BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
August 20, 2010

Directions: There are three parts to this exam. Part 2 has an answer sheet and for Parts 1 and 3, you will need a bluebook. Please follow the instructions for each part carefully. Write your **Alias** on the answer sheet for Part 2 and on Parts 1 and 3 write your **Alias, part number, question number(s)** on the front of each blue book.

Please read the entire exam before writing anything.

Part I. This part consists of a single question that you are required to answer.

An individual has a utility function $U = x_1 x_2$, where x_1 is the number of times per month he travels downtown to attend a concert and x_2 is his monthly consumption of bread. Travelling downtown to attend a concert entails a monetary outlay of p_1 dollars and a time outlay of t_1 hours. Consuming a unit of bread entails a monetary outlay of a dollar but no time outlay. The individual has T hours per month available for concerts and work. Work pays an hourly wage of w and is the individual's only source of income.

(a) Set out the individual's utility maximization problem and derive his demand functions for concerts and bread.

(b) Initially, $t_1 = 6$, $p_1 = 40$, $w = 10$ and $T = 200$. A transportation improvement is possible which would lower t_1 to 5, but which would have to be paid for by imposing a lump sum tax of \$10 per month on every consumer. Calculate the compensating variation that the individual would realize if the transportation improvement along with the tax were put into effect.

PART II

Allocate about two hours for this part. Please answer all questions in this section. Mark your answers on the special answer sheet provided.

Each of the following questions has several possible answers. Mark all the correct answers or statements, and only the correct ones. Important: There may be no correct answers, several correct answers, and it may even happen that all answers are correct!

Your grade for this part (out of 100) will be

$$2x + 6y + 2$$

where x is the number of correct answers you'll give in question 1 and y is the number of correct answers you'll give in questions 2–4.

1. Society is composed of n individuals. For each of the following social aggregation rules, check which properties of Transitivity, Unanimity, IIA, and No Dictatorship they satisfy. Mark them with ✓ in the table. If a property is not always satisfied then don't mark it. In all cases assume that no individual is indifferent between any two options (although society may be indifferent).

   (a) For every x and y, $x \succeq y$ iff there is a person i such that for all $z \neq x, y$, $x \succ_i z \succ_i y$.

   (b) For every x and y, $x \succ y$ iff exactly $n-1$ individuals strictly prefer x to y. Otherwise, $x \sim y$.

   (c) For every x and y, $x \succ y$ iff the majority prefers y to x (assume an odd number of individuals).

   (d) For every x and y, $x \succ y$ iff the majority strictly prefers x to y and $x \succ_1 y$ (here too assume an odd number of individuals).

2. Which of the following statements is correct? Mark them.

   (a) If all preferences are homothetic, then WE is unique.

   (b) If $u_1(x^1, x^2) = \min\{x^1, x^2\}$ then there must be many different WE price vectors.

   (c) Suppose $u_i = u_j$. If in the WE price vector p^*, $p^* \cdot \omega_i > p^* \cdot \omega_j$, then it is possible that $u_i(\omega_i) < u_j(\omega_j)$, but it must still be the case that $u_i(x_i^*) > u_j(x_j^*)$. (As always, the WE is denoted (x^*, p^*). For "True" answer, both parts of the statement must hold).

(d) If $\omega^m = \omega^\ell$, then in WE commodities m and ℓ have the same price.

(e) Brouwer's fixed point theorem is used in proving the equal treatment in the core theorem.

3. (a) The function $u(x) = \ln x$ satisfies CRRA (constant relative risk aversion).

   (b) The preferences $(100, 1) \succ (0, \frac{1}{4}; 100, \frac{1}{2}; 200, \frac{1}{4}) \succ (0, \frac{1}{2}; 200, \frac{1}{2})$ are inconsistent with expected utility theory.

   (c) Insurance markets require that not all clients of a company have the same utility function. If this condition is not satisfied, the market will collapse and no one will buy insurance.

4. (a) Sellers may prefer first price auctions to second price auctions even though in first price auction consumers offer less than their true price.

   (b) If person 1 is a fully discriminating monopolist and person 2 is a price taker, then person 2 has an incentive to misrepresent his preferences.

   (c) The decision maker is facing a probability $p < 1$ of losing a property of value L. There is a price $q^* < 1$ such that if the price of insurance (per dollar) is higher than q^* the decision maker will buy no insurance.

PART III

Micro Theory Comprehensive Exam - Summer 2010
M. Utku Ünver
Suggested Time: 60 minutes

An object will be auctioned among $n > 2$ bidders. Each bidder $i = 1, ..., n$ has a private signal v_i that is the value of the object to the bidder. Other bidders $j \neq i$, who do not observe v_i, believe that i's value is distributed with respect to the distribution function $F(v_i) = v_i$, which has full support on $[0, 1]$. The auction method is the first-price sealed bid auction (i.e., the bidders simultaneously submit their bids; an even lottery determines the winner among the highest bidders; and winner gets the object paying the price he submitted, while all other bidders have payoff 0). Each bidder has a payoff function that is quasi-linear in value and price when he wins (i.e., $v_i - b_i$ is the payoff of bidder i, if he wins and pays b_i).

(1) 75 pt. Find a symmetric Bayesian-Nash equilibrium of this game; show your steps of calculation. [HINT: Each bidder i's equilibrium strategy is linear in v_i.]

