

Stata tip 38: Testing for groupwise heteroskedasticity

Christopher F. Baum
Department of Economics
Boston College
Chestnut Hill, MA 02467
baum@bc.edu

A natural source of heteroskedasticity in many kinds of data is *group membership*: observations in the sample may be a priori defined as members of groups, and the variance of a series may differ considerably across groups. This concept will also apply to the errors from a linear regression. The assumption of homoskedasticity in the relationship may reasonably hold within each group, but not between groups. This assumption most commonly arises in cross-sectional datasets. In economic data, for instance, the groups may correspond to firms in different industries or workers in different occupations. It could also apply in a time-series context: for instance, the variance of daily temperature may not be constant over the four seasons. In any case, a test for heteroskedasticity of this sort should take this a priori knowledge into account.

How might we test for groupwise heteroskedasticity in a variable or in the errors from a regression? In the context of regression, if we can argue that each group's regression equation satisfies the classical assumptions (including that of homoskedasticity), the s^2 computed by `regress` (see [R] `regress`) is a consistent estimate of the group-specific variance of the disturbance process. For two groups, an F test may be constructed, with the larger variance in the numerator; the degrees of freedom are the residual degrees of freedom of each group's regression. Conducting an F test is easy if both groups' residuals are stored in one variable, with a group variable indicating group membership (in this case 1 or 2). The third form of `sdtest` may then be used, with the `by(groupvar)` option, to conduct the F test.

What if there are more than two groups across which we wish to test for equality of disturbance variance, for instance, a set of 10 industries? We may then use the `robvar` command (see [R] `sdtest`), which like `sdtest` expects to find one variable containing each group's residuals, with a group membership variable identifying them. The `by(groupvar)` option is used here as well. The test conducted is that of [Levene \(1960\)](#) labeled as W_0 , which is robust to nonnormality of the error distribution. Two variants of the test proposed by [Brown and Forsythe \(1974\)](#), which uses more robust estimators of central tendency (e.g., median rather than mean), W_{50} and W_{10} , are also computed.

We illustrate groupwise heteroskedasticity with state-level data: 1 observation per year for each of the six states in the New England region of the United States for 1981–2000. We first apply `robvar` to the state-level population series to examine whether the variance of population is constant across states.

```
. use http://www.stata-press.com/data/imeus/NEdata
. robvar pop, by(state)
```

state	Summary of pop		Freq.
	Mean	Std. Dev.	
CT	3276614.5	81452.212	20
MA	6030915.5	178354.76	20
ME	1212718.1	46958.538	20
NH	1094238.9	94362.302	20
RI	1000209.9	29548.701	20
VT	562960.65	31310.625	20
Total	2196276.3	1931629.4	120

```
W0 = 13.856324 df(5, 114) Pr > F = 0.00000000
W50 = 11.820938 df(5, 114) Pr > F = 0.00000000
W10 = 13.306895 df(5, 114) Pr > F = 0.00000000
```

All forms of the test clearly reject the hypothesis of homoskedasticity across states' population series: hardly surprising when the standard deviation of Massachusetts' (MA) population is six times that of Rhode Island (RI).

We now fit a linear trend model to state disposable personal income per capita, `dpipc`, by regressing that variable on `year`. The residuals are tested for equality of variances across states with `robvar`.

```
. regress dpipc year
```

Source	SS	df	MS			
Model	3009.33617	1	3009.33617	Number of obs =	120	
Residual	806.737449	118	6.83675804	F(1, 118) =	440.17	
Total	3816.07362	119	32.0678456	Prob > F =	0.0000	
				R-squared =	0.7886	
				Adj R-squared =	0.7868	
				Root MSE =	2.6147	

dpipc	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
year	.8684582	.0413941	20.98	0.000	.7864865	.9504298
_cons	-1710.508	82.39534	-20.76	0.000	-1873.673	-1547.343

```
. predict double eps, residual
```

```
. robvar eps, by(state)
```

state	Summary of Residuals		Freq.
	Mean	Std. Dev.	
CT	4.167853	1.3596266	20
MA	1.618796	.86550138	20
ME	-2.9841056	.93797625	20
NH	.51033312	.61139299	20
RI	-.8927223	.63408722	20
VT	-2.4201543	.71470977	20
Total	-6.063e-14	2.6037101	120

```
W0 = 4.3882072 df(5, 114) Pr > F = 0.00108562
W50 = 3.2989851 df(5, 114) Pr > F = 0.00806751
W10 = 4.2536245 df(5, 114) Pr > F = 0.00139064
```

The hypothesis of equality of variances is soundly rejected by all three `robvar` test statistics, with the residuals for Connecticut, Massachusetts, and Maine possessing a standard deviation considerably larger than those of the other three states.

References

- Brown, M. B., and A. B. Forsythe. 1974. Robust tests for the equality of variances. *Journal of the American Statistical Association* 69: 364–367.
- Levene, H. 1960. Robust tests for equality of variances. In *Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling*, ed. I. Olkin, S. G. Ghurye, W. Hoeffding, W. G. Madow, and H. B. Mann, 278–292. Menlo Park, CA: Stanford University Press.