. Does it hold for this economy? Why or why not?

C. Consider a simple exchange economy that is the same as in A. except that Yolanda has a utility function of the form $U = a^2 + b^2$ and Zacharias has a utility function of the form $U = \max(a,b)$.
a) Show the set of Pareto optimal allocations in the Edgeworth exchange box.
b) Indicate two competitive equilibrium allocations.
c) State the First Theorem of Welfare Economics. Does it apply to this economy? Why or why not?
d) State the Second Theorem of Welfare Economics. Does it apply to this economy? Why or why not?

Part III

Allocate about one hour for this part. Please answer all questions in this section. Mark your answers on the special answer sheet provided.

Each of the following questions has several possible answers. Mark all the correct answers, and only the correct answers. Important: There may be no correct answers, several correct answers, and it may even happen that all answers are correct!

All 14 questions have the same value of 7 points. You'll get 2 points bonus if you answer at least half of them correctly.

1. Society needs to allocate \$100 between n individuals with the utility functions $u_1, \ldots, u_n$. Which of the following allocations $\alpha = (\alpha_1, \ldots, \alpha_n)$ can be the **only** social optimum under weighted utilitarianism $\sum a_i u_i$ where $a_1, \ldots, a_n > 0$?

   (a) $n = 2$, $u_1(x) = 2x$, $u_2(x) = 3x - 5$, $\alpha = (60, 40)$.

   (b) $n = 2$, $u_1(x) = 2x$, $u_2(x) = 3x - 5$, $\alpha = (40, 40)$.

   (c) $n = 2$, $u_1(x) = 2x$, $u_2(x) = 3x - 5$, $\alpha = (100, 0)$.

   (d) $n = 2$, $u_1(x) = \sqrt{x}$, $u_2(x) = 2\sqrt{x}$, $\alpha = (60, 40)$.

   (e) $n = 2$, $u_1(x) = \sqrt{x}$, $u_2(x) = 2\sqrt{x}$, $\alpha = (100, 0)$.

   (f) $n = 2$, $u_1(x) = x^2$, $u_2(x) = x_2$, $\alpha = (50, 50)$

   (g) $n = 4$, $u_i(x) = ix$, $\alpha = (10, 20, 30, 40)$.

2. Which of the following can be the preferences of an expected utility maximizer with a strictly increasing utility function?

   (a) $(200, \frac{1}{18}; 100, \frac{1}{9}; 40, \frac{2}{9}; 0, \frac{11}{18}) \succ (100, \frac{1}{3}; 40, \frac{2}{3}) \succ (200, \frac{1}{12}, 0, \frac{11}{12})$.

   (b) $(5M, 0.1; 1M, 0.89; 0, 0.01) \succ (1M, 1)$ (M stands for million).

   (c) $(6000, \frac{5}{6}; 0, \frac{1}{6}) \succ (5000, 1)$.

   (d) $(3000, 1) \succ (6000, \frac{1}{2}; 0, \frac{1}{2})$ together with $(-6000, \frac{1}{2}; 0, \frac{1}{2}) \succ (-3000, 1)$.

3. Mark each of the following statements that is true.

   (a) An expected utility maximizer with differentiable utility function will buy full insurance iff the price of insurance equals the probability of loss.

   (b) $u(x) = \frac{1-e^{-x}}{1-e}$ represents constant absolute risk aversion.

(c) The function

$$u(x) = \begin{cases} \sqrt{x} & x \leqslant 100 \\ 10 + \sqrt{x - 100} & x > 100 \end{cases}$$

represents risk aversion.

Part IV

Allocate about 60 minutes for this part (Game Theory).
Answer the following three questions.

1. True or False: **Explain.**

   (a) The revelation principle says that for any Bayesian game and any Bayesian Nash equilibrium of the game, we can construct a new type-reporting Bayesian game in which truth-telling is weakly dominant strategy for each player.

   (b) In finitely repeated games, if all players have strictly dominant strategies then there is unique subgame perfect equilibrium in which all players play their dominant strategies in all stage games. However, if players do not have strictly dominant strategies, then even in a finitely repeated games, there are other subgame perfect equilibria if the stage game is repeated many times.

   (c) Consider an n person strategic form game that is solvable by iterative eliminations of strictly dominated strategies (there is a unique survivor strategy profile s*). Suppose now that players move sequentially in some particular prefixed order (say in the order of players 1, 2, 3, ..., n, or in the order of players 2, 5, 3, 1, ... etc.) keeping the payoff structure intact. Then, any subgame perfect equilibrium in any sequential move game attains the same outcome s*.