(2) 25 pt. If, instead of the first-price format, the object were auctioned through a second-price auction or an ascending-price English auction (i.e., an open dynamic auction where price publicly increases continuously starting from 0 and each bidder's strategy at each point in time is to drop out or continue at the current level of price; once a bidder drops out he cannot continue to the game; the winner is the agent who stays in while all others drop out and he pays the price at which the last agent drops out), which of the three auction formats would raise more (ex-ante) expected revenue for the auctioneer at a symmetric monotonic Bayesian-Nash equilibrium? Why? Briefly explain.

BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
June 11, 2010

Directions: There are three parts to this exam. Part 2 has an answer sheet and for Parts 1 and 3, you will need a bluebook. Please follow the instructions for each part carefully. Write your **Alias** on the answer sheet for Part 2 and on Parts 1 and 3 write your **Alias, part number, question number**(s) on the front of each blue book.

Please read the entire exam before writing anything.

PART I

Part I. This part consists of a single question that you are required to answer.

A consumer maximizes the utility function $U = x_1^{1/2} x_2^{1/2} x_3$. Prices and income are given by $p_1 = 50$, $p_2 = 5$, $p_3 = 1$ and $I = 400$. The individual has a manufacturer's coupon for \$40 off the purchase of 4 units of commodity 1. The individual has only one coupon; use of the coupon requires purchasing 4 units; and the \$40 cost reduction is off of the total cost, as opposed to a reduction per unit of commodity 1.

(a) Determine whether or not the individual will choose to use the coupon (Note: In this and the following part of the problem, certain roots, such as square roots, may arise which are best not rounded off until the final step of your computations).

(b) Calculate the individual's compensating and equivalent variations (CV and EV) for having the coupon.

PART II

Allocate about two hours for this part. Please answer all questions in this section. Mark your answers on the special answer sheet provided.

Each of the following questions has several possible answers. Mark all the correct answers or statements, and only the correct ones. Important: There may be no correct answers, several correct answers, and it may even happen that all answers are correct!

Your grade for this part (out of 100) will be

$$2x + 6y + 4$$

where x is the number of correct answers you'll give in question 1 and y is the number of correct answers you'll give in questions 2–4.

1. The production cost of a public good x is given by $c(x) = x^2$. There are n individuals in society, and they announce the differentiable (but not necesarily monotonic) utility functions $m + u_1(x), \ldots, m + u_n(x)$, where m is money. The functions $u_1, \ldots, u_n$ are defined over the range $[0, \bar{x}]$ (society cannot produce more than $\bar{x}$). Also, society has $\overline{m} > \bar{x}^2$ dollars to pay for the production of x and to be allocated between the n individuals.

   For each of the following production/allocaton mechanism, decide whether it is Pareto efficient, symmetric (in the sense that identical individuals will receive the same outcome), and incentive compatible. In the table provided, mark by ✓ only those properties that are satisfied. A property is satisfied if it is satisfied for all possible utility profiles. In all cases, society will produce x^* units of the public good and will give person i m_i dollars.

   (a) $x^* = \bar{x}/2$, $m_i = \overline{m}/n$, $i = 1, \ldots, n$.

   (b) x^* solves $2x^* = \sum u_i'(x^*)$, $m_i = (\overline{m} - (x^*)^2)/n$, $i = 1, \ldots, n$.

   (c) x^* solves $(x^*)^2 = \sum u_i'(x^*)$, $m_i = 0$, $i = 1, \ldots, n$.

   (d) x^* solves $\sum u_i'(x^*) = 0$, $m_1 = \overline{m} - (x^*)^2$, $m_i = 0$, $i = 2, \ldots, n$.

2. Which of the following statements is correct?

   (a) If the economy has no Walrasian equilibrium, then its core is empty.

   (b) If two individuals have the same utility function and the same initial endowments, then in all efficient allocations they receive the same outcome.

   (c) There are economies with more than 2 individuals where the set of WE allocation is equal to the core.

   (d) If all individuals have the same strictly quasi concave and differentiable utility functions, then the economy has only one WE.

3. (a) There are $n > 3$ individuals numbered $1, \ldots, n$, and all have single-peaked preferences over $\Re$. The social ranking $x \succeq y$ iff a majority of the individuals $\{1, 2, 3\}$ weakly prefers x to y satisfies all of Arrow's axioms except for universal domain.

   (b) Harsanyi utilitarianism violates IIA.

   (c) Sen's liberalism violates IIA.

   (d) Arrow, Sen, Harsanyi, and Diamond won the Nobel prize in economics.

4. (a) John holds a ticket for lottery X. Jane paid him y dollars, and he gave her the ticket. Both are happy. If $y > \mathrm{E}[X]$, then they are not all risk loving.

(b) $u(x) = e^{-\alpha x}$, $0 < \alpha < 1$ represents CARA.

(c) If the two strictly concave functions u and v represent CARA, then so does uv.

(d) If X dominates Y by both FOSD and SOSD, then $X = Y$.

PART III

Micro Theory Comprehensive Exam - Spring 2010

Utku Unver

60 minutes

Suggestion: Divide your time according to the worth of each question.

