

**BOSTON COLLEGE**  
**Department of Economics**

EC 771B  
Econometrics  
Spring 2001  
Prof. Baum

FINAL EXAM  
9 May 2001

Answer all questions. Total of 170 points. Partial credit given for partial answers.

1. (40 pts) Briefly define each of the following terms in one or two sentences.
  - a. Two-way ANOVA
  - b. Condition number of  $X'X$
  - c. Davidson-MacKinnon  $J$  test (*Greene 302-303; mentioned in lecture*)
  - d. Random effects model
  - e. Arellano-Bond estimator
  - f. Breusch-Pagan test
  - g. KPSS test
  - h. Cointegrating vector

2. (15 pts) Explain how you would derive consistent estimates of the parameters of the model:

$$\begin{aligned}y_t &= \alpha + \beta x_t + \gamma y_{t-1} + \delta y_{t-2} + \epsilon_t \\ \epsilon_t &= \rho \epsilon_{t-1} + u_t\end{aligned}$$

where  $u_t$  is *i.i.d.* Normal.

Since the model has both AR(1) errors and lagged dependent variables, OLS will be inconsistent. An instrumental variables approach would use  $x_{t-1}$  and  $x_{t-2}$  as instruments for  $y_{t-1}$  and  $y_{t-2}$ , respectively. Residuals from the IV estimator can be used to derive a consistent estimate of  $\rho$ , and the quasi-differenced model may then be estimated via feasible GLS:

$$y_t - \hat{\rho} y_{t-1} = \alpha(1 - \hat{\rho}) + \beta(x_t - \hat{\rho} x_{t-1}) + \gamma(y_{t-1} - \hat{\rho} y_{t-2}) + \delta(y_{t-2} - \hat{\rho} y_{t-3}) + u_t.$$

3. (30 pts) In a 1972 article, Williams examined the determinants of Jamaican coffee supply, employing a traditional production function in which capital (the number of coffee trees) and labor determine output. He used an auxiliary function to determine the desired number of trees as a function of the price of bananas  $PB_t$  (an alternative crop) and the price of cocoa  $PC_t$  (a complementary crop). This gives desired output  $S_t$  as a function of  $P_t$ , the price of coffee,  $PB_t$ ,  $PC_t$ , and  $W_t$ , average weekly earnings of farmhands.

a. If the model is specified as  $\ln S_t$  as a linear function of the logarithms of  $P_t$ ,  $PB_t$ ,  $PC_t$ , and  $W_t$ , how would you interpret the coefficients? What econometric problems might you expect to encounter? How would you address these problems? Be explicit.

The coefficients are estimates of the supply elasticities of the regressors. This formulation might exhibit autocorrelation in the errors if equilibrium cannot be reached every year, which is very plausibly the case in this context where trees take time to become productive. An estimator that allowed for AR(1) errors might be called for.

b. An alternative approach would consider the “time-to-build” involved with placing farmland into coffee production, implying the model

$$\ln S_t = \varphi\alpha + \varphi\beta_1 \ln P_{t-1} + \varphi\beta_2 \ln PB_{t-1} + \varphi\beta_3 \ln PC_{t-1} + \varphi\beta_4 \ln W_{t-1} + (1 - \varphi) \ln S_{t-1} + \epsilon_t$$

What is the economic rationale for this approach? How do you interpret the coefficient  $\varphi$ ? What are the expected signs on the  $\beta$  coefficients?

This partial adjustment model is a rate of change formulation derived from  $\frac{S_t}{S_{t-1}} = \left(\frac{S_t^*}{S_{t-1}}\right)^\varphi$  where  $S_t^*$  is the desired level of  $S$  and  $0 \leq \varphi \leq 1$ . Taking logs, this model multiplies the desired level of  $S_t$  by  $\varphi$ , expressing realised  $S_t$  as a convex combination of desired  $S$  and last period’s actual  $S$ . The coefficient  $\varphi$  is the fraction of the gap closed per period. The coefficients on  $P$  and  $PC$  are expected to be positive, and the coefficients on  $PB$  and  $W$  are expected to be negative.

c. In estimating the model of part (b), the author found that the Durbin-Watson statistic was 1.90. Is this cause for concern? Why or why not?

The Durbin-Watson test should not be used on this model, given the presence of a lagged dependent variable, since it will be biased toward 2.0 (non-rejection). The Durbin  $h$  statistic should be used to test for AR(1).

d. In estimating the model of part (b), the author estimated the coefficient on  $\ln S_{t-1}$  to be -0.8607 with a standard error of 0.406. Is this cause for concern? Why or why not?

This is an implausible value for that coefficient, since a negative value for  $(1 - \varphi)$  implies a value of  $\varphi$  in excess of unity, implying a dynamically unstable process. This result strongly suggests that the dynamic model as estimated is misspecified.

e. In a specification similar to part (b), but lacking the lagged  $S$  term and the corresponding  $\varphi$ s, the author reported an estimated value for  $\beta_1$  of 0.7741 with a standard error of 0.255. Based on these results, is coffee supply elastic or inelastic?

Since the confidence interval (at any level of significance) includes values on both sides of unity, supply could be either elastic or inelastic with respect to price.

4. (25 pts) Consider a SUR (seemingly unrelated regression) model with two equations in which  $y_1$  and  $y_2$  are each regressed on the same regressor matrix  $X$ . Prove that the point estimates derived from SUR will be identical to those derived by application of OLS to each equation in turn.

See Greene, pp. 616-617.

5. (35 pts) Write an essay discussing the advantages and disadvantages of employing panel, or longitudinal, data over analysis based on pure cross-section, time-series or pooled cross-section/time-series data sets.

6. (25 pts) Write an essay discussing the importance of unit roots in empirical macroeconomic analysis.