Price Stability in Open Economies*

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Abstract

This paper studies the conditions under which price stability is the optimal policy in a two-country open-economy model with imperfect competition and price stickiness. Special conditions on the levels of country-specific distortionary taxation and the intratemporal and intertemporal elasticities of substitution need to be satisfied. These restrictions apply to both cooperative and non-cooperative settings. Most importantly, we show that cooperative and non-cooperative solutions do not converge despite market completeness and producer currency pricing.

1 Introduction

There is a large consensus among policymakers and students of monetary policy that price stability should be the main objective of a Central Bank. This is a desirable goal insofar as it can induce an efficient allocation of resources across different uses and times. An increasing literature on monetary policy evaluation has started to address the issue of optimal monetary policy in stochastic general-equilibrium models with monopolistic competition and price stickiness.

In closed-economy models, the case for price stability is quite robust. Its desirability is associated with the possibility of reproducing the fluctuations that would

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arise in a flexible-price world. Under ex-ante commitment and isoelastic preferences, a policy of price stability reproduces the flexible-price allocation, as shown in Goodfriend and King (2000). On the other hand, under discretion, the policymaker has an incentive to inflate the economy and eliminate the existing monopolistic distortions. Only when an appropriate taxation subsidy eliminates this 'inflation bias', can a finite discretionary equilibrium with price stability exist and the allocation reproduces the flexible-price one, as shown in Woodford (1999a).

The focus of the present paper is to investigate the conditions under which price stability is the optimal outcome in a two-country open economy model. We present a standard dynamic general equilibrium model in which each country is specialized in a production of a continuum of differentiated goods, prices of the final goods are sticky and producer currency pricing holds. Domestic and international markets are complete. In our open economy framework, price stability refers to the stabilization of the domestic producer price level in each country. The resulting allocation reproduces the flexible-price equilibrium.

The open-economy case enriches the analysis by allowing for a strategic interaction between different policymakers. We study the allocations that result from various forms of interaction. We consider the efficient solution as well as strategic equilibria in which monetary policymakers can either commit or act in a discretionary way.

In general, in a two-country open-economy model, the argument for price stability relies on 'knife-edge' conditions. Several interesting results emerge.

In general the flexible price allocation is not efficient unless for specific combination of degrees of monopolistic competition corrected by the distortionary taxation and weights used in the social welfare function. Otherwise, multiple distortions will exist and the sticky-price frictions can be used to improve upon the flexible-price allocation *even from a centralized perspective*. Other special conditions for which the flexible price allocation is always efficient include the case of unitary intratemporal elasticity of substitution (like in Devereux and Engel, 2000, and Obstfeld and Rogoff, 2001) or the case in which the intratemporal elasticity of substituion is equal to the intertemporal elasticity of substitution (like in Corsetti and Pesenti, 2001).

We then move to the strategic game between policymakers. Under an ex-ante commitment solution, there are gains from cooperation even in a world where goods and financial markets are perfectly integrated, although even in the cooperative case, the equilibrium exists only in restricted circumstances, and there is no presumption that the gains from cooperation are either feasible or large. This result is new and contrasts the main intuition provided by Obstfeld and Rogoff (2001). Indeed, the conditions for price stability are a subset of the conditions under which price stability is the efficient solution. In particular the equalization of the degrees of monopolistic distortions across countries is not sufficient in implementing price stability as a decentralized allocation. In the producer-currency pricing case, Corsetti and Pesenti (2001b), Devereux and Engel (2000), and Obstfeld and Rogoff (2000) the flexible-price allocation is always efficient and can also be reached in a decentralized context. Their key assumption is the unitary intratemporal elasticity of substitution between home and foreign produced goods.

Under discretion, there exists only one finite rational expectation equilibrium with price stability. Differently from the closed-economy case, the monopolistic distortions should not be completely neutralized. Still, there exists an inflationary bias because policymakers would like to reduce the distortions associated with monopoly power in production. However, in an open-economy model, a deflationary bias might also arise due to the incentive to use the terms of trade strategically. In some cases, each country could be better off by deflating and worsening the terms of trade, leaving the burden of production to the other country without overly sacrificing consumption.[1] In our stochastic context, when appropriate distortionary taxes balance these contrasting forces, the resulting equilibrium is the flexible-price allocation.