2. Consider the following public good provision game. There are a public good and a numeraire private good. There are n identical consumers who are endowed with the same amount of numeraire private good (but no public good endowment). The amount of private good endowment is large enough to assure an interior solution. One unit of public good can be produced by one unit of private good: that is, if each consumer i contributes x_i units of private good, then $y = \sum_{i=1}^{n} x_i$ units of public good can be produced. Consumer i's payoff function is

$$u_i(y, x_i) = y - \frac{1}{2}x_i^2 = \sum_{j=1}^{n} x_j - \frac{1}{2}x_i^2,$$

where y is amount of public good and x_i is the contribution of private good made by i (that is, $\frac{1}{2}x_i^2$ is disutility from the contribution).

(a) Find a Nash equilibrium of this game. Anything special about this Nash equilibrium?

(b) Find a symmetric Pareto efficient allocation. Show that the above Nash equilibrium is Pareto inefficient.

(c) Consider an infinitely repeated game with a common discount factor $\delta \in (0, 1)$. In order to support the symmetric efficient allocation repeated forever, what is the critical value for δ?

3. Consider an (independent-value) n-person auction problem (the one we studied in the homework problem). Now consider the following two variations of the second price sealed bid auction.

(i) Suppose that **the second highest bidder gets the object** (unlike in the second price sealed bid auction in which the highest bidder gets the object) and pays **her bid (the second price)** to the auctioneer.

(ii) Suppose that **the highest bidder gets the object** and pays **the lowest price (the n-th highest price instead of the second highest price in the second price sealed bid auction)** to the auctioneer.

(a) Is truth-telling a Bayesian Nash equilibrium in these auction problems? State your answer and explain clearly for each auction problem.

(b) Do these two auction problems attain the same expected revenues for the auctioneer? Explain logically by utilizing what you have learned in class.

Part III Answer Sheet

Mark your answers below.

1. a b c d e f g

2. a b c d

3. a b c

BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
August 19, 2005

Directions: There are three parts to the exam. Please follow the instructions for each part of the exam carefully. Write the answer to each part in separate bluebooks except for part three. Please write your answers on part three-on the exam itself-and separate it from your bluebooks.

Write your alias, part number and question number(s) on the front of each blue book for Part I and II and write your alias on Part III.

Please read the entire exam before writing anything.

Part I. Answer both questions from this part. The suggested time for this part is 1 hour.

1. The context of this question is the two-good case of the standard consumer model; you can take this to be the two-good case of the general model of consumer choice developed in the first module of Micro 1.

Determine which of the following is invariant to a monotone increasing transformation of a utility function. In each case, derive your answer.

(a) The value of $\partial V/\partial I$ (V denotes indirect utility) under a given price-income combination.

(b) The value of CV, the compensating variation measure of welfare change, for a fall in p_1.

2. A firm has a production function $y = f(x_1, x_2)$ which is homogeneous of degree one. The price of input 2 is parametric. However, the price that the firm faces for input 1 increases linearly with x_1.

(a) Determine whether the firm's expansion path is linear or not.

(b) Consider the elasticity of the firm's cost function with respect to y. Intuitively, is this elasticity greater than, equal to, or less than one? Prove your answer for the case of the Leontief production function $y = \min\{x_1, x_2\}$.

(c) What does your intuitive answer to (b) imply about the profitability (i.e., are profits positive, negative or zero?) of marginal cost pricing? Support your answer with a proof.

PART II

Allocate about 120 minutes for this part (GE and GT).
Answer the following six questions.

1. Consider an Edgeworth box economy with the following information: $\succeq_1$ is represented by a utility function $u_1(x_{11}, x_{21}) = \min\{2x_{11}, x_{21}\}$, $\succeq_2$ is represented by a utility function $u_2(x_{12}, x_{22}) = x_{12} \times x_{22}$, $\omega_1 = (1, 0)$ and $\omega_2 = (0, 2)$.

   (a) Draw an Edgeworth box with all the relevant information.

   (b) Calculate Walrasian equilibrium (if any).

   (c) Point out (i) Pareto efficient set, (ii) the core, and (iii) Walrasian equilibrium set in the box.

