

# Estimating the Feasible Economic Gains From International Portfolio Diversification

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## **Abstract**

International portfolio diversification is explored from the standpoint of constructing simple feasible optimal portfolios, and estimating the economic gains from moving from domestic to international equities. These experiments attempt to estimate how obvious the gains are to a simple “rule of thumb” investor who is considering a global portfolio as an alternative to a purely domestic strategy. The results show that from the perspective of a U.S. investor the gains from diversification are quite small and unstable. They were large during the 70's, and early 80's, but the last 15 years, have made the simple justifications for international diversification much weaker.

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# 1 Introduction

One of the major puzzles in international finance is the home bias puzzle.<sup>1</sup> The fact that residents of countries do not take advantage of the the gains from internationally diversified portfolios remains a disturbing challenge to modern financial theory, and it would appear to be a blatant example of modern portfolio theory being ignored by a large group of consumers.

This paper is in the spirit of Cochrane (1989) in looking at the purely domestic investor as following a kind of “rule of thumb”.<sup>2</sup> This investor needs to be shaken from this entrenched strategy by seeing the obvious gains to holding some international securities in his or her portfolio. The objective is to estimate the gains of moving to nearby international portfolios estimated using various optimization techniques and several different data sets. The most important of these is to make portfolio decisions out of sample, using only returns that have been observed in the past. These portfolios constitute feasible strategies for either the international or domestic investor. This approach is designed to be an initial estimate of how easy it would be for simple learning algorithms to figure out how to build diversified portfolios given the observed time series.<sup>3</sup> If nearby simple modifications toward international diversification do not make significant utility improvements, then it is not clear if learning to diversify will be a easy case to make to consumers, or a learning dynamic that they will naturally find.<sup>4</sup>

This paper stresses the problems in estimating optimal portfolios given noisy financial data in a simple learning environment. Several other recent papers have examined the statistical problems in diversifying internationally.<sup>5</sup> Other authors have looked at the more general issue of whether a set of domestic assets can span the international portfolios.<sup>6</sup> The general finding is that from various statistical perspectives the home bias puzzle is smaller than previously thought.

This paper diverges from many of the earlier studies by leaving the mean/variance framework and optimizing CRRA utility functions directly. This allows for possible deviations from multivariate normality in the returns distributions. It also could potentially account for unusually large probabilities for large

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<sup>1</sup>One of the earliest estimates of home bias is in French & Poterba (1991). See Lewis (1999) for a recent survey. Also, Obstfeld & Rogoff (2001), cite it as one of the six major puzzles of international finance.

<sup>2</sup>See Brandt (1999) for recent approaches to utility based decisions and estimation for dynamic portfolio strategies, and West, Edison & Cho (1993) for strategies employing GARCH volatility estimates.

<sup>3</sup>This is similar in spirit to the experiments on learning optimal consumption policies performed in Allen & Carroll (2001). Also, Pastor (2000) employs a Bayesian learning mechanism to the home bias puzzle, and finds that that the observed portfolio weights can only be supported by extreme beliefs about the return distributions.

<sup>4</sup>It is important to contrast this with experimental evidence suggesting that learning equilibrium in an experimental CAPM world may be very easy, Bossaerts & Plott (2000).

<sup>5</sup>Among these are Britten-Jones (1999), Gorman & Jorgensen (1997), and Hasan & Simaan (2000).

<sup>6</sup>Examples of this include Errunza, Hogan & Hung (1999).

moves to occur simultaneously in global markets.<sup>7</sup> Actual utility based estimates will be compared with mean/variance approximations to determine the importance of relaxing mean/variance assumptions.

Section two outlines the utility based methodology. Section three presents the results, and section four concludes.

## 2 Utility Comparisons

Utility based comparisons are made using a myopic CRRA utility function of power form. For a random return sequence for some portfolio,  $\tilde{r}_p$ , the expected utility is given by

$$U(\tilde{r}_p) = E\left(\frac{1}{1-\gamma}(1 + \tilde{r}_p)^{1-\gamma}\right). \quad (1)$$

where  $\gamma$  is the coefficient of relative risk aversion. For  $\gamma = 1$  the function becomes log utility. Consumers construct optimal portfolios over their feasible set of securities to maximize this utility. The set is constrained in various experiments as is the data used for maximization. For all investors short selling and borrowing is prohibited. For domestic investors the investment set includes only a U.S. equity index, and the Tbill return. International investors can chose between these securities and differing sets of foreign equity (but not risk free) assets. In other words, the investor seeks to maximize,

$$U = E\left(\frac{1}{1-\gamma}(1 + \sum_j \omega_j \tilde{r}_j)^{1-\gamma}\right), \quad (2)$$

The portfolio weights are constrained to be positive, and to sum to 1. They are further constrained to be multiples of 0.1. This is required by the fact that the optimization is using simple search over portfolios and it is necessary to keep the number of potential portfolios under control as the number of securities increases.

For comparison with mean/variance optimization some experiments will use the second order Taylor series expansion of the utility function evaluated around zero. This corresponds to most of the literature on portfolio optimization, and should give the same results given multivariate normal return distributions. Part of the reason for using this numerical search procedure is that it will reveal if the deviations from normality in the data are economically relevant.

The optimal portfolios are determined using varying sets of returns. In the simplest case the optimal portfolio is determined over the entire sample. This gives a single set of optimal portfolio weights,  $\omega_j$ . This

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<sup>7</sup>See Longin & Solnik (2001) for research on extreme events in asset market returns.

experiment is not a feasible one for investors since it uses future data as part of the optimization process. A true out of sample optimization is also performed by estimating the optimal portfolio weights using all time periods before the current decision date,  $t$ .<sup>8</sup> This yields a time series of optimal portfolios weights,  $\omega_{j,t}$  which is used to construct the portfolio returns. A second feasible method uses a rolling 5 year band which proceeds the current decision date. This builds optimal portfolios from the past 5 years of data, and rolls along this historical 5 year period as the decision point moves through the series.

Once optimal portfolios have been determined they need to be compared. There are many possible measures that one might use to compare two risky portfolios. This study uses one designed to be consistent with the optimization framework. A compensating variation type of measure is used to compare domestic and global portfolios. Assume that  $\tilde{r}_d$  and  $\tilde{r}_w$  are the returns from the domestic and global strategies respectively. Define the return adjustment as the value  $x$  such that

$$U(\tilde{r}_d) = U(\tilde{r}_w - x). \quad (3)$$

This would be the certainty return that should be removed from the global returns to make the investor indifferent between the two portfolios. One reason for looking at this is that it can be thought of as the hurdle that would need to be cleared to move a stubborn domestic investor to a fully diversified position. If there were some friction or cost to moving,  $c$ , then it would not be optimal to move to an international portfolio as long as  $x < c$ .<sup>9</sup>

Estimates of precision are found using a bootstrap procedure. The two portfolios  $r_d$  and  $r_w$  are taken as given, and then simultaneously scrambled with replacement. This means that each random draw pulls an observation from each series maintaining the cross-sectional correlations, but destroying any intertemporal dependence. Standard deviations on the return adjustment are reported along with the probability that  $x > 0$ , or that there is an actual utility gain to diversification.

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<sup>8</sup>In all experiments the first 5 years of data are set aside as an initialization set. Portfolio tracking begins in year 6, using the first five years to determine optimal weights. As the investor moves into the future, additional data is added to the set used to construct portfolios.