Finally we show that under the same conditions that support the discretionary equilibrium, quadratic approximations of country-specific welfare can be correctly evaluated by relying only on log-linear approximation to the structural equilibrium conditions.[2] Linear-quadratic analyses in open-economy models are then appropriate only under special conditions. More accurate approximations are needed, as in Sims (2000).

The structure of the work is the following: section 2 presents the model emphasizing the main assumptions; section 3 studies the closed-economy limiting case; section 4 discusses the conditions under which the flexible-price allocation is efficient in our open-economy framework; section 5 discusses the strategic solutions, while section 6 provides the conditions under which linear-quadratic models are appropriate in

[1] This result has been emphasized in a perfect foresight model by Corsetti and Pesenti (2001a) and Tille (2000).

[2] See Clarida, Gali and Gertler, 2001 for an application of our results.

open-economy models. Section 7 concludes.

2 A two country open economy model

The model belongs to a recent class of stochastic general equilibrium models with imperfect competition and price stickiness that have been used for positive and normative analysis.[3] In this section we emphasize the main structure of the model and its crucial assumptions. We consider an open-economy model with two countries, Home and Foreign. They produce a continuum of goods indexed on the intervals $[0, n)$ and $[n, 1]$, respectively. In each country there is a continuum of economic agents, with population size set equal to the range of produced goods: home and foreign households lie on the interval $[0, n)$ and $[n, 1]$, respectively. Each agent is a monopolist in producing a single differentiated good. The preferences of a generic household j belonging to country H are given by

$$U_t^j = \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(C_\tau^j) - V(y_\tau^j, z_\tau) \right] \right\},$$

where E_t is the expectation conditional on the information at time t; β is the intertemporal discount factor, with $0 < \beta < 1$. U is an increasing concave function in the consumption index C, while V is an increasing convex function of y. y^j denotes the production of the differentiated good produced by agent j, while z is a country-specific shock. Preferences of a generic household belonging to country F are identical, with the exception that variables specific to country F are denoted with a star. The consumption index C, which is common across countries, is defined as

$$C = \left[n^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \ \theta > 0$$

where C_H and C_F are consumption bundles of the home- and foreign-produced goods, respectively; θ denotes the intratemporal elasticity of substitution between C_H and C_F. We have that

$$C_H \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_o^n c(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}},$$

[3] Closed economy models are described in Goodfriend and King (1997), King and Wolman (1998), Woodford (1996, 1999a). Corsetti and Pesenti (2001b), Devereux and Engel (2000), Obstfeld and Rogoff (1998,2001) consider open-economy models.

where $c(h)$ and $c(f)$ are consumptions of the generic differentiated goods produced in country H and F, respectively; σ is the elasticity of substitution across goods produced within a country, where $\sigma > 1$. The appropriate consumption-based price index that corresponds to the above specification of preferences is

$$P = \left[n P_H^{1-\theta} + (1-n)\left(P_F\right)^{1-\theta} \right]^{\frac{1}{1-\theta}},$$

with P_H and P_F given by

$$P_H = \left[\left(\frac{1}{n}\right) \int_o^n p(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}}, \qquad P_F = \left[\left(\frac{1}{1-n}\right) \int_n^1 p(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}},$$

where $p(h)$ is the price in units of currency H of a generic differentiated good h produced in country H, while $p(f)$ is the price in units of currency H of a generic good f produced in country F.

The nominal exchange rate, S, is defined as the price of the foreign currency in terms of home currency. All goods are traded and the law of one price holds. Thus $p(h) = S \cdot p^*(h)$ and $p(f) = S \cdot p^*(f)$. Given the law of one price and the fact that the consumption index C is common across countries, purchasing power parity holds, i.e. $P = SP^*$ and $P_H = SP_H^*$, $P_F = SP_F^*$.