2. Consider an Edgeworth box economy with the following information: $\succeq_1$ is represented by a utility function $u_1(x_{11}, x_{21}) = \max\{x_{11}, x_{21}\}$, $\succeq_2$ is represented by a utility function $u_2(x_{12}, x_{22}) = x_{12} + x_{22}$, $\omega_1 = (2, 2)$ and $\omega_2 = (3, 1)$.

   (a) Draw an Edgeworth box with all the relevant information.

   (b) Calculate Walrasian equilibrium (if any).

   (c) Point out (i) Pareto efficient set, (ii) the core, and (iii) Walrasian equilibrium set in the box.

3. True or False: **Explain the reasonings.**

   (a) Suppose that each consumer can consume only an integer amount of each commodity. Even in this economy, the proof of the first welfare theorem works.

   (b) In an Edgeworth box economy (2 person, 2 commodities), the core is always nonempty unlike the Walrasian equilibrium.

   (c) Consider Edgeworth box economies in which consumers have identical strongly monotonic and strictly convex preferences (2 person). Then, Walrasian equilibrium is unique.

4. True or False: **Explain the reasonings.**

   (a) Consider a variation of second-price sealed bid auction: *the second highest bidder wins the object* and pays his/her bid. Although truth-telling is no longer a weakly dominant strategy, the expected revenue of this auction mechanism is still the same as the one of the first- and second-price sealed bid auction by the revenue equivalence theorem.

   (b) In two person two strategy strategic form games, if there are always odd number of equilibria (including equilibria with mixed strategies).

   (c) If Nash equilibrium in pure strategies is unique, then the game is solvable by iterative eliminations of strictly dominated strategies.

5. Consider a two-bidder first price sealed bid auction problem. Each bidder's willingness-to-pay is distributed over $[0, 1]$ uniformly, and is i.i.d. **Suppose that the auctioneer can set a reservation price** $r \in [0, 1]$. Reservation price r means that if every bidder's bid is less than r, then the auctioneer retains the object.

   (a) Find a symmetric Bayesian equilibrium for given r.

   (b) Calculate the expected revenue for given r.

   (c) Find the optimal reservation price for the auctioneer.

6. Recall the job market signalling game studied in class. In a diagram, describe a perfect Bayesian equilibrium that satisfies Requirement 5 yet does not satisfy Requirement 6. Explain how all the requirements (R1, 2, 3, 5) are met, but R6 is not.

PART III

Alias:____________________________

Allocate about one hour for this part. Please answer all questions in this section. Mark your answers on this form.

Each of the following questions has several possible answers. Mark all the correct answers, and only the correct answers. Important: There may be no correct answers, several correct answers, and it may even happen that all answers are correct!

1. For each of the following preference relations over lotteries, mark all the axioms it satisfies. In all cases, $X = (x_1, p_1; \ldots; x_n, p_n)$ and $Y = (y_1, q_1; \ldots; y_m, q_m)$.

   (a) $X \succeq Y$ iff

      i. $\sum p_i \sqrt[3]{x_i} \geqslant \sum q_i \sqrt[3]{y_i}$

      and

      ii. $\sum p_i x_i \geqslant \sum q_i y_i$

      A. Completeness

      B. Transitivity

      C. Continuity

      D. Independence

   (b) $X \succeq Y$ iff

   $$\frac{\sum p_i x_i^2}{\sum p_i e^{x_i}} \geqslant \frac{\sum q_i y_i^2}{\sum q_i e^{y_i}}$$

      A. Completeness

      B. Transitivity

      C. Continuity

      D. Independence

   (c) $X \succeq Y$ iff $\sum p_i u(x_i) \geqslant \sum q_i u(y_i)$ where

   $$u(x) = \begin{cases} x & x \leqslant 0 \\ e^x & x > 0 \end{cases}$$

      A. Completeness

      B. Transitivity

      C. Continuity

      D. Independence

2. Mark <u>only</u> the correct statements.

   (a) Arrow's impossibility theorem proves Pareto to be inconsistent with either transitivity or IIA.

   (b) For any $n \geqslant 3$ (not necessarily odd) there is an n person society with such preferences that majority rule leads to a violation of transitivity.

   (c) Arrow's impossibility theorem does not hold when there are only two individuals. (Be careful: The question is not whether we proved it only for $n \geqslant 3$, but whether it true only for $n \geqslant 3$).

BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
June 10, 2005

Directions: There are three parts to the exam. Please follow the instructions for each part of the exam carefully. Write the answer to each part in separate bluebooks except for part two. There is a separate answer sheet for that part.

Write your alias, part number and question number(s) on the front of each book/answer sheet.