<sup>9</sup>It is also possible to compare portfolios from the perspective of equivalent variation. In this case the domestic returns would be augmented by a fixed amount,  $y$ , until they were equivalent, in utility terms, to the international portfolios. This measure doesn't have as nice an interpretation as the compensating variation in terms of producing the hurdle that needs to be overcome to diversify. Also, in most cases this measure was equal to the compensating variation measure due to the lack of curvature in the utility function.

## 3 Empirical Results

### 3.1 Data Sources and Summaries

The portfolio comparisons are made using the MSCI global indices as equity benchmarks and the U.S. 90 day Treasury Bill (TBill) as a risk free proxy. All data are sampled at monthly frequency in U.S. dollars, and are adjusted for inflation using the U.S. CPI index. The full sample extends from January 1970 through Sept 2001 for a total of 380 monthly observations. This paper concentrates on the returns starting in January 1975 as a benchmark because most of the out of sample portfolio construction experiments will set aside the first 5 years of data to allow for some initial history to begin the optimal portfolio construction.

The initial tests concentrate on the simple equity holdings of the U.S. portfolio, and the rest of the developed world which is represented by the MSCI Europe, Asia, Far East, and Canada index, (EAFE + Canada). Later tests will include individual indices for Europe and Asia. Summary statistics are presented in table 1. Monthly returns are compounded out to one year, and the monthly standard deviation is multiplied by  $\sqrt{12}$  to approximate its annual value. The US returns are in the rows labeled, US, and the rest of the world is labeled, Foreign. The full sample displays very similar properties for both of the international equity portfolios. Their mean returns are close to 10 percent, and standard deviations are in the neighborhood of 0.15. Both exhibit some excess kurtosis which is well known for equity returns. The US correlation with the rest of the world is quite large at 0.55. It is also clear that in terms of means, the equity returns dominate the risk free asset which only is able to average a return of about 2 percent during the full sample. The final row labeled Equal, displays the properties of an equal weighted equity portfolio composed of the foreign and domestic parts. It shows a moderate amount of variance reduction, and an obviously large correlation with the US portfolio.

The sub-samples display some very interesting features which suggest the series are not as similar in different periods. Excess kurtosis continues, and equity standard deviations continue in the range of 0.15. However, the mean returns diverge greatly. During the first sub-sample, the foreign mean return rises to over 16 percent, while the US falls to only 9.5 percent. In the second sub-sample, the US return rises to 12 percent, and the foreign falls to only 2.5 percent. Much of this shortfall can be attributed to the poor performance of the Japanese market over this period. This performance difference will have a serious impact on the results here. Later experiments will look at the European and Asian markets separately, to further explore the impact of the return differences.

Figures 1 give a graphical preview of how the values in table 1 impact results on international diversifi-

Table 1: *Return Summary Statistics*

|                       |       |       |        |       |       |
|-----------------------|-------|-------|--------|-------|-------|
| Jan 75 - Sept 2001    |       |       |        |       |       |
| US                    | 0.107 | 0.153 | -0.430 | 4.937 | 1.000 |
| TBill                 | 0.021 | 0.009 | 0.246  | 3.723 | 0.098 |
| Foreign               | 0.092 | 0.168 | -0.078 | 3.721 | 0.552 |
| Equal                 | 0.100 | 0.142 | -0.384 | 4.820 | 0.869 |
| Jan 75 - May 1988     |       |       |        |       |       |
| US                    | 0.095 | 0.162 | -0.451 | 5.606 | 1.000 |
| TBill                 | 0.020 | 0.011 | 0.300  | 2.768 | 0.079 |
| Foreign               | 0.165 | 0.167 | -0.128 | 4.276 | 0.529 |
| Equal                 | 0.129 | 0.144 | -0.426 | 5.851 | 0.870 |
| June 1988 - Sept 2001 |       |       |        |       |       |
| US                    | 0.123 | 0.143 | -0.389 | 3.648 | 1.000 |
| TBill                 | 0.021 | 0.005 | -0.235 | 3.124 | 0.151 |
| Foreign               | 0.025 | 0.168 | -0.036 | 3.241 | 0.593 |
| Equal                 | 0.073 | 0.139 | -0.364 | 3.673 | 0.874 |

Annualized real return summary statistics. US is MSCI United States portfolio. TBill is US 90 day TBill real return. Foreign is MSCI EAFE+Canada. Equal is an equal weighted portfolio of the US and Foreign. All returns are in real US dollar terms, and include reinvested dividends.

cation. Figure 1 plots the means and standard deviations for various equity only portfolios varying portfolio weights from the U.S. to the EAFE+Canada. The sample is set from 1970-1996, and shows the traditional argument for the gains from international portfolio diversification. Assuming mean variance preferences, it is clear that a U.S. investor with any risk aversion would prefer to diversify, since this would move to higher returns, and lower variances. The picture changes dramatically when the next 4.5 years of data are added in figure 2. Here, the gains to diversification are not as clear. There is clearly risk reduction in the diversified portfolios, but this comes at the cost of reducing expected returns. The gains are now sensitive to consumer preferences, and attitudes toward risk. The simple graphical case for diversification is no longer as strong as it was before.

### 3.2 Return Adjustments

This section estimates the economic gains to international diversification using the various benchmark securities. Table 2 looks at gains from optimal portfolios constructed using information from the entire 75-2001 sample. The optimal portfolio is constructed both using the restricted set of the domestic equity and Tbill returns, and it is expanded by adding the EAFE+Canada index to these two securities.<sup>10</sup> As described in section 2, the return adjustments should be thought of as the constant return that would be subtracted from

<sup>10</sup>In this study investment in foreign risk free securities is not allowed. This opens up a large a complicated area which includes potential foreign exchange hedging and prediction. The purpose of this paper is to concentrate on very simple strategies.

the international portfolio to leave the global investor equally well off as the domestic only investor. One can think of this as an upper bounded in terms of “return cost” that an investor could withstand while still leaving himself better off with a move to an internationally diversified portfolio. If the number is negative it indicates that the international portfolio performed worse, in utility terms, than the domestic one.

The first row of table 2 displays adjustment values in annual percentage units. The columns correspond to the differing levels of risk aversion indicated in the table. A gain of zero, as in the first column, indicates that the investor would optimally chose to hold none of the international portfolio. As the coefficient of relative risk aversion increases the gains increase, although not monotonically. Even the largest gain, which corresponds to  $\gamma = 4$ , is only 0.2 percent per year. A lack of international diversification at a point estimate this low seems to easily be explained by many types of frictions in the individual investment process. The accuracy, and significance of these values is explored in the next two rows labeled, Std, and Prob( $U_w > U_d$ ). Std. presents a bootstrapped estimate of the standard deviation of the adjustment number. The bootstrap is constructed by taking the series of the optimal domestic and international portfolios as given. These returns are then scrambled with replacement 500 times, and a new adjustment number is derived at each iteration.<sup>11</sup> This is used to build a distribution where the standard deviation is estimated. The distribution is also used to estimate the probability that the compensation is positive, or that the optimal world portfolio is preferred to the domestic. This number is a kind of confidence indication as to how likely one can be, given current data, that the move to the international portfolio will yield a utility gain. These numbers indicate a large amount of uncertainty about both the magnitude, and the sign of the utility gains from international diversification. There are no preferences for which the probability of a gain exceeds 0.63.

The second panel, labeled MV Utility, repeats the previous results using a second order Taylor series expansion on the utility function. This corresponds to traditional mean/variance portfolio optimization. The results here are very similar to the first panel, indicating that the deviations from multi-variate normality in the series do not appear to be having a large impact, or that the returns are small enough that the utility approximation is a very good one.

