Solutions to Problem Set 1 (Due September 18)

EC 228 01, Fall 2013 — Prof. Baum, Mr. Lim

Maximum number of points for Problem set 1 is: 40

Problem A.7.

(i) (2 pts.) By exponentiating the left and right sides of the equation we get:

$$Salary = e^{10.6+.027exper}$$

Therefore, for $exper = 0$, we have $Salary_1 = e^{10.6} \approx 40134.84$, and for $exper = 5$, $Salary_2 = e^{10.6+.027*5} = e^{10.735} \approx 45935.80$.

(ii) (2 pts.) $\frac{Salary_2 - Salary_1}{Salary_1} \approx \ln Salary_2 - \ln Salary_1 = .735 - .6 = .135$

(iii) (2 pts.) $\frac{Salary_2 - Salary_1}{Salary_1} = \frac{45935.80 - 40134.84}{40134.80} \approx .144 > .135$

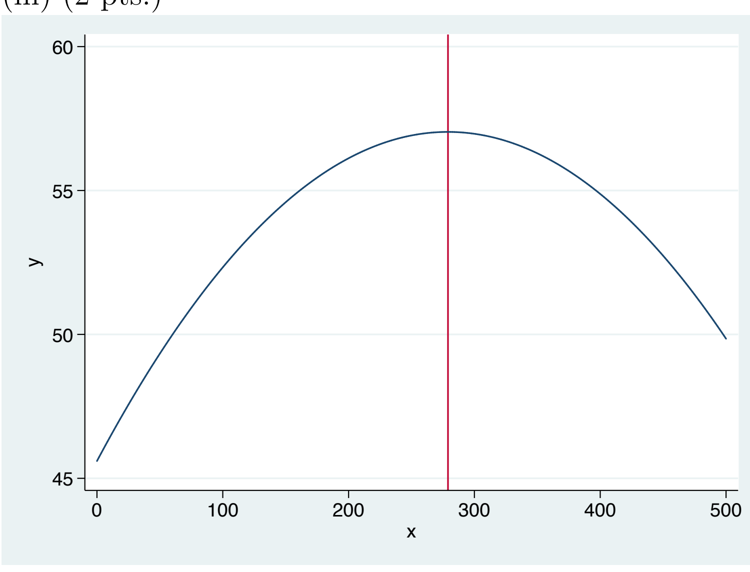
Problem A.10.

(i) (2 pts.) It means that the average score is 45.6 when the class size is zero. It is not of much interest because we are not interested in the case of no class.

(ii) (2 pts.) $\dfrac{\partial score}{\partial class} = 0.082 - 2 \times 0.000147 \times class = 0 \implies class = 278.91 \implies 279$ students.

The highest score $= score(class = 279) = 45.6 + 0.082 \times 279 - 0.000147 \times (279)^2 = 57.04$.

(iii) (2 pts.)



(iv) (2 pts.) They don't seem likely have a deterministic relationship because there are probably other factors affecting the score beyond the class size.

Problem B.2.

(i) (2 pts.) P$(X \leq 6) = P[(X-5)/2 \leq (6-5)/2] = P(Z \leq 0.5) \approx 0.692.$, where Z denotes a Normal(0,1) random variable. [We obtain $P(Z) \leq 0.5$ from Table G.1]

(ii) (2 pts.) $P(X > 4) = P[(X-5)/2 > (4-5)/2] = P(Z > -0.5) = P(Z \leq 0.5) \approx 0.692.$

(iii) (2 pts.) $P(|X-5| > 1) = P(X-5 > 1) + P(X-5 < -1) = P(X > 6) + P(X < 4) \approx (1-0.692) + (1-0.692) = 0.616$, where we have used answers from parts (i) and (ii).

Problem B.4.

(3 pts.) $Pr(X \geq .6) = 1 - Pr(X < .6) = 1 - F(.6) = 0.3520.$

Problem C.1.

(i) (2 pts.) This is just a special case of what we covered in the text, with $n = 4: E(\bar{Y}) = \mu$ and $Var(\bar{Y}) = \sigma^2/4$.

(ii)(2 pts.) $E(W) = E(Y_1)/8 + E(Y_2)/8 + E(Y_3)/4 + E(Y_4)/2 = \mu[(1/8) + (1/8) + (1/4) + (1/2)] = \mu(1+1+2+4)/8 = \mu$, which shows that W is unbiased.
Because the Y_i are independent with the same variance σ^2 and because $Var(aY) = a^2Var(Y)$ for any constant a and variable Y,

$$\begin{aligned} Var(W) &= Var(Y_1)/64 + Var(Y_2)/64 + Var(Y_3)/16 + Var(Y_4)/4 = \\ &= \sigma^2((1/64) + (1/64) + (4/64) + (16/64)) = \sigma^2(22/64) = \sigma^2(11/32). \end{aligned} \tag{1}$$

(iii) (2 pts.) Because $11/32 > 8/32 = 1/4, Var(W) > Var(\bar{Y})$ for any $\sigma^2 > 0$, so $\bar{Y}$ is preferred to W because each is unbiased.

Problem C.6.

(i) (1 pt.) $H_0 : \mu = 0$

(ii) (1 pt.) $H_1 : \mu < 0$

(iii) (2 pts.) The standard error of $\bar{y}$ is $s/\sqrt{n} = 466.4/30 \approx 15.55$. Therefore, the t statistic for testing $H_0 : \mu = 0$ is $t = \bar{y}/se(\bar{y}) = -32.8/15.55 \approx -2.11$. We obtain the p-value as

$P(Z \leq -2.11)$, where $Z \sim$ Normal(0,1). These probabilities are in Table G.1: p-value=.0174. Because the p-value is below .05, we reject H_0 against the one-sided alternative at the 5 percent level. We do not reject at the 1 percent level because p-value = $.0174 > .01$.

(iv) (1 pt.) The estimated reduction, about 33 ounces, does not seem large for an entire year's consumption. If the alcohol is beer, 33 ounces is less than three 12-ounces cans of beer. Even if this is hard liquor, the reduction seems small. (On the other hand, when aggregated across the entire population, alcohol distributors might not think the effect is so small.)

(v) (1 pt.) The implicit assumption is that other factors that affect liquor consumption - such as income, or changes in price due to transportation costs, are constant over the two years.

Problem 7

(a) (1 pt.) tab female. There are 252 females (indicated by a 1 in the female column) and 274 males (indicated by a zero in the female column) in this dataset.

(b) (1 pt.) summ wage if female. The average wage for a female is 4.587659.

(c) (1 pt.) summ wage if female, detail. The median wage (50^{th} percentile, or `r(p50)`) for a female is 3.75.

(d) (1 pt.) tab nonwhite. There are 54 nonwhites in this dataset.

(e) (1 pt.) summ wage if nonwhite returns the mean wage for nonwhites, which is 5.475926, summ wage if nonwhite==0 gives the mean wage for whites 5.944174. Thus nonwhites earn, on average, 0.468248 less than whites.