Read the entire exam before writing anything.

Part I. Answer both questions from this part. The suggested time for this part is 1 hour.

1. The context of this question is the two-good case of the standard consumer model; you can take this to be the two-good case of the general model of consumer choice developed in the first module of Micro 1.

Determine which of the following is invariant to a monotone increasing transformation of a utility function. In each case, derive your answer.

(a) The marginal rate of substitution between commodities at a point (x_1, x_2) in commodity space.

(b) The sign of $\frac{\partial^2 U(x_1, x_2)}{\partial x_1^2}$.

(c) The sign of $\frac{\partial^2 V(p_1, p_2, I)}{\partial I^2}$.

(d) The value of CS, the traditional Marshallian consumer's surplus welfare change measure, for a fall in p_1.

2. Consider again the standard two-good consumer model, but this time with the additional assumption that the utility function is homothetic. What does the additional assumption of homotheticity imply about the income elasticity of demand for each commodity? Prove your answer.

Part 2

Allocate about one hour for this part. Please answer all questions in this section. Mark your answers on the special answer sheet provided.

Each of the following questions has several possible answers. Mark all the correct answers, and only the correct answers. Important: There may be no correct answers, several correct answers, and it may even happen that all answers are correct!

1. Society has 53 members and is facing k possible social policies. Consider the following social ranking rule, and mark the axioms it satisfies.

   Check first for all pairs of social options over which everyone agrees, and make social preferences to agree with individual preferences on these pairs. Next, make the following list of pairs (x_1, x_2), ..., (x_1, x_k), (x_2, x_3), ..., (x_2, x_k), ..., (x_{k-1}, x_k). Start with the first pair, and go down the list. If a pair is already decided, go to the next. If it is not yet decided, check it you can decide the social preferences on this pair by using transitivity from already decided pairs. For example, suppose you already found out that $x_5 \succ x_8$ and $x_8 \succ x_2$, and you reached the pair (x_2, x_5). Then decide $x_5 \succ x_2$ and move to the next pair. If the pair is undecided and cannot be decided by transitivity, use majority rule.

   (a) Unanimity.
   (b) Transitivity.
   (c) IIA.
   (d) Non dictatorial.

2. Mark the correct statements.

   (a) Risk loving implies $R_A \leqslant 0$.
   (b) For $x \geqslant 1$, risk aversion implies $R_R(x) \geqslant R_A(x)$.
   (c) If $u(0) = v(0) = 0$, and for all $x > 0$, $u(x) > v(x)$, then for all $x > 0$, $R_A^u(x) \leqslant R_A^v(x)$.
   (d) If u and v have the same first, second, and third order derivatives, (that is, for all x, $u'(x) = v'(x)$, $u''(x) = v''(x)$, and $u'''(x) = v'''(x)$), then u satisfies increasing relative risk aversion iff v satisfies increasing relative risk aversion.

3. Which of the following set of preferences is inconsistent with expected utility theory?

(a)

$$(1000, 1) \succ (5000, 0.1; 1000, 0.89; 0, 0.01)$$

and

$$(5000, 0.98; 0, 0.02) \succ (5000, 0.78; 1000, 0.22)$$

(b)

$$(3000, 1) \succ (6000, 0.5; 0, 0.5)$$

and

$$(-6000, 0.5; 0, 0.5) \succ (-3000, 1)$$

(c)

$$(50, 1) \succ \left(2, \tfrac{1}{2}; 4, \tfrac{1}{4}; 8, \tfrac{1}{8}; \ldots; 2^n, \tfrac{1}{2^n}; \ldots\right)$$

PART 3

Allocate about 120 minutes for this part (GE and GT).
Answer the following six questions. You can choose one of two questions in Problem 6.

1. Consider an Edgeworth box economy with the following information: $\succsim_1$ is represented by a utility function $u_1(x_{11}, x_{21}) = x_{11} \times x_{21}$, $\succsim_2$ is represented by a utility function $u_2(x_{12}, x_{22}) = x_{12} + \frac{1}{2}x_{22}$, $\omega_1 = (0, 1)$ and $\omega_2 = (1, 0)$.

    (a) Draw an Edgeworth box with all the relevant information.

    (b) Calculate Walrasian equilibrium (if any).

    (c) Point out (i) Pareto efficient set, (ii) the core, and (iii) Walrasian equilibrium set in the box.

2. The first welfare theorem in an exchange economy. An exchange economy with private ownership is a list $(I, (X_i, \omega_i, \succsim_i)_{i \in I})$, where $X_i = \mathbb{R}^L_+$.

    (a) Define a feasible allocation.

    (b) Define a Pareto efficient allocation.

    (c) Define a Walrasian equilibrium.

    (d) State the first welfare theorem.

    (e) Fill the underlined parts of the proof of the first welfare theorem. "Suppose that x is a *Walrasian equilibrium allocation* and that there is a feasible allocation x' that Pareto-dominates x. Then, for all $i \in I$, ____________________ follows, and for some $\tilde{\imath} \in I$, ____________________ follows. Thus, we have ________________. Under the assumption of ____________________, we know the following: "If x_i is a maximal for $\succsim_i$ in $\{x'_i \in \mathbb{R}^L_+ : p \cdot x'_i \leq p \cdot \omega_i\}$. Then, for any $\tilde{x}_i \in \mathbb{R}^L_+$ with $\tilde{x}_i \succsim_i x_i$, we have $p \cdot \tilde{x}_i \geq p \cdot \omega_i$." Thus, we have ____________________. This contradicts with ________________."

3. True or False: **Explain the reasonings.**

   (a) In a production economy, suppose that each firm's profit is distributed by its workers equally instead that it is taken by shareholders (labor-managed firms). Each worker chooses a job that maximizes the sum of wage income and profits (assume homogenous labor and working condition). In this economy, a market equilibrium is still Pareto efficient.

   (b) In an Edgeworth box economy (2 person, 2 commodities), the Walrasian equilibrium always exists as long as consumers' preferences and endowments are identical, even if preferences are nonconvex.

   (c) Consider Edgeworth box economies with strong monotonicity and strictly convex preferences (2 person). If there are multiple Walrasian equilibria then the set of Walrasian equilibria is a strict subset of the core.

4. True or False: **Explain the reasonings.**

   (a) Consider a third-price sealed bid auction: that is, the highest bidder pays the third price. Since telling the truth is still the dominant strategy, the expected revenue is less than the one under the second-price sealed bid auction.

   (b) In two person two strategy strategic form games, if there are two Nash equilibria in pure strategies then there must be another Nash equilibrium in strictly mixed strategies.

   (c) A Nash equilibrium of a strategic form game is a strategy profile such that no player wants to change her strategy from it. A perfect Bayesian equilibrium in a signalling game is also a strategy profile such that no player wants to change her strategy from it, but the definition of strategy is no longer an action. For Sender, it is a function from a type set to a message set (an action set for Sender), and for Receiver, it is a function of a message set to an action set (an action set for Receiver).

5. Consider the following two person strategic form game.

	L	M	R
T	$5,5$	$0,12$	$2,2$
M	$12,0$	$1,1$	$1,-1$
B	$2,2$	$-1,1$	$4,4$

(a) Find all Nash equilibria including mixed strategy equilibria.

(b) Finitely repeated game. Can we support (T, L) on an SPNE path? If so, how many times must the game be repeated?

(c) Infinitely repeated game with a common discount factor $\delta \in (0,1)$. Can we support a path $\mathbf{a} = \{(T,L),(T,L),...\}$? If so, what is the critical value for δ?

6. Answer **one of the following two questions.**

(a) Consider a Bayesian game $(N,(A_i,T_i,p_i,u_i)_{i\in N})$. State formally and explain the revelation principle. Explain the importance of it by providing an example.

(b) Consider a signalling game (S,R,M,A,T,p,u_S,u_R). We introduce a new refinement criterion (R7) that is a modification of the Intuitive Criterion (R6). For simplicity, we assume that $|T| = 2$: $T = \{t_1, t_2\}$. (R7) goes as follows: "Suppose that for type $t_k \in T$, a message $m_i \in M$ is equilibrium-dominated, then $\mu(t_k|m_i) = 0$ *if possible* (unless m_i is also equilibrium-dominated for the other type). Moreover, if $m_i \in M$ is not equilibrium dominated for any type, then $\mu(t_k|m_i) = p(t_k)$ (prior belief)." Is (R7) stronger or weaker than (R6)? Or is there no logical relationship between the two conditions? Discuss equilibria that satisfy (R7) in the Spence signalling game. Do you find anything in relation to a screening problem a la Rothschild and Stiglitz (1976, QJE)?

PART 2

Alias________________________________

Mark your answers below.

1. a b c d

2. a b c d

3. a b c