The final panel, labeled equal weighted, fixes the international portfolio as an equal weighted portfolio between the U.S. and foreign equity positions. Investors construct a portfolio out of this equity position along with the U.S. Tbill return. This is compared to the U.S. portfolio as before. This attempts to capture a simple “rule of thumb” in international diversification. If an investor held strong beliefs about the distribution of returns having the same expected returns and variances, this would be the optimal portfolio

<sup>11</sup>The scrambling destroys any intertemporal dependence in the series, but it maintains the cross-sectional dependencies.

to hold.<sup>12</sup> The values show that moving to the equal weighted portfolio reduce the utility for all the investors. The smallest losses,  $-0.02$  percent per year, come from the  $\gamma = 10$  investor who gains the most from the risk reduction. The probabilities of a gain which are all less than 0.5 except for  $\gamma = 10$  show that this rule of thumb diversification would appear sub-optimal.

Table 2: *International Diversification Gains: Full Sample*

| CRRA                | 1      | 2      | 4      | 10     |
|---------------------|--------|--------|--------|--------|
| Power Utility       |        |        |        |        |
| Return Adjustment   | 0.000  | 0.018  | 0.200  | 0.093  |
| Std                 | 0.000  | 0.303  | 0.549  | 0.346  |
| Prob( $U_w > U_d$ ) | 0.000  | 0.498  | 0.628  | 0.606  |
| MV Utility          |        |        |        |        |
| Return Adjustment   | 0.000  | 0.019  | 0.203  | 0.098  |
| Std                 | 0.000  | 0.297  | 0.606  | 0.351  |
| Prob( $U_w > U_d$ ) | 0.000  | 0.516  | 0.600  | 0.572  |
| Equal Weighted      |        |        |        |        |
| Return Adjustment   | -0.546 | -0.370 | -0.065 | -0.020 |
| Std                 | 1.454  | 1.532  | 1.250  | 0.544  |
| Prob( $U_w > U_d$ ) | 0.368  | 0.394  | 0.440  | 0.510  |

Diversification gains. Gains from shifting to international portfolios measured as return adjustments, or compensations making international investors as well off as before diversification. Adjustments are reported as annual percentages. Std's are estimated over 500 bootstrap iterations. Prob( $U_w > U_d$ ) is the estimated probability of a utility gain moving to the international portfolio.

Though the gains to diversification presented so far are only in the range of small to moderate they are a kind of upper bound, since they use information to form portfolios that the investor would not have access to. Table 3 displays true out of sample gains. Optimal portfolios are determined using all information up until the date in which the investment is made. The first five years of the data, 1970-74, are used as the initial start up period. This procedure generates a portfolio which changes over time since new information is available. Figure 3 shows the time evolution of the optimal portfolios for  $\gamma = 2$ . There is a clear shift toward U.S. equity over time as the returns of the U.S. market during the 1990's become part of the investment decision making process. The reported return adjustments in table 3 now turn further negative as one should expect. One exception is the equal weighted portfolio which reports larger gains then before. The most dramatic is for the  $\gamma = 10$  case where the return adjustment is roughly 0.2 percent per year, and the probability of a utility gain is estimated at 0.728.

The final panel of the table tries another rule of thumb strategy where the optimal portfolio is determined using a rolling window of the last 5 years. This might be a reasonable strategy for the investor who believes

<sup>12</sup>This analysis is ignoring correlations with the relatively risk free TBill returns which is not unreasonable.

that the means, variances, and covariances in the distributions are changing over time. The performance of this strategy is miserable with large negative return adjustments, and very low utility gain probabilities. This is obviously only one type of rolling window strategy, but it is a common benchmark that investors use.

Table 3: *International Diversification Gains: Out of Sample*

| CRRA                | 1      | 2      | 4      | 10     |
|---------------------|--------|--------|--------|--------|
| Power Utility       |        |        |        |        |
| Return Adjustment   | -0.264 | 0.017  | -0.355 | 0.132  |
| Std                 | 2.898  | 2.354  | 1.542  | 0.579  |
| Prob( $U_w > U_d$ ) | 0.476  | 0.458  | 0.450  | 0.596  |
| MV Utility          |        |        |        |        |
| Return Adjustment   | -0.267 | -0.050 | -0.286 | 0.091  |
| Std                 | 2.761  | 2.396  | 1.410  | 0.582  |
| Prob( $U_w > U_d$ ) | 0.482  | 0.506  | 0.410  | 0.572  |
| Equal Weighted      |        |        |        |        |
| Return Adjustment   | -0.357 | 0.305  | -0.039 | 0.195  |
| Std                 | 1.473  | 1.282  | 0.889  | 0.338  |
| Prob( $U_w > U_d$ ) | 0.390  | 0.606  | 0.458  | 0.728  |
| Rolling 5 Year      |        |        |        |        |
| Return Adjustment   | -0.959 | -1.231 | -1.220 | -0.793 |
| Std                 | 2.188  | 2.214  | 1.941  | 1.069  |
| Prob( $U_w > U_d$ ) | 0.316  | 0.286  | 0.268  | 0.210  |

Diversification gains. Gains from shifting to international portfolios measured as return adjustments, or compensations making international investors as well off as before diversification. Adjustments are reported as annual percentages. Std's are estimated over 500 bootstrap iterations. Prob( $U_w > U_d$ ) is the estimated probability of a utility gain moving to the international portfolio. Portfolio weights are determined out of sample, using only past return information to determine optimal weights.

The mean variance properties of the optimal portfolios both in and out of sample are shown in figures 4 and 5. The figures repeat the results from tables 3 and 2, where the in sample investors optimally chose to invest very close to the domestic portfolios, and the out of sample investors actually do slightly worse.

### 3.3 Robustness Checks

This section analyses the robustness of the previous results using several methods including sub-samples, equity only portfolios, and annual returns. Table 4 displays the gains in the two halves of the series, 1975 - May 1988 and June 1998 - September 2001. All estimates are made using power utility both in and out of sample as indicated. The first sub-sample displays very strong gains to diversification with percentage adjustments in the range of 2 – 6 percent. It seems very reasonable that these are the kinds of gains that need to be taken very seriously by investors. The utility gain probabilities are all near 0.90. The second panel shows that moving out of sample does not in any way reduce the gains to international diversification.

These first two panels clearly show the classic case for international diversification as is usually perceived.

Moving to the second sub-sample the situation changes completely. The in sample gains are all zero indicating that the investor is choosing to stay with the domestic portfolio for all levels of risk aversion. Out of sample the performance is poor with compensation levels on the order of  $-5$  percent per year, and utility gain probabilities all less than 0.05. The case for diversification on this restricted sub-sample would be a very difficult one to make.

Figure 6 repeats the previous mean/standard deviations pictures for the two sub-samples. The picture visually repeats the results of table 4 in showing strong diversification gains during the first sub-sample, and no gains during the second sub-sample.

Table 4: *Sub-sample Diversification Gains*

| CRRA                                    | 1      | 2      | 4      | 10     |
|---|--------|--------|--------|--------|
| January 1975 - May 1988 (in sample)     |        |        |        |        |
| Return Adjustment                       | 6.435  | 6.384  | 5.488  | 2.243  |
| Std                                     | 4.585  | 4.474  | 4.205  | 1.940  |
| Prob( $U_w > U_d$ )                     | 0.926  | 0.936  | 0.896  | 0.870  |
| January 1975 - May 1988 (out sample)    |        |        |        |        |
| Return Adjustment                       | 8.010  | 7.047  | 3.536  | 1.652  |
| Std                                     | 4.432  | 3.736  | 2.199  | 0.764  |
| Prob( $U_w > U_d$ )                     | 0.964  | 0.980  | 0.952  | 0.980  |
| June 1998 - September 2001 (in sample)  |        |        |        |        |
| Return Adjustment                       | 0.000  | 0.000  | 0.000  | 0.000  |
| Std                                     | 0.000  | 0.000  | 0.000  | 0.000  |
| Prob( $U_w > U_d$ )                     | 0.000  | 0.000  | 0.000  | 0.000  |
| June 1998 - September 2001 (out sample) |        |        |        |        |
| Return Adjustment                       | -5.793 | -6.160 | -5.156 | -3.089 |
| Std                                     | 3.341  | 2.994  | 2.730  | 1.462  |
| Prob( $U_w > U_d$ )                     | 0.050  | 0.028  | 0.026  | 0.018  |

Diversification gains. Gains from shifting to international portfolios measured as return adjustments, or compensations making international investors as well off as before diversification. Adjustments are reported as annual percentages. Std's are estimated over 500 bootstrap iterations. Prob( $U_w > U_d$ ) is the estimated probability of a utility gain moving to the international portfolio.

One possible criticism of the analysis so far is that the EAFE Canada portfolio forces the international investor to have some weight on the falling Japanese equity market. Table 5 extends the previous results by allowing the international investor to chose between three portfolios, US, Europe, and Asia. All these are represented by the corresponding MSCI indices. This complicates the portfolio decision slightly since the global investor now choses between 4 different securities including the U.S. Tbill return. The table shows that in sample, adding this dimension of flexibility improves the overall diversification gains. For the middle

range of  $\gamma = 2$ , and  $\gamma = 4$  the return adjustment are 0.3, and 0.5 percent respectively. The probability of a gain is 0.58, and 0.69 in these two cases.

The difficulty with adding more securities is that it increases the potential for in sample bias since the degrees of freedom increase. This bias can be observed by moving to the second panel which shows the out of sample results. The gains are reduced as before to being negative for 3 out of the 4 risk aversion levels. One interesting exception is for  $\gamma = 2$  where the gains remain moderate at 0.32 percent per year. The gain probability is greatly reduced to only 0.54 indicating only a 50/50 chance of utility improvement from diversification. Overall, the additional portfolio flexibility is not yielding an improvement to diversification.

Table 5: *Diversification Gains From Multi-Region Portfolios*

| CRRA                | 1      | 2     | 4      | 10     |
|---------------------|--------|-------|--------|--------|
| In Sample           |        |       |        |        |
| Return Adjustment   | 0.154  | 0.302 | 0.484  | 0.195  |
| Std                 | 1.158  | 1.116 | 1.084  | 0.540  |
| Prob( $U_w > U_d$ ) | 0.542  | 0.578 | 0.686  | 0.616  |
| Out Sample          |        |       |        |        |
| Return Adjustment   | -0.232 | 0.319 | -0.547 | -0.077 |
| Std                 | 3.638  | 3.374 | 2.552  | 0.896  |
| Prob( $U_w > U_d$ ) | 0.486  | 0.544 | 0.396  | 0.498  |

Diversification gains. Gains from shifting to international portfolios measured as return adjustments, or compensations making international investors as well off as before diversification. Adjustments are reported as annual percentages. Std's are estimated over 500 bootstrap iterations. Prob( $U_w > U_d$ ) is the estimated probability of a utility gain moving to the international portfolio.

These experiments are concentrating on sensible rules of thumb for investing and diversifying. It is possible that in forming such a rule, investors might decide that the risk free Tbill return is dominated by equity returns, and should be ignored. This is explored in table 6. Here the domestic benchmark investor is fully invested in the equity portfolio. The diversified investor chooses between only the domestic equity and foreign equity portfolios. In sample, this change increases the gains. The investor needs some way to reduce risk, and international diversification is the only channel in this case. The gains are moderate, and increasing as risk aversion increases. For  $\gamma = 10$  the return adjustment is 1.8 percent per year, and the gain probability is 0.87 which suggests a strong incentive to hold some international equity. All the gains are reduced out of sample. However, the 0.77 percent adjustment for  $\gamma = 10$  still appears large, but the gain probability is reduced to only 0.63.

Finally, table 7 takes the simple rule policy to an extreme. Investment decisions are made using only annual return data. Obviously, this reduces sample sizes, but it is intended to replicate the decision processes

Table 6: *Diversification Gains From Equity Only Portfolios*

| CRRA                | 1      | 2      | 4      | 10    |
|---------------------|--------|--------|--------|-------|
| In Sample           |        |        |        |       |
| Return Adjustment   | 0.000  | 0.018  | 0.287  | 1.757 |
| Std                 | 0.000  | 0.297  | 0.916  | 1.545 |
| Prob( $U_w > U_d$ ) | 0.000  | 0.522  | 0.606  | 0.868 |
| Out sample          |        |        |        |       |
| Return Adjustment   | -1.131 | -1.049 | -0.834 | 0.773 |
| Std                 | 2.666  | 2.832  | 2.526  | 2.628 |
| Prob( $U_w > U_d$ ) | 0.334  | 0.344  | 0.404  | 0.626 |

Diversification gains. Gains from shifting to international portfolios measured as return adjustments, or compensations making international investors as well off as before diversification. Adjustments are reported as annual percentages. Std's are estimated over 500 bootstrap iterations. Prob( $U_w > U_d$ ) is the estimated probability of a utility gain moving to the international portfolio.

of long term investors. It is possible that certain patterns at monthly horizons, such as deviations from normality, will become insignificant at the annual horizon. To keep things simple only equity portfolios are used as in the previous table. The in sample gains appear to be comparable to the monthly case, and there is very little change in shifting to the mean variance approximation. Once again, when examining the feasible out of sample gains, the numbers reveal no utility performance enhancement with losses for all risk levels. These probably also reflects the fact that the out of sample test must begin with only five years of data which gives the investor only 5 data points to start optimizing on.

Table 7: *Annual Horizon Gains Equity Only Portfolios: Annual Return Compensation*

| CRRA                | 1      | 2      | 4      | 10     |
|---------------------|--------|--------|--------|--------|
| Power Utility       |        |        |        |        |
| Return Adjustment   | 0.000  | 0.039  | 0.234  | 0.508  |
| Std                 | 0.000  | 0.362  | 0.729  | 0.924  |
| Prob( $U_w > U_d$ ) | 0.000  | 0.538  | 0.626  | 0.696  |
| MV Utility          |        |        |        |        |
| Return Adjustment   | -0.021 | 0.028  | 0.174  | 0.423  |
| Std                 | 0.376  | 0.718  | 1.051  | 1.366  |
| Prob( $U_w > U_d$ ) | 0.472  | 0.476  | 0.558  | 0.600  |
| Out of sample       |        |        |        |        |
| Return Adjustment   | -1.193 | -1.191 | -1.323 | -4.613 |
| Std                 | 3.497  | 3.094  | 3.062  | 4.225  |
| Prob( $U_w > U_d$ ) | 0.338  | 0.386  | 0.326  | 0.228  |

Diversification gains. Gains from shifting to international portfolios measured as return adjustments, or compensations making international investors as well off as before diversification. Adjustments are reported as annual percentages. Std's are estimated over 500 bootstrap iterations. Prob( $U_w > U_d$ ) is the estimated probability of a utility gain moving to the international portfolio.

The analysis has concentrated so far on simple CRRA utility functions. It is possible that the gains to international diversification are real, but are coming from parts of the distribution that are not seen by these preferences. An example might be a fear of extreme losses which puts a heavy weight on the extreme left tail of the distribution. It would be difficult to explore a large set of preferences like this, but a small graphical experiment is performed in figure 7. This figure shows the estimated value at risk, VaR, for varying levels for both the domestic and equal weighted domestic/foreign portfolios.<sup>13</sup> The figure shows very little difference in the left tail. There is an observable gain at the extreme, where the domestic portfolio picks up a large loss corresponding to Oct 1987. This loss is reduced in the equal weighted portfolio. The figure shows some other places where the diversified portfolio may improve slightly on the VaR estimates, but the magnitude is small, and it is not clear these are significant changes. From the perspective of this figure it may be difficult to find preferences weightings that could bring back the international diversification gains. The distributions appear to be too close, and also the samples out in the distribution tails get very small.

## 4 Conclusions

These results show that gains to international diversification for U.S. investors using simple international portfolio combinations are not very large or easy to find. These gains are reduced by the dramatic instabilities in international returns over time, and their persistence. They are also reduced by considering true out of sample performance estimates.

From the U.S. perspective this agrees with the literature suggesting that the home bias puzzle is not significant, either statistically or economically. The dramatic sub-sample variations suggest that the international investor is posed with some very difficult time series issues. Simple rule of thumb search methods cannot overcome these obstacles to optimal portfolio construction. This can be considered an overall deterrent to even trying to diversify internationally since simple learning algorithms would get blocked before getting out of the starting gate. However, it also asks questions about many more sophisticated portfolio tools, and whether these deliver better performance, and are simple enough that they could be easily discovered.<sup>14</sup>

All of the tests performed here failed in finding a significant difference between the full power utility function, and its mean/variance approximation. This is an interesting result in that it supports the long held view that multivariate normality was a fair economic approximation to multivariate return distributions.

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<sup>13</sup>VaR here is just the left quantile on the return distribution.

<sup>14</sup>Michaud (1998) contains extensive information on improving traditional portfolio optimization. Also, more sophisticated strategies should also include the possibility to fully or partially hedge the foreign exchange exposure.

These results do not solve all under diversification puzzles observed in the data. An interesting extension to this will be to move to other countries where the gains may be larger and the portfolio compositions just as biased as in the U.S.<sup>15</sup> However, one of the major aspects of home bias, the fondness of U.S. investors for their own securities, seems much less disturbing. It is important to realize that many financial decisions need to be made in environments which are so noisy that they greatly restrict the abilities of consumers to achieve optimum benchmarks. When this problem is further compounded with the dynamics of learning, the lack of international diversification is not too surprising.

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<sup>15</sup>Home bias has also been noted in purely domestic situations in Coval & Moskowitz (1999).

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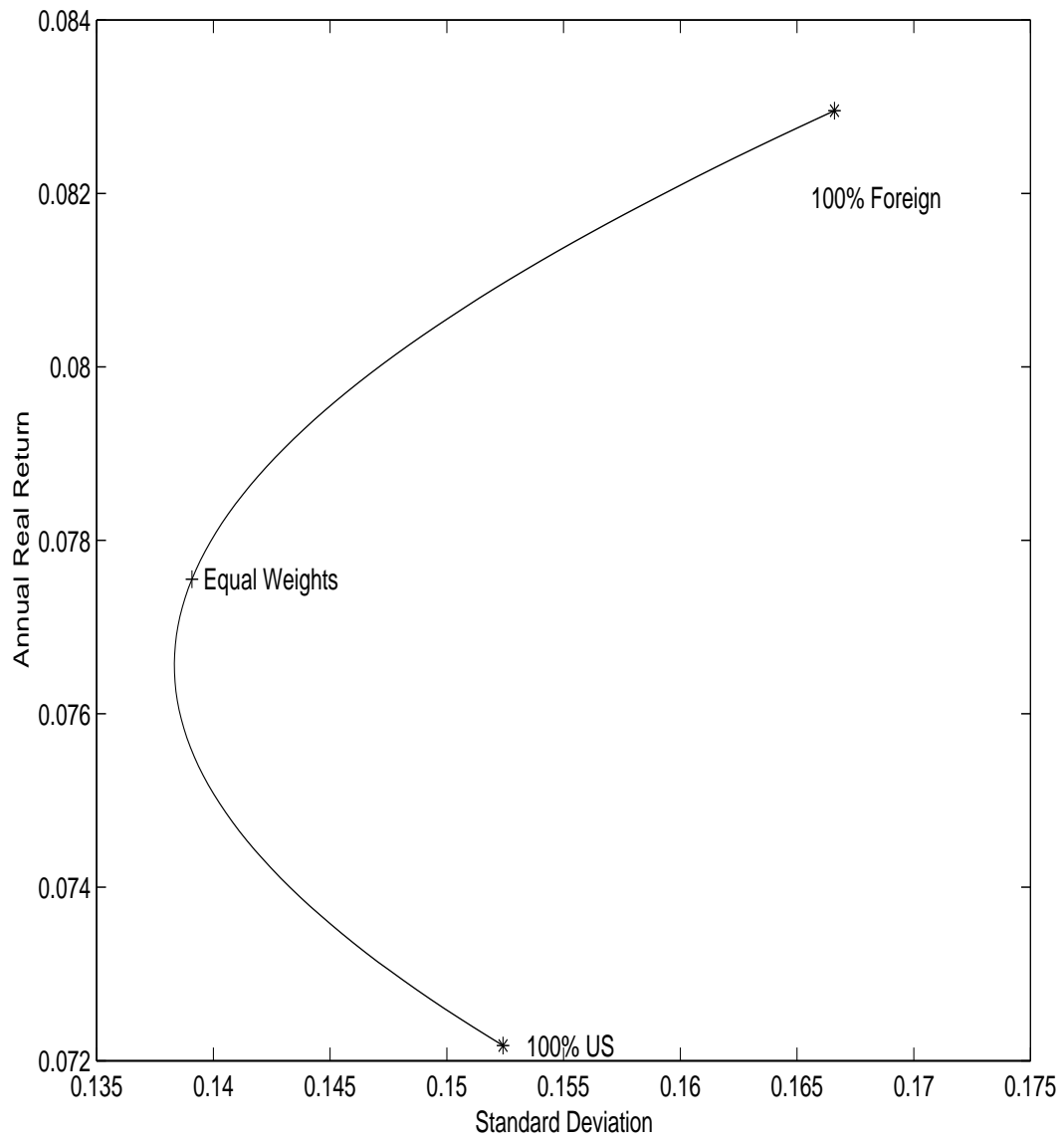


Figure 1: US versus Global Portfolio Performance (1970-1996)

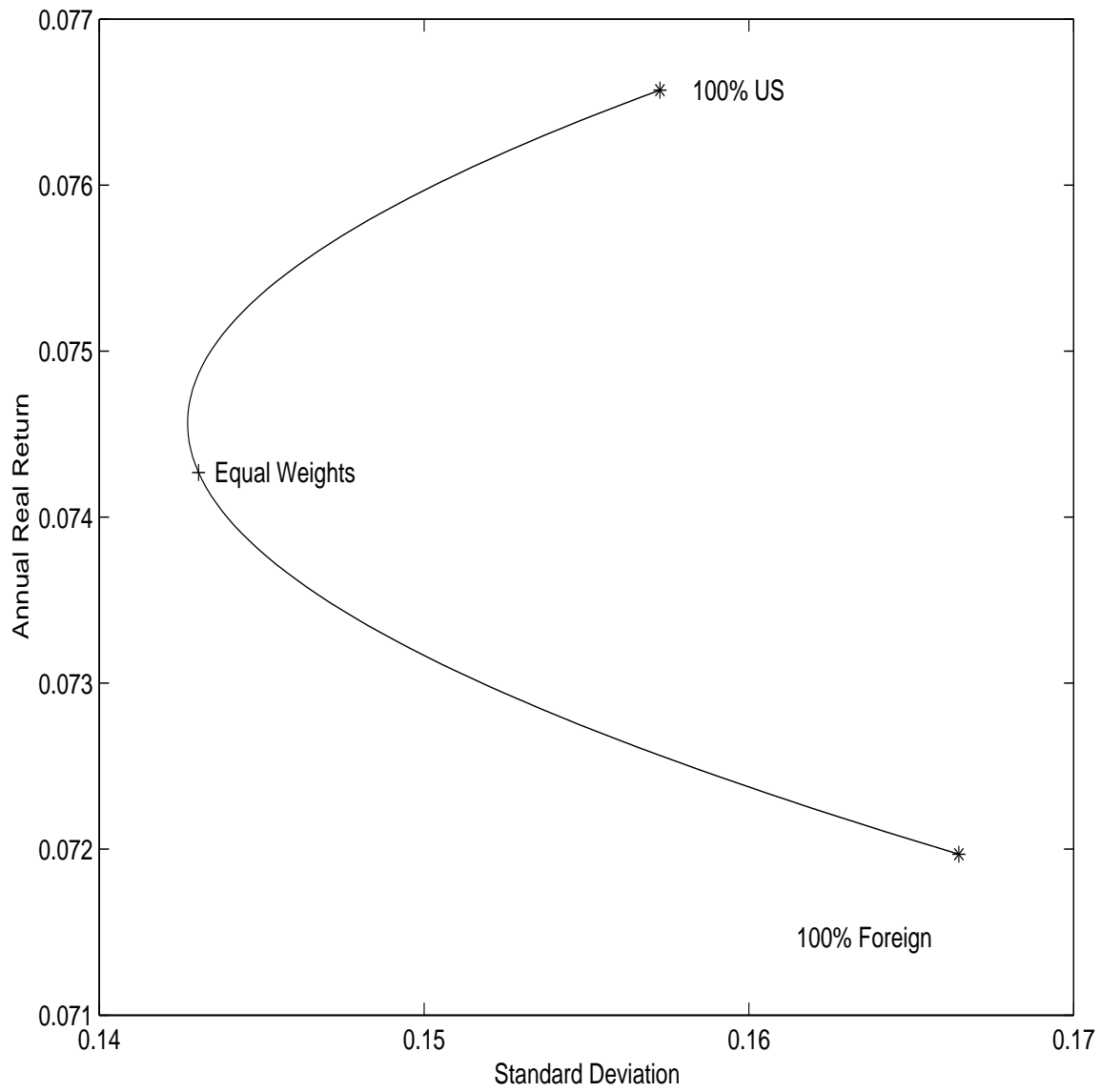


Figure 2: US versus Global Portfolio Performance (1970-2001(Sept))

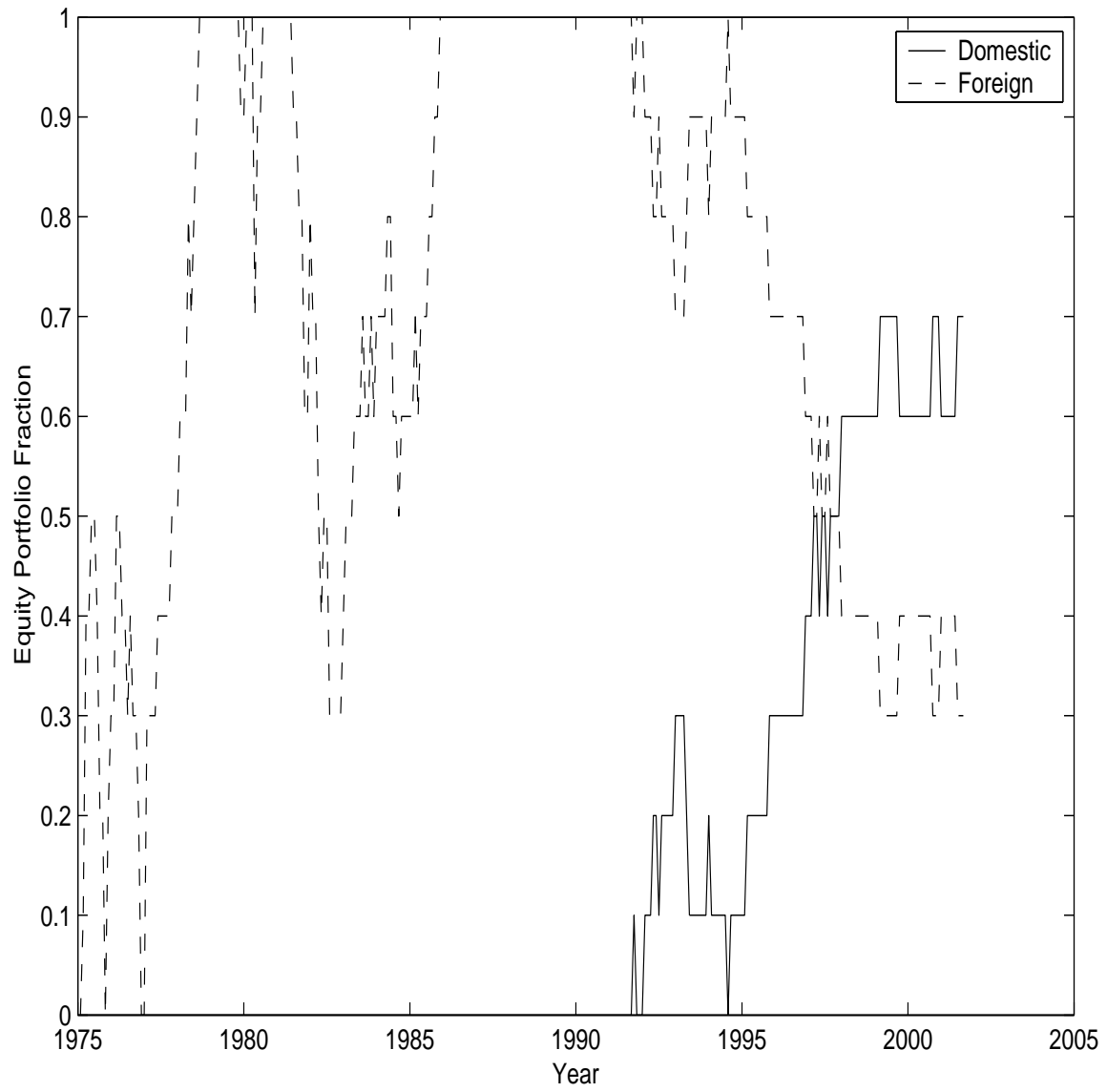


Figure 3: **Portfolio Composition Time Series:**  $\gamma = 2$

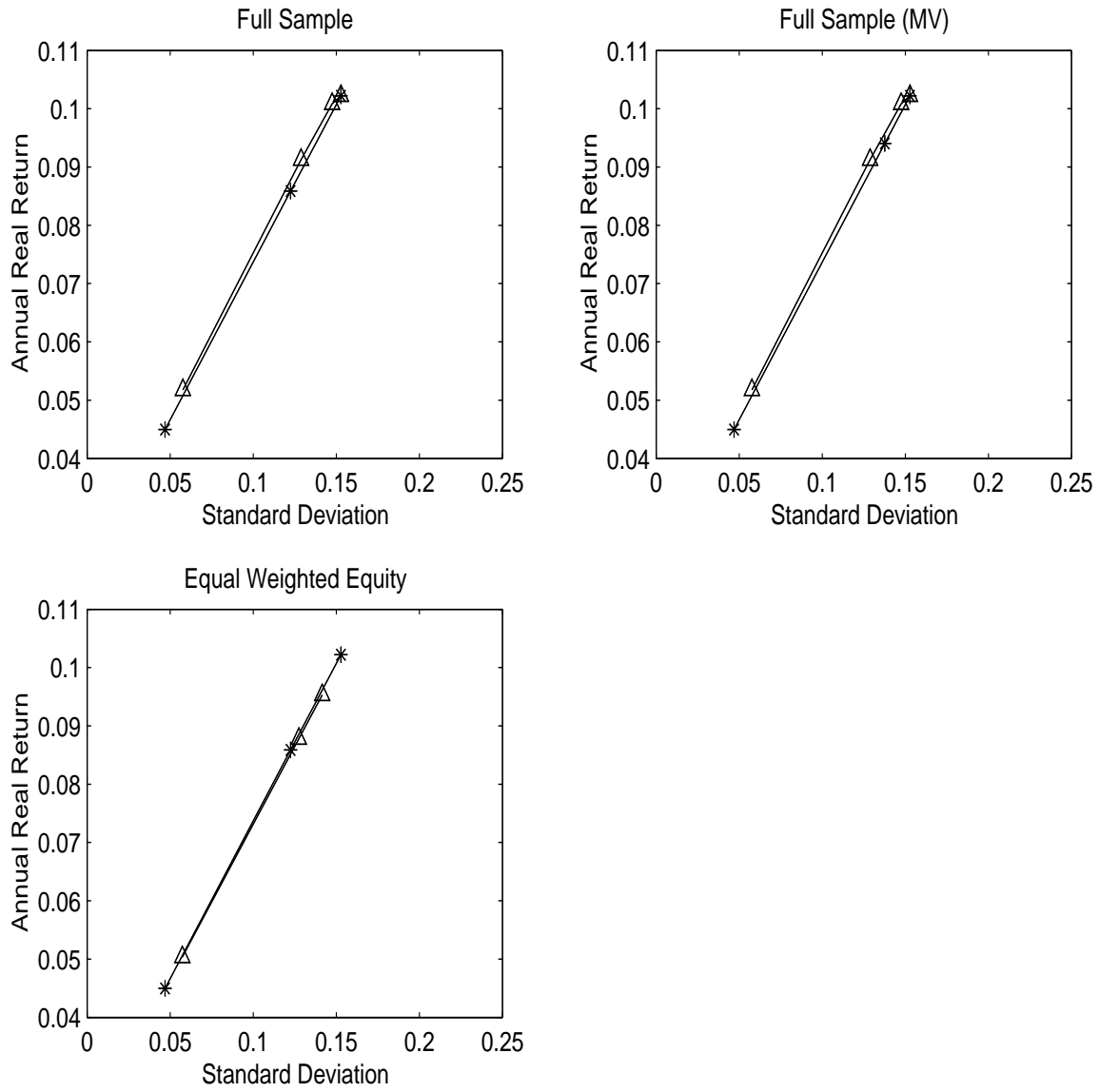


Figure 4: In Sample Mean/Std's for Varying Risk Aversion

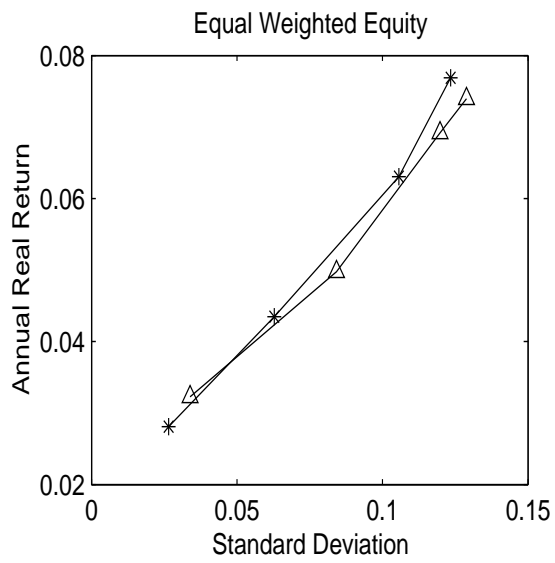
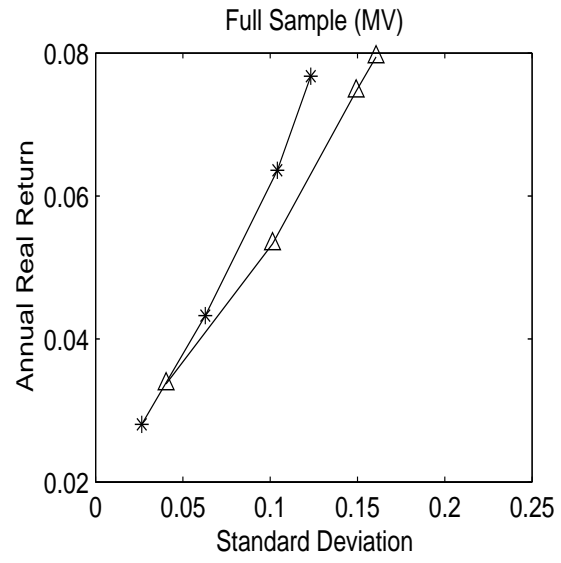
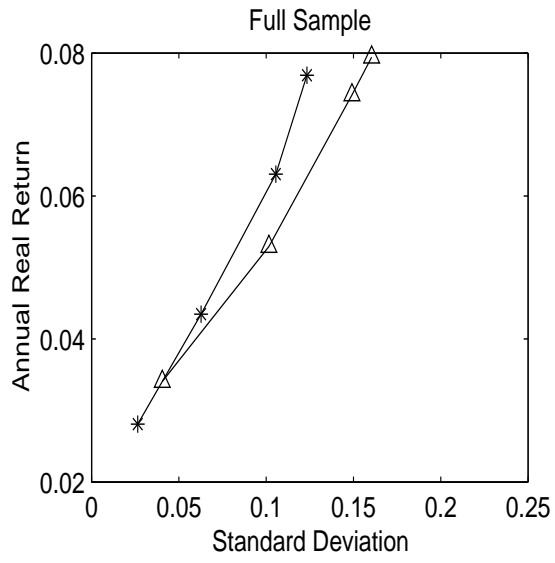