Given the structure of preferences, the demands of the generic goods h and f are given by

$$y^d(h) = \left[\frac{p(h)}{P_H}\right]^{-\sigma} \left[\frac{P_H}{P}\right]^{-\theta} C^W, \qquad y^d(f) = \left[\frac{p(f)}{P_F}\right]^{-\sigma} \left[\frac{P_F}{P}\right]^{-\theta} C^W \tag{1}$$

where C^W is world consumption defined as $C^W \equiv nC + (1-n)\,C^*$.

We assume market completeness both at domestic and international level. Given the producer-currency-pricing assumption and the fact that preferences are symmetric across countries, complete markets implies that there is perfect consumption risk-sharing, i.e. $C = C^* = C^W$.

We do not model money explicitly, but we interpret this model as a cash-less limiting economy, in the spirit of Woodford (1998), in which the role of money balances in facilitating transactions is negligible. [4]

[4] We discuss more formally our cash-less limiting economy in the appendix. Our model can be interpreted as the limiting case in which the relative importance of the service flow from real money balances in the utility function goes to zero.

2.1 Flexible price allocation

Households act as monopolists in selling their differentiated goods. We first focus on the flexible price allocation. A generic seller h that belongs to country H chooses her price $p(h)$ in order to maximize the function

$$\Psi_t = (1-\tau)\lambda_t p_t(h) y_t^d(h) - V(y_t^d(h), z_t) \tag{2}$$

where $y^d(h)$ is defined by (1), while τ is a country-specific proportional tax[5] on firms' revenue; λ_t is the marginal utility of nominal income at time t, with $\lambda_t = U_C(C_t)/P_t$. The optimal price-setting decision will be identical across all sellers within a country. In the symmetric equilibrium, the price-setting conditions for country H and F imply

$$(1-\Phi)U_C(C_t)\frac{P_{H,t}}{P_t} = V_y\left(\left[\frac{P_{H,t}}{P_t}\right]^{-\theta} C_t, z_t\right), \tag{3}$$

$$(1-\Phi^*)U_C(C_t)\frac{P_{F,t}}{P_t} = V_y\left(\left[\frac{P_{F,t}}{P_t}\right]^{-\theta} C_t, z_t^*\right). \tag{4}$$

in all contingencies and at all times t. Equations (3) and (4) combined with the definition of the consumption-based price index P determine the level of consumption and relative prices under the flexible price allocation. We have defined the overall monopolistic distortions corrected by distortionary taxation –the variables Φ and Φ^* for country H and F respectively– as

$$(1-\Phi) \equiv \frac{\sigma-1}{\sigma}(1-\tau), \qquad (1-\Phi^*) \equiv \frac{\sigma-1}{\sigma}(1-\tau^*),$$

where $\sigma/(\sigma-1)$ indicates the mark-up that arises from the monopolistic competition. When $\Phi = 0$, the monopolistic distortions are completely eliminated by an appropriate taxation subsidy. An intuitive interpretation of equations (3) and (4) follows from noting that real marginal costs are defined by

$$\begin{aligned} mc_t &= \frac{\left(\frac{P_{H,t}}{P_t}\right)^{-1} V_y\left(\left[\frac{P_{H,t}}{P_t}\right]^{-\theta} C_t, z_t\right)}{U_C(C_t)}, \\ mc_t^* &= \frac{\left(\frac{P_{F,t}}{P_t}\right)^{-1} V_y\left(\left[\frac{P_{F,t}}{P_t}\right]^{-\theta} C_t, z_t^*\right)}{U_C(C_t)}, \end{aligned}$$

[5] These proportional taxes are rebated to the consumer through lump-sum transfers.

for countries H and F, respectively. In the flexible-price allocation, real marginal costs are proportional to the level implied by the overall degrees of monopolistic competition. When $\Phi = \Phi^* = 0$ the resulting allocation reproduces the competitive one since mark-ups are completely eliminated.