1. 25 pts Consider a two player game played on a rectangular array of $m \times n$ cells numbered (i, j), where $1 \leq i \leq m$ and $1 \leq j \leq n$, and m and n are finite integers exceeding 1. All cells are initially unoccupied. Players take turns to occupy an additional cell, with the rule that if cell (i, j) has been previously occupied by either player, then neither player can subsequently occupy any cell (i_0, j_0) for which both $i_0 \geq i$ and $j_0 \geq j$. The player forced to occupy $(1, 1)$ is the loser.

Is there a player with a winning strategy? If so, is it the first mover or the second mover? Prove or disprove.

HINT: Do not try to construct a winning strategy.

2. 50 pts

Consider an n firm homogeneous product industry where firm i produces output q_i at cost $c_i(q_i) = cq_i$ for some constant $c > 0$. Price is: $p = \alpha - \beta Q$ where $Q = q_1 + ... + q_n$, and $\alpha > 0$, $\beta > 0$ are constants. Suppose that $\alpha - c > 0$.

25 pts (a) What are the firms' outputs in the Cournot equilibrium?

25 pts (b) Suppose that there are $n = 2$ firms, and each firm i has cost $c_i(q_i) = c_i q_i$ where c_1 is private information to firm 1, while c_2 is common knowledge to both firms. Suppose that c_1 is distributed in $[0, 1]$ with a continuous density that has a p.d.f. $f(c_1) = \frac{2\beta + c_1}{2\beta + 1/2}$, and this is common knowledge. First firm 1 finds out c_1 and c_2 is announced to both players. Then each firm announces some quantity to produce. Find the Cournot Bayesian-Nash equilibria of this game.

3. 25 pts The following mechanisms always satisfy which of given properties? Only write in your answer sheet the correct properties, no explanation is needed.

(a) Groves Mechanism in the quasi-linear domain:

Strategy-proof, Efficient, Individually rational, Coalitionally strategy-proof, Nonbossy

(b) Serial Dictatorship in the indivisible resource allocation without transfers domain under strict preferences

Strategy-proof, Pareto efficient, Individually rational, Coalitionally strategy-proof, Non-bossy

(c) Uniform Rule in the allocation of a perfectly divisible good with single-peaked preferences

Strategy-proof, Pareto efficient, Individually rational, Coalitionally strategy-proof, Non-bossy

(d) First-Price Auction in the quasi-linear domain:

Bayesian incentive-compatible, Efficient, Individually rational, Non-bossy, Strategy-proof

BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
August 21, 2009

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For Part 2, write your Alias, part number, question number(s) on the front of each blue book.

Please read the entire exam before writing anything.

PART 1

Allocate about three hours for this part. Answer all questions in this section. Mark your answers on the special answer sheet provided.

In all questions you have to fill in a table. Check carefully the instructions provided.

Your grade for this part (out of 100) will be

$$4 + 4x$$

where x is the number of correct answers in all questions together.

1. Society needs to allocated the bundle $x = (x^1, \ldots, x^k)$ between n individuals. Each of these individuals has no other resources and each of them will declare his preferences over $\Re_+^k$ which must be strictly convex. For mechanisms (a)–(d), check which of properties 1–4 are satisfied. In the table on the last page, mark + is this property must be satisfied, ∘ if it may be satisfied but it may also not be satisfied, and × if this property is never satisfied.

Mechanisms

(a) Give everything to person 1, zero to everyone else.

(b) Give everyone the same share, that is, $\frac{x}{n}$.

(c) Allocate x to $x_1, \ldots, x_n$ such that the allocation is efficient and $u_1(x_1) = \ldots = u_n(x_n)$. Remark: Such an allocation exists, and it is unique.

(d) Divide it equally between all individuals i who satisfy for all j, $u_i(x) \geqslant u_j(x)$.

Properties

1. Efficiency
2. Individual rationality
3. Symmetry
4. Dominant strategy

2. For each of the utility functions (a)–(d), decide which of the properties 1–4 it satisfies. In the table on the last page, mark + is this property must be satisfied, $\circ$ if it may be satisfied but it may also not be satisfied, and $\times$ if this property is never satisfied. All functions are defined (only) on $\Re^n_+$.

Utilities

(a) $u(x^1, \ldots, x^k) = \sum \alpha^j \ln x^j$, $\alpha^1, \ldots, \alpha^i > 0$.

(b) $u(x^1, \ldots, x^k) = \sum u^j(x^j)$ where $u^1, \ldots, u^k$ are strictly concave.

(c) $u(x, y) = e^x e^{\sqrt{y}}$

(d) $u(x^1, \ldots, x^k) = \sum (x^j)^j$ (sum x^j to the power of j).

Properties

1. Homotheticity
2. Strict quasi concavity
3. Quasi linearity
4. Strict monotonicity (that is, $\forall j$, $x^j \geqslant y^j$ and there exists j such that $x^j > y^j$ imply $u(x) > u(y)$).

3. For each of the preferences over lotteries that are represented by the functions (a)–(d), decide which of the properties 1–4 it satisfies. In the table on the last page, mark + is this property must be satisfied, ∘ if it may be satisfied but it may also not be satisfied, and × if this property is never satisfied. All functions are defined (only) on $\Re^n_+$. As usual, $X = (x_1, p_1; \ldots; x_n, p_n)$.