Figure 5: **Out of Sample Mean/Std's for Varying Risk Aversion**

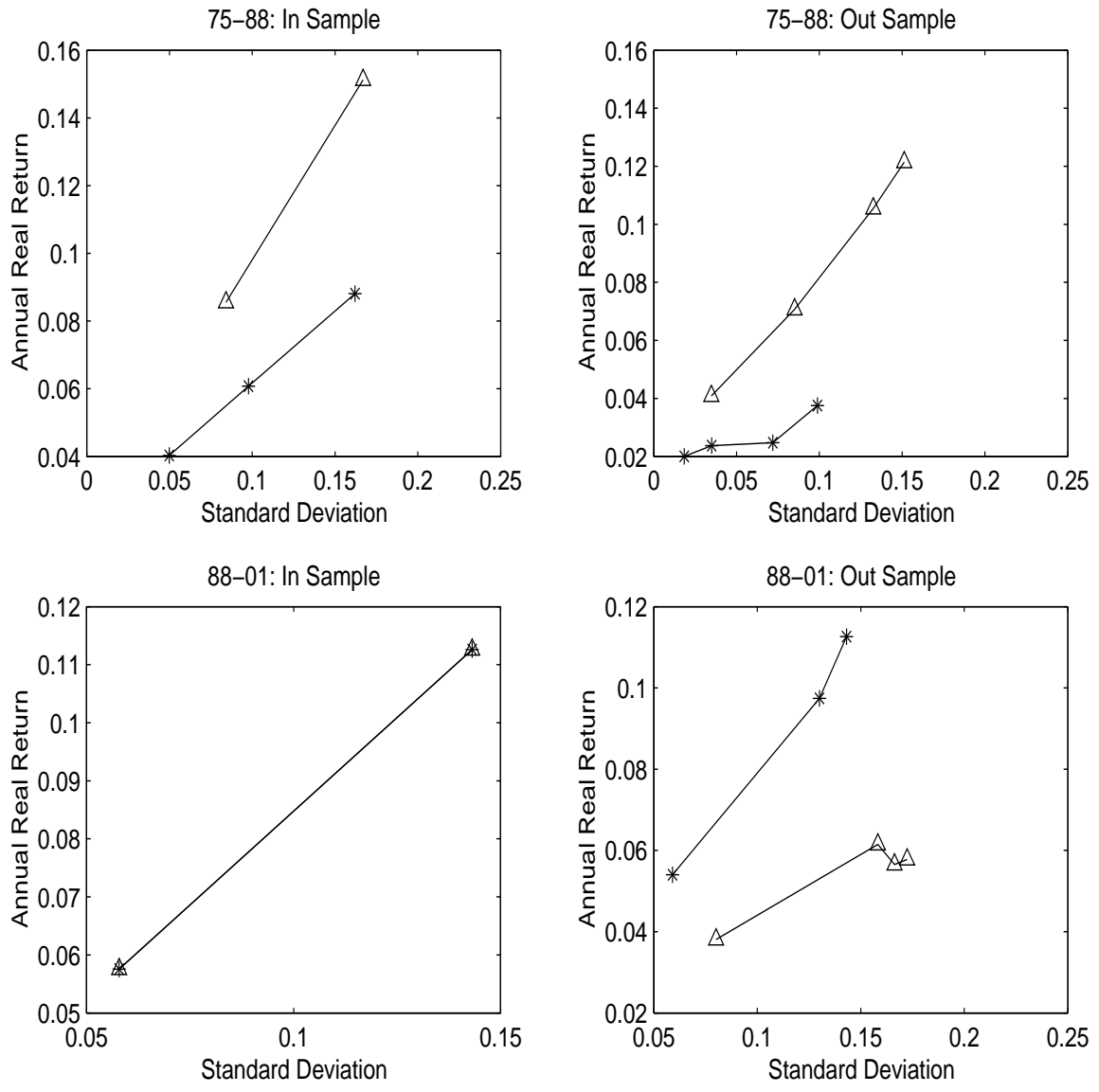


Figure 6: Mean/Std's for Varying Risk Aversion: Sub-samples

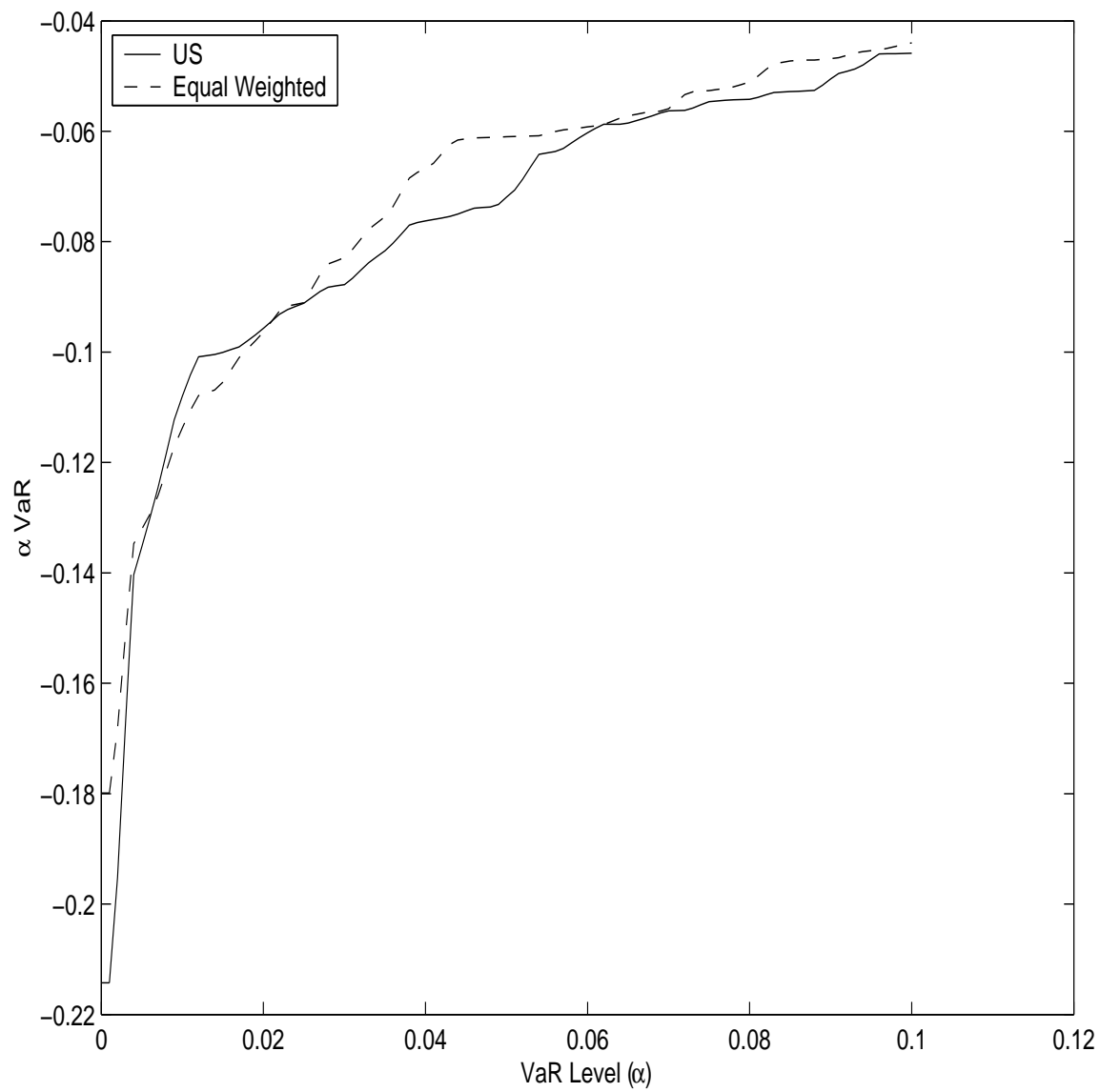


Figure 7: **VaR Estimates:** US versus Equal Weighted Equity