2.2 Welfare Analysis

In this work, we assume that the monetary authorities are benevolent and maximize expected households' utility. The welfare criteria for the home and foreign policy-makers are defined as

$$W_t \equiv \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} w_\tau \right\}, \qquad W_t^* \equiv \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} w_\tau^* \right\}, \tag{6}$$

where w_t and w_t^* are the instantaneous average utility flows among all the households belonging to countries H and F, respectively:

$$w_\tau \equiv U(C_\tau) - \frac{\int_0^n V(y_\tau(h), z_\tau)dh}{n}, \qquad w_\tau^* \equiv U(C_\tau) - \frac{\int_{1-n}^1 V(y_\tau(f), z_\tau^*)df}{1-n}.$$

2.3 Preferences specification

We assume that $U(.)$ and $V(.)$ are isoelastic functions of the form

$$\begin{aligned} U(C_t) &\equiv \frac{(C_t)^{1-\rho}}{1-\rho}, \\ V(y_t^j, z_t) &\equiv \frac{z_t (y_t^j)^\upsilon}{\upsilon} \text{ if } j \in H, \quad V(y_t^j, z_t^*) \equiv \frac{z_t^* (y_t^j)^\upsilon}{\upsilon} \text{ if } j \in F, \end{aligned}$$

where ρ is the intertemporal elasticity of substitution in consumption, with $\rho > 0$ while $\eta \equiv \upsilon - 1$, with $\upsilon \geq 1$, is the elasticity of labor supply. Goodfriend and King (2000) analyze their closed-economy case within this class of preferences. In a two-country open economy model, Corsetti and Pesenti (2001b) assume $\rho = \upsilon = \theta = 1$, Devereux and Engel (2000) assume $\upsilon = \theta = 1$, Obstfeld and Rogoff (2001) assume $\theta = 1$. The latter work includes also non-tradable goods in the consumption index, which becomes a Cobb-Douglas index of tradable and non-tradable goods.

3 Commitment and discretion in closed economy

Here we discuss the closed-economy case, which can be obtained from the model presented in the previous section by making n equal to 1. We abstract from the strategic interaction between different policymakers.

First, we consider the case in which all prices are set one period in advance. The optimal pricing decision of a generic firm j in setting its price p_t^j for time t with the information set at time $t-1$ implies that

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi)\frac{U_C(C_t)}{P_t}p_t^j - V_y(y_t^j, z_t)\right]y_t^j\right\} = 0, \quad \text{for all } j \tag{7}$$

where

$$y_t^j = \left(\frac{p_t^j}{P_t}\right)^{-\sigma} C_t.$$

An intuitive interpretation of condition (7) is that prices are set to keep average real marginal costs constant. According to (7), there is a unique choice of p_t^j. All firms will set the same price, $p_t^j = P_t$ and $y_t^j = Y_t = C_t$. We can then re-write (7) as

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi)U_C(Y_t) - V_y(Y_t, z_t)\right]Y_t\right\} = 0, \tag{8}$$

at each time t. Under ex-ante commitment, the policymaker maximizes the welfare function $W_{t,}$ as in (6), with the information set at time $t-1$, under the sequence of constraints as in (8), implied by optimal price-setting for period t onwards. The utility flow w_τ is in this case

$$w_\tau = U(Y_\tau) - V(Y_\tau, z_\tau).$$

Proposition 1 *Within the class of preferences of section 2.3, in a closed-economy model, the flexible-price allocation is constrained efficient under ex-ante commitment.*

The proof can be found in Goodfriend and King (2000).[6] An intuition for this result follows from the observation that in this model there are two distortions, price

[6] In a model in which transaction services are not negligible, Adao et al. (2001) have studied the conditions under which the flexible-price allocation is constrained efficient under a general class of utility functions and shocks. In their paper they focus on the commitment case in a closed economy framework.