Representation functions

(a) Expected utility with the vNM function
$$u(x) = \begin{cases} x & x < 10 \\ 5 + \frac{x}{2} & x \geqslant 10 \end{cases}$$

(b) Expected utility with the vNM function $u(x) = x^\alpha$, $\alpha \in (0, 1)$.

(c) Mean variance: $V(X) = f(\mu_X, \sigma_X)$.

(d) Quadratic utility:
$$V(X) = \sum_i \sum_j p_i p_j \varphi(x_i, x_j)$$
where $\varphi(x, y) = \varphi(y, x)$.

Properties

1. Risk aversion
2. Constant absolute risk aversion
3. The decision maker will buy full insurance if and only if the price of dollar insurance equals the probability of loss.
4. The decision maker's preferences are consistent with the paradox of Allais (that is, $(5, 0.1; 0, 0.9) \succ (1, 0.11; 0.0.89)$ but $(1, 1) \succ (5, 0.1; 1, 0.89; 0, 0.01)$).

PART 2

Micro Theory Comprehensive Exam

Utku Ünver

60 minutes

Total 100 pts.

Suggestion: Divide your time according to the worth of each question.

1.50 pts. Consider an n firm homogeneous product industry where firm i produces output q_i at cost $c_i(q_i) = cq_i$ for some constant $c > 0$. Price is: $p = \alpha - \beta Q$ where $Q = q_1 + ... + q_n$, and $\alpha > 0$, $\beta > 0$ are constants. Suppose that $\alpha - c > 0$.

25 pts (a) What are the firms' outputs, prices and profits in the Cournot equilibrium? What happens as $n \to \infty$? Comment.

25 pts (b) If two firms merge, then the merged firm has marginal cost of c, just as before, and they share their merged firm profit using some division rule which is common knowledge. Consider the following extensive game: Firm 1 proposes a merger to one of the other $n-1$ firms or does not offer merger to any other firm. All firms observe firm 1's action. Then in the second stage the firm which receives an offer (if there is one) accepts or declines a merger with firm 1. All other firms observe this action. Then they play a Cournot game in the third stage. If at a subgame perfect equilibrium of this game, we observe a merger between firm 1 and one of the other $n-1$ firms, then what does n satisfy (i.e., what is the range of n)? Show your work.

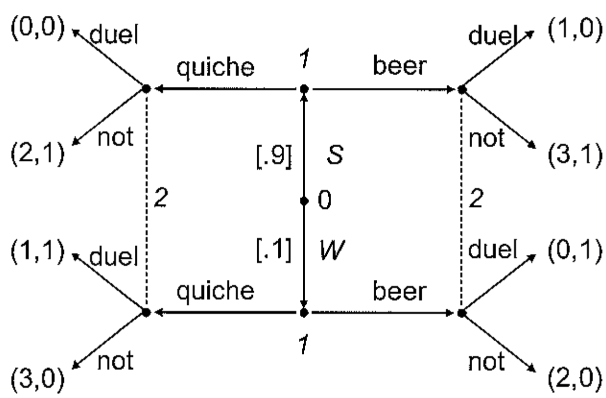
2. 50 pts. Show your work:

15 pts (a) Find all subgame-perfect Nash equilibria of the following game.

15 pts (b) Find all perfect-Bayesian equilibria of the following game.

10 pts (c) Find all sequential equilibria of the following game.

10 pts (d) Which of the above equilibria satisfy the intuitive criterion?



ALIAS: ____________________

Mark your answers below.

1.

	1	2	3	4
a				
b				
c				
d				

2.

	1	2	3	4
a				
b				
c				
d				

3.

	1	2	3	4
a				
b				
c				
d				

BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
June 5, 2009

Directions: There are four parts to this exam. Parts 1, 2 & 3 have an answer sheet and for Part 4, you will need a bluebook. Please follow the instructions for each part carefully. Write your **Alias** on the answer sheet for Parts 1, 2 and 3.

For Part 4, write your **Alias, part number, question number(s)** on the front of each blue book.

Please read the entire exam before writing anything.

PARTS 1, 2, and 3

Allocate about three hours for this part. Please answer all questions in this section. Mark your answers on the special answer sheet provided.

Each of the following questions has several possible answers. Mark all the correct answers or statements, and only the correct ones. Important: There may be no correct answers, several correct answers, and it may even happen that all answers are correct!

Your grade for this part (out of 100) will be

$$4 + 4x + 8y$$

where x is the number of correct answers in question 1–4 and y is the number of correct answers in question 5.

1. (a) The following choice function c satisfies the weak axiom of revealed preferences: $X = \{1, \ldots, 100\}$, for $\varnothing \neq A \subset X$,

$$c(A) = \begin{cases} \max\{x : x \in A\} & |A| \text{ is odd} \\ \\ \min\{x : x \in A\} & |A| \text{ is even} \end{cases}$$

where $|A|$ is the number of items in A.

(b) $X = [0, 60)$ (the set of numbers x such that $0 \leqslant x < 60$). For $x, y \in X$, $x \succeq y$ iff one of the following happens:

i. $x \geqslant y$ and $x - y \leqslant 30$

ii. $x < y$ and $y - x > 30$

The relation $\succeq$ is transitive.

(c) $X = \{1, \ldots, 100\}$. Define $x \succeq y$ iff $x - y$ is a positive prime number. Then $\succeq$ is a complete relation.

(d) $X = \{(x, y) : x, y \text{ are integers}\}$. Define $(x, y) \succeq (x', y')$ iff $x > x'$, or $x = x'$ and $y \geqslant y'$. Then $\succeq$ cannot be represented by a real function. That is, there is no function $u : X \to \Re$ such that $(x, y) \succeq (x', y')$ iff $u(x, y) \geqslant u(x', y')$.

2. Consider the utility function $u(x, y) = x + xy + y$

(a) This function is quasi concave.

(b) The demand function for x is given by

$$D^x(p_x, p_y, m) = \frac{p_y - p_x + m}{2p_x}$$

where p_x and p_y are the prices of x and y and m is the consumer's income (all in dollars).

(c) If $m = 50$, $p_x = 10$, and $p_y = 60$, then $D^y(p_x, p_y, m) = 0$.

(d) By Walras law, such utility functions cannot be used in the analysis of Walrasian equilibrium.

3. In all parts, assume weak quasi concave utility functions.

   (a) If an outcome in the core is efficient, then it must be Walrasian equilibrium.

   (b) In Walrasian equilibrium, identical consumers receive the same outcome.

   (c) Walrasian equilibrium satisfies the No Envy condition: For all i and j, i weakly prefers his equilibrium outcome to that of j.

   (d) Walrasian equilibrium satisfies the following condition: There are no i and j such that $x_j^* \succ_i x_i^*$ and $x_i^* \succ_j x_j^*$.

4. (a) The relation $\succsim$ defined by "$F \succsim G$ iff F weakly dominates G by FOSD" is transitive.

   (b) The relation $\succsim$ defined by "$F \succsim G$ iff F weakly dominates G by FOSD" is complete.

   (c) If F dominates H by FOSD and G dominates H by FOSD, then $\frac{1}{2}F + \frac{1}{2}G$ dominates H by FOSD.

   (d) If F does not dominate H by FOSD and G does not dominate H by FOSD, then $\frac{1}{2}F + \frac{1}{2}G$ does not dominate H by FOSD.

5. (a) A risk averse expected utility decision maker has \$$m$ which he wants to allocate between two assets, A and B. Each unit of asset A yields \$$\alpha$ if event E happens, 0 otherwise, while each unit of asset B yields \$$\beta$ if E does not happen, 0 otherwise. The probability of E is p. The prices of units of A and B are \$$r$ and \$$s$, respectively. If $\alpha ps = \beta(1-p)r$, then after optimizing, the decision maker doesn't care whether E happens or not.

(b) The following is known about an expected utility decision maker.

i. His vNM utility function is

$$u(x) = \begin{cases} \sqrt{x} & x \leqslant 100 \\ 5 + \frac{1}{2}\sqrt{x} & x \geqslant 100 \end{cases}$$

ii. His wealth is w, and he is facing the risk of losing L of it with probability $p > 0$. (With probability $1 - p$, nothing will be lost).

iii. The price of insurance, per unit, is $q > p$.

iv. He buys full insurance.

Then $w - L < 100 < w$.

(c) If u and v are CRRA (constant relative risk aversion) vNM functions, then so is uv.

(d) Let u and v be vNM functions. Define $X \succeq Y$ iff

$$\min\{\mathrm{E}[u(X)], \mathrm{E}[v(X)]\} \geqslant \min\{\mathrm{E}[u(Y)], \mathrm{E}[v(Y)]\}$$

Then $\succeq$ satisfies the independence axiom.

Micro Theory Comprehensive Exam - Spring 2009
Part 4 - Game Theory and Information Economics
60 minutes

1.50 pts In many markets, consumers have switching costs. Consider the following simple model of such a market: Two firms A and B simultaneously and non-cooperatively set prices in a single period for a commodity that they can each produce at zero cost. Each consumer has at most demand of 1 for this indivisible good. There are $n + s$ customers, where $n > 0$ and $s > 0$ all with reservation price R, i.e., they do not buy the commodity, if the price exceeds R. Because of switching costs, $s/2$ customers can only buy from A and $s/2$ can only buy from B. The n "new" customers buy from the cheapest firm, if at all.

25 pts (a) Show that there are no pure strategy equilibria.

25 pts (b) Find a symmetric mixed-strategy Nash equilibrium in which each firm i =A,B chooses price p_i according to a common cumulative distribution function $F(p_i)$. (Hint: F is continuous, with no atoms, and supported over an interval of prices. Now, for a given p_i, calculate expected profit of firm i. What can you say about all prices in the support of the mix? What value does p_i take when $F(p_i) = 1$? You should now be able to find F. Over what interval of prices do the firms mix?)

2. 50 pts Consider an infinitely repeated game where the stage game is:

	L	R
U	9, 9	1, 10
D	10, 1	7, 7

Players discount the future using the common discount factor δ.

9 pts (a) What outcomes in the stage-game are consistent with Nash equilibrium play?

9 pts (b) Let v_1 and v_2 be the average repeated game payoffs to Row Player (Player 1) and Column Player (Player 2) respectively. Draw the set of feasible (average) payoffs from the repeated game, explaining any normalization you use.

9 pts (c) Are all the payoffs in the feasible set obtainable from mixed-strategy combinations in the stage-game? (That is, for every point in the feasible set, can you find a p such that $p \in [0, 1]$ and a q such that $q \in [0, 1]$ that will give those expected payoffs from a single play?)

9 pts (d) What are the players' minmax values? Show the individually rational feasible set.

14 pts (e) Find a subgame-perfect Nash equilibrium in which the players obtain the $(9, 9)$ payoff each period forever. What restrictions on δ are necessary?

ALIAS:____

Mark your answers below--for Parts 1, 2, and 3

1. a b c d

2. a b c d

3. a b c d

4. a b c d

5. a b c d

BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
August 15, 2008

Directions: There are two parts. Please follow the instructions for each part carefully. Write the answer to Part I and Part II in a separate bluebook.

Write your alias, Part number, question number(s) on the front of each blue book

Please read the entire exam before writing anything.

PART I

Allocate about three hours for this part. Please answer all questions in this section. All questions have the same value.

1. For each of the following statements decided whether it is true or false. Explain your answers. Answers without an explanation will receive no points.

   (a) In WE, two individuals with different utility functions and different initial endowments cannot receive the same allocation.

   (b) If the functions $u : \Re^n_+ \to \Re_+$ and $v : \Re^n_+ \to \Re_+$ represent homothetic preferences, then so does $u \cdot v$.

   (c) Under utilitarianism, two individuals with the same utility function will receive the same outcome.

2. (a) Write the axioms and the statement of Arrow's impossibility theorem.

   (b) Write no more than one page on the meaning and importance of this theorem.

3. $u_1(x, y) = xy$, $u_2(x, y) = \min\{x, y\}$, $u_3(x, y) = x + y$.
   $\omega_1 = (10, 0)$, $\omega_2 = (0, 10)$, $\omega_3 = (10, 10)$.

   (a) What is the set of efficient allocations?

   (b) What is the core?

   (c) What is the set of WE?

   (d) Suppose 3 is a monopolist, what prices will he set for x and y? (He cannot discriminate between 1 and 2).

4. For each of the following statements decided whether it is true or false. Explain your answers. Answers without an explanation will receive no points.

   (a) There are vNM utilities u and v, and there is $\alpha > 0$ such that for all w, $R^v_A(w) = \alpha R^u_A(w)$. Then for all lotteries X and Y, $\mathrm{E}[u(X)] \geqslant \mathrm{E}[u(Y)]$ iff $\mathrm{E}[v(X)] \geqslant \mathrm{E}[v(Y)]$.

(b) A decision maker has the following preferences: $(200, 1) \succ (300, \frac{1}{3}; 200, \frac{1}{3}; 100, \frac{1}{3})$ and $(-300, \frac{1}{3}; -200, \frac{1}{3}; -100, \frac{1}{3}) \succ (-200, 1)$. Then he must violate at least one of the axioms of expected utility theory.

(c) It follows from the independence axiom that if $X \succ Y$ (X and Y are lotteries), then for all $p \in (0, 1)$, $X \succ (X, p; Y, 1-p) \succ Y$.

PART II

Allocate about 60 minutes for this part (Game Theory).
Answer the following two questions.

1. Consider a symmetric Bertand duopoly problem with heterogeneous commodities. Suppose that firm $i, j \in \{1, 2\}$ with $i \neq j$ has following demand functions:

$$D_i(p_i, p_j) = 1 - p_i + \frac{1}{2}p_j,$$

where p_i and p_j are prices of firms i and j, respectively. Marginal cost of production is $c = \frac{1}{4}$.

(i) Write down firm i's profit function. (5 points)

Static Game

(ii) Supposing two firms choose prices simultaneously, formally write down the Bertand duopoly problem as a strategic form game. (5 points)

(iii) Draw best response curves and iso-profit curves of firms in a figure clearly. (12 points)

(iv) Calculate a Bertand equilibrium. (8 points)

Sequential Move Game

(v) Now, suppose that firm 1 chooses p_1 first, then by observing it firm 2 chooses p_2. Draw a game tree of this sequential move game. (5 points)

(vi) Point out an equilibrium allocation of the sequential move game in the best-response figure. Which firm's price is higher? Which firm's profit is higher? Discuss. (20 points)

Repeated Game

(vii) Find joint profit maximizing allocation (cartel strategy profile). (7 points)

(viii) Consider a infinitely repeated game of the above with a common discount factor $\delta \in (0, 1)$. Find a condition on δ that supports the joint profit maximizing allocation (repeatedly forever). (8 points)

2. A city office is planning to build a bridge, and is trying to choose a construction company that builds it. There are two construction companies $\{1, 2\}$ that can build the bridge. Company's building cost of the bridge is a random variable $\mathbf{c}$ that is independently distributed over the interval $[0, 1]$ with density $f(c)$, which is a common knowledge. Company i's actual (realized) building cost of the bridge is denoted by c_i. Each company's building cost is a private information. The two firms submit asking price to the city office simultaneously. The lower price company gets the job and builds the bridge at its building cost by receiving its asking price. Before firms submit their asking prices, the city office announces the cut-off cost $\bar{c} \in (0, 1)$ to the firms: the city gives the job to one of the firms only if it submits its asking price less than or equal to $\bar{c}$.

   (a) Calculate a symmetric Bayesian equilibrium strategy. (15 points)

   (b) What is the counterpart of the second price sealed bid auction? Is the truth-telling strategy a weakly dominant strategy in this particular problem? (8 points)

   (c) What is the counterpart of the revenue-equivalence theorem? (7 points)

BOSTON COLLEGE
Department of Economics

Microeconomics Theory Comprehensive Exam
June 6, 2008

Directions: There are two parts. Please follow the instructions for each part carefully. Write the answer to Part I in a separate bluebook and please use the answer sheet for Part II.

Write your alias, Part I, question number(s) on the front of each blue book used and write your alias on the Part II answer sheet.

Please read the entire exam before writing anything.

PART I

Allocate about 60 minutes for this part.
Answer all three questions.

1. (Bayesian Nash Equilibrium) Everybody knows that truth-telling is the equilibrium in the second price sealed-bid auction. Is there any direct relationship between the above fact and the revelation principle? Explain clearly.

2. (Nash equilibrium) Consider the following two player game: $N = \{1,2\}$, $S_1 = S_2 = [0,6]$, and

$$\begin{aligned} u_1(s_1,s_2) &= -s_2(s_1 - \frac{s_2}{2} - 1)^2, \\ u_2(s_1,s_2) &= -s_1(s_2 - \frac{s_1}{2} - 1)^2. \end{aligned}$$

Answer the following questions.

   (a) Does this game satisfy the conditions required in Debreu's existence theorem for Nash equilibrium?

   (b) Find all pure strategy Nash equilibria by drawing the best response curves in a diagram.

3. (subgame perfect equilibrium) Consider the following two-person game. There are multi-periods ($t = 1, 2, 3, ...$), and players 1 and 2 play alternately. Player 1 gets her turn in odd periods, player 2 gets her turn in even periods. There are two urns A and B with rewards inside. In period 1, urns A and B have \$4 and \$1, respectively, and every period the amount of money in each urn is doubled. That is, in period t, urns A and B have $\$2^{t+1}$ and $\$2^{t-1}$, respectively. In each period, the player who has turn can end the game by taking urn A or can pass her turn. In the former case (the player chooses "end"), the other player can take urn B, and the game ends. In the latter case (the player chooses "pass"), then the rewards are doubled and the next period comes. Answer the following questions.

   (a) Suppose that $t \in \{1, 2, ..., 10\}$, and in period 10, there is no option of "pass." What is subgame perfect equilibrium?

(b) Now, the main problem. Suppose that there is no definite ending period in this game, but the game ends stochastically. In the beginning of each period, with a constant probability $p > \frac{1}{2}$, the option of passing disappears (which means that the game has to end in that period, and the player who has her turn in that period takes urn A and the other takes urn B). Consider the following strategy: Whenever a player gets her turn, she passes unless the option of "pass" disappears. Under what value of p, is the symmetric strategy profile of the above strategy a subgame perfect Nash equilibrium?

PART II

Allocate about three hours for this part. Please answer all questions in this section. Mark your answers on the special answer sheet provided.

Each of the following questions has several possible answers. Mark all the correct answers, and only the correct answers. Important: There may be no correct answers, several correct answers, and it may even happen that all answers are correct!

All XX questions have the same value of Y points.

1. Which of the following preference relations can be represented by a real function? In all cases, the preferences are over a set A.

   (a) $A = \Re^4_+$, $x = (x_1, \ldots, x_4) \succeq y = (y_1, \ldots, y_4)$ iff $x_1 x_2 > y_1 y_2$, or $x_1 x_2 = y_1 y_2$ and $x_3 + x_4 \geqslant y_3 + y_4$.

   (b) $A = \Re^2_+$, $x = (x_1, x_2) \succeq y = (y_1, y_2)$ iff $x_1 - x_2 \geqslant y_1 - y_2$.

   (c) $A = \Re$, $x \succ y$ iff x is rational and y is irrational. If x and y are both rational or both irrational, then $x \sim y$.

   (d) $A = \Re$. $x \succeq y$ iff $[x] \geqslant [y]$, where for $x \in \Re$, $[x]$ is largest integer z such that $z \leqslant x$.

2. Which of the following functions is quasi concave?[1] Mark them in the answer sheet.

   (a) $u : \Re^3_+ \to \Re$ is given by $u(x_1, x_2, x_3) = x_1 x_2 + x_1 x_3 + x_2 x_3$.

   (b) $u : \Re^n \to \Re$ is given by

   $$u(x_1, \ldots, x_n) = \begin{cases} \min\{x_i\} & \min\{x_i\} \leqslant 1 \\ 1 + (x_1 - 1) \times \ldots \times (x_n - 1) & \min\{x_i\} > 1 \end{cases}$$

   (c) $u : \Re \to \Re$ is given by $u(x) = x + x^3$.

   (d) $u : \Re^2_+ \to \Re$ is given by $u(x_1, x_2) = -x_1^2 - x_2^2$.

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[1] A function is quasi concave if for all x, the set $\{y : u(y) \geqslant u(x)$ is concave.

3. Mark the correct statements.

(a) The function $u : \Re^3_+ \to \Re$, given by

$$u(x_1, x_2, x_3) = \begin{cases} x_1^2 + x_2^2 + x_3^2 & x_1 \geqslant x_2 \geqslant x_3 \\ \\ x_1x_2 + x_1x_3 + x_2x_3 & \text{otherwise} \end{cases}$$

is homothetic.

(b) Let $X = x_1 < \ldots < x_k$. The median of X, $m(X)$, is given by

$$m(X) = \begin{cases} x_{k/2} & k \text{ is even} \\ \\ x_{(k+1)/2} & k \text{ is odd} \end{cases}$$

Let $A = \{1, \ldots, 100\}$. Define a choice function c over all subsets of A such that for $X \subseteq A$, $c(X) = m(X)$. This function cannot be rationalized by a preference relation over A.

(c) The utility function $u(x_1, \ldots, x_n)$ implies nondifferentiable demand functions.

(d) A preference relation satisfying: "If $x \succ y$ and $y \succ z$ then $x \succ z$" is transitive.

4. Mark the correct statements.

(a) WE $\subseteq$ Core $\subseteq$ PE.

(b) The set of WE allocations is convex.

(c) The core is convex.

(d) The set of efficient allocations is convex.

5. Below is the proof of the equal-treatment-in-the-core theorem we had in class. The lines, alas, are not in the right order. What is the right order of the lines?

   (a) We show that the coalition S consisting of all agents # 1 in the n types can be improved upon x.
   (b) $\sum_{i=1}^{n} \bar{x}_i = \sum_{i=1}^{n} \frac{1}{r} \sum_{j=1}^{r} x_i^j =$
   (c) If not all of type 1 agents receive the same outcome,
   (d) $\frac{1}{r} \sum_{i=1}^{n} r\omega_i = \sum_{i=1}^{n} \omega_i$
   (e) Let x_i^j, $i = 1, \ldots, n$, $j = 1, \ldots, r$ be the outcome of agent j in type i.
   (f) $\bar{x}_i = \frac{1}{r} \sum_{j=1}^{r} x_i^j$
   (g) then $\bar{x}_1 \succ_1 x_1^1$
   (h) For each type, assume, wlg, that x_i^1 is a worst of its type.
   (i) which is the average basket of type i.
   (j) That is, for all j, $x_i^j \succeq_i x_i^1$.
   (k) Hence feasibility.
   (l) Assume that agents of type 1 do not all receive the same outcome.
   (m) and if $\forall j \, x_1^j \succeq_1 x_1^1$,
   (n) We'll do it through $\{\bar{x}_1, \ldots, \bar{x}_n\}$ where
   (o) $\frac{1}{r} \sum_{i=1}^{n} \sum_{j=1}^{r} x_i^j = \frac{1}{r} \sum_{i=1}^{n} \sum_{j=1}^{r} \omega_i^j =$

6. Each of n individuals announces his utility function from a costless public good $a \in [0, 1]$. Which of the following mechanisms is DS (dominant strategy)?

   (a) Choose an allocation that maximizes the sum of utilities. If there are several such points, pick one at random.
   (b) Let person n be a dictator.
   (c) Produce 0.37 units of the public good.
   (d) Produce $a = \min\{a_i^*\}$, where a_i^* is the minimal quantity of the public good person i finds to be optimal (assume utility functions have to be continuous).

7. Which of the following statement is true?

   (a) Define the following preference relation over lotteries: $X \succeq Y$ iff $F_X(5) \leqslant F_Y(5)$.[2] Then $\succeq$ satisfies the independence axiom.

   (b) The condition "If $X \succeq Y$ then for all $\alpha \in [0,1]$, $X \succeq (X, \alpha; Y, 1-\alpha) \succeq Y$" is equivalent to the independence axiom.

   (c) Expected utility is the only theory to satisfy the independence axiom.

   (d) By the independence axiom, if $X \sim Y$, then for all $\alpha \in [0,1]$, $(X, \alpha; Y, 1-\alpha) \sim (X, 1-\alpha; Y, \alpha)$.

8. Mark the correct statements.

   (a) If the vNM utility u represents risk aversion and h is an increasing concave function, then $v = h \circ u$ too represents risk aversion.

   (b) If the vNM functions u and v represent risk loving, then so does the function $w(x) = \min\{u(x), v(x)\}$.

   (c) An expected utility maximizer cannot buy both insurance and a lottery ticket.

   (d) The vNM function $u(x) = 1 - e^{-\alpha x}$ represents CARA (constant absolute risk aversion).

9. Mark the correct statements.

   (a) Harsanyi's axioms are consistent with the social welfare function $W(p) = \sum [v_i(p)]^3$, where $v_i(p) = [\sum p_j u_i(x_j)]^{1/3}$. In this question, p is a lottery over all possible social policies.

   (b) Arrow's impossibility theorem fails when the domain of preferences are lotteries and all individuals are expected utility maximizers.

   (c) The social ranking $x \succ y$ iff for the lowest i such that $x \not\sim_i y$, $x \succ_i y$ violates IIA (Independence of Irrelevant Alternatives axiom)

   (d) Arrow, Harsanyi, and Sen won the Nobel prize in Economics.

[2] F_X and F_Y are the distribution functions of the lotteries X and Y, respectively.

ANSWER SHEET FOR PART II

ALIAS: ____

Mark your answers below.

1. a b c d

2. a b c d

3. a b c d

4. a b c d

5. ___ ___ ___ ___ ___ ___ ___ ___

___ ___ ___ ___ ___ ___ ___

6. a b c d

7. a b c d

8. a b c d

9. a b c d