stickiness and the monopoly power. The latter produces an inefficient level of output. In this case, the efficient policy coincides with the competitive allocation where the marginal utility of consumption is equated to the marginal disutility of producing the goods. Under ex-ante commitment, the monetary policymaker binds itself not to 'inflate' the economy systematically. The other remaining distortion is price stickiness which prevents the efficient adjustment to the perturbations that affect the economy. Under sticky prices, productivity shocks would not have any effect on the economy since production is demand determined. A procyclical monetary policy can remove the sticky-price distortion by making production as if it were supply determined and then achieving the efficient equilibrium. By applying an argument familiar to the theory of uniform optimal taxation, constant elasticities are necessary for mark-up constancy to be optimal.[7]

Goodfriend and King (2000) call the policy that reproduces the flexible-price allocation neutral, making a case for price stability. Indeed an appropriate definition of price stability can implement such an allocation. To this end, we introduce the concept of *notional price*.[8] The notional price is defined as the price that a supplier would choose in principle, if she were free to choose a price in a certain period t independently of past prices and of the prices that would be chosen in the future. In fact, the notional price for a generic period t, p_t^N, satisfies

$$(1-\Phi)U_C(Y_t)\frac{p_t^N}{P_t} = V_y\left(\left(\frac{p_t^N}{P_t}\right)^{-\sigma} Y_t, z_t\right). \tag{9}$$

In particular, with isoelastic functions, (9) implies that

$$\frac{Y_t}{Y_t^n} = \left(\frac{1}{1-\Phi}\right)^{\frac{1}{\rho+\eta}} \left(\frac{p_t^N}{P_t}\right)^{\frac{1+\sigma\eta}{\rho+\eta}}, \tag{10}$$

where Y_t^n represents the natural rate of output, which would arise under flexible prices ($Y_t^n \equiv z_t^{\frac{1}{\rho+\eta}}$). At a generic time t output can deviate from its natural rate if the notional price at time t differs from the average actual price for that period. In this context we can then properly define price stability.

[7]With time-varying elasticities and with public expenditure shocks, the proposition 1 does not apply. It can be restored by assuming time-varying taxation, which implies a time-varying Φ.

[8]We are grateful to Mike Woodford for this hint.

Definition 2 *With prices all fixed one-period in advance, price stability is defined as the equivalence between the notional price and the average actual price in all contingencies and at all times.*

We can then restate proposition 1 in the following way.

Proposition 3 *Within the class of preferences of section 2.3, in a closed-economy model, price stability is the optimal policy under ex-ante commitment. This allocation coincides with the one under flexible-price.*

The optimal allocation can be implemented by setting the notional prices equal to the actual average price in all contingencies and at all times.

Moreover, a policy specified in terms of notional prices can determine the average actual price at each time t. Indeed, substituting the expression for Y_t, derived from (10), into (8), it results that prices P_t, which are preset at time $t-1$, depend only on the joint distribution of $\{p_t^N, Y_t^n\}$. Moreover P_t is homogenous of degree 1 in p_t^N. Once P_t are determined, then the actual realization of p_t^N determines the actual level of output Y.[9]

Under discretion, the policymaker maximizes welfare at a generic time t taking in consideration the incentive compatibility constraints given by (8) only from periods $t+1$ onwards. The only optimal condition that changes is that at period t, where we obtain

$$U_C(Y_t) = V_y(Y_t, z_t). \tag{11}$$

At time t, when prices are taken as given, the level of output is then pushed up to the competitive allocation, in which the marginal utility of consumption is equated to the disutility of output. Once prices are set then a policymaker that acts under discretion finds optimal to surprise price setters and set the notional price according to

$$p_t^N = \left(\frac{1}{1-\Phi}\right)^{\frac{1}{1+\sigma\eta}} P_t$$

achieving then condition (11). This is not a rational expectation equilibrium: once P_t adjusts to the new notional price level, then the monetary authority has still the incentive to surprise price setters. This incentive to inflate is different from the

[9] Using the consumers' Euler equation, one can retrieve the interest rate adjustment needed in order to control the notional price.

inflationary bias that one finds in the Barro-Gordon model. In our context it is not a rational expectations equilibrium.

Proposition 4 *Within the class of preferences of section 2.3, in a closed-economy model, the only discretionary equilibrium is when* $\Phi = 0$.

Indeed when $\Phi = 0$, i.e. when an appropriate subsidy offsets the monopolistic distortions, the incentive to inflate disappears. In this case the notional price will be set equal to the average actual price and the fluctuations of the economy will reproduce the flexible-price fluctuations. This discretionary equilibrium coincides with a particular solution under ex-ante commitment. Price stability can be enforced in a time-consistent equilibrium if an appropriate subsidy eliminates the monopolistic distortions.

The simple case presented here is revealing about the type of solution that would occur under a more complicated price-setting mechanism. In particular, one can assume a context in which part of the sellers pre-set their prices while the other part sets their prices in a flexible way. It can be shown that even in this context the optimal solution under ex-ante commitment can be reached by a policy of price stability. Instead if the policymaker re-optimizes at each period, the incentive to inflate remains, but will be smaller in magnitude since inflation here will create a cost of dispersion of demand across goods that are produced according to the same technology. Once again the only discretionary equilibrium would be one in which monopolistic distortions are completely offset. This argument applies also to a more complicated price-setting mechanism as in the model of Calvo (1983). Indeed Woodford (1999b) has shown, in a neighborhood of the competitive allocation, that if the policymakers can commit in a 'timeless perspective' way, they will avoid to inflate the economy pursuing then the price stability policy. His 'timeless perspective' view on the commitment corresponds to our ex-ante commitment in which the price-setting condition, for the period in which the commitment is taken, is considered as an incentive compatibility constraint.[10] Woodford (1999a) further shows that the discretionary equilibrium coincides with price stability when the monopolistic distortions are completely offset.

[10] King and Wolman (1999) obtain the same result in a model with contract à la Taylor.

4 Price stability as an efficient equilibrium in open economy

The open-economy context enriches the analysis by allowing for a strategic interaction between policymakers. We focus on the price-setting mechanism where prices are fixed one-period in advance. In this case the optimal choice of the price for period t maximizes the expected value of (2) using $t-1-$information, i.e. $\mathrm{E}_{t-1}\Psi_t$. The optimal price-setting decision implies

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi)U_C(C_t)\frac{P_{H,t}}{P_t} - V_y\left(\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t, z_t\right)\right]\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t\right\} = 0, \qquad (12)$$

for country H, while

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi^*)U_C(C_t)\frac{P^*_{F,t}}{P^*_t} - V_y\left(\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\theta} C_t, z^*_t\right)\right]\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\theta} C_t\right\} = 0 \qquad (13)$$

for country F.

Before undertaking the issue of strategic interaction between policymakers, we examine the conditions under which the flexible-price allocation is the constrained efficient policy. In particular we focus on the central planner's problem who maximizes a weighted average of expected utility of home and foreign consumers[11]

$$E_{t-1}\left\{nW_t + (1-n)W^*_t\right\}, \qquad (14)$$

where the weights are given n and $1-n$, for the home and foreign country, respectively. Another way to look at this issue is to ask whether price stability is the efficient policy. Following our closed-economy example, price stability is defined as the equalization between notional producer price and the average actual producer price in a country. The notional producer price, $p^N_{H,t}$, for country H is defined as

$$(1-\Phi)U_C(C_t)\frac{p^N_{H,t}}{P_t} = V_y\left(\left(\frac{p^N_{H,t}}{P_{H,t}}\right)^{-\sigma}\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t, z_t\right),$$

while for country F, $p^{*N}_{F,t}$, is

$$(1-\Phi^*)U_C(C_t)\frac{p^{*N}_{F,t}}{P^*_t} = V_y\left(\left(\frac{p^{*N}_{F,t}}{P^*_{F,t}}\right)^{-\sigma}\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\theta} C_t, z^*_t\right).$$

[11] In the appendix, we briefly present the more general case in which the weights differ from the country size.

From the above conditions, one can observe that our notion of price stability in both countries implements the flexible-price allocation, as described by the equations (3) and (4). Under an ex-ante commitment solution the efficient allocation is obtained by maximizing (14) under the constraints given by (12) and (13) and the constraint on the price indexes

$$1 = n\left(\frac{P_{H,t}}{P_t}\right)^{1-\theta} + (1-n)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{1-\theta}, \tag{15}$$

where we have used the law of one price and the assumption of symmetric preferences, i.e. $P^*_{F,t}/P^*_t = P_{F,t}/P_t$.

Proposition 5 *When shocks are symmetric, i.e. $z_t = z^*_t$ in all contingencies and at all times, price stability in both countries, i.e. the flexible-price allocation, is always constrained efficient. When the shocks are asymmetric, price stability in each country is always constrained efficient if Φ=Φ^*; otherwise it should be either $\theta = 1$ or $\theta = \rho^{-1}$ for any given Φ and Φ^*.*

Proof. The proof is in the Appendix ■

Even if the efficient equilibrium has been extensively studied in the literature, this proposition adds further insights on the conditions that have to be satisfied in order for the flexible-price allocation to be efficient. In the producer-currency-pricing case, Devereux and Engel (2000) and Obstfeld and Rogoff (2001) have found that the flexible-price allocation is *always* efficient, independently of the degrees of monopolistic competition and the weights used in the global welfare function, (14). [12] The common crucial assumption is the unitary intratemporal elasticity of substitution, θ.

However, Obstfeld and Rogoff (2001) have shown that their result breaks down when they assume an intertemporal elasticity of substitution in consumption, ρ, different from the unitary value. In their model, given the presence of non-tradable goods in the consumption index, $\rho \neq 1$ implies imperfect consumption risk sharing at an international level. It follows that, in a cooperative solution, there is a trade-off between the distortion coming from nominal rigidities and the one arising from imperfect risk-sharing in tradable goods: the flexible-price equilibrium is no longer

[12] Corsetti and Pesenti (2001b) and Devereux and Engel (2000) have shown that with local currecny pricing the flexible-price allocation is not achievable in the centralized equilibrium.

efficient. However, in the absence of non-tradable goods, their result holds even if $\rho \neq 1$.[13]

Our first result states that the flexible price allocation is the efficient response to common (i.e. symmetric) shocks. This result confirms previous findings (see Obstfeld and Rogoff, 2001).

New results emerge when we consider asymmetric shocks. The key departure from previous work is a non-unitary intratemporal elasticity of substitution between home and foreign goods. The flexible-price allocation is always constrained efficient if the overall degrees of monopolistic distortions are equalized across countries.[14] Price stability reproduces the flexible-price allocation and the exchange rate moves in order to accommodate asymmetric shocks. On the other hand, when the degrees of monopolistic competition are different, price stability requires either $\theta = 1$ or $\theta = \rho^{-1}$.

As a first step, we explain why condition Φ=Φ^* is required for the flexible-price allocation to be efficient.

We observe that equations (12) and (13) can be written as[15]

$$\begin{aligned} \mathrm{E}_{t-1}\{\Lambda_t\} &= \mathrm{E}_{t-1}\left\{\left[\frac{(1-\Phi)}{\upsilon}\frac{U_C(C_t)}{P_t}P_{H,t}Y_{H,t} - V\left(Y_{H,t}, z_t\right)\right]\right\} = 0, \\ \mathrm{E}_{t-1}\{\Lambda_t^*\} &= \mathrm{E}_{t-1}\left\{\left[\frac{(1-\Phi^*)}{\upsilon}\frac{U_C(C_t)}{P_t^*}P_{F,t}^*Y_{F,t}^* - V\left(Y_{F,t}^*, z_t^*\right)\right]\right\} = 0, \end{aligned}$$

where Y_H and Y_F^* are appropriate indexes of aggregate production, for country H and F respectively. Comparing the above equations with (2), we observe that Λ_t and Λ_t^* can be interpreted as 'national' profits in units of utility. Here the overall degrees of monopolistic distortions act as a tax over the revenues of the 'national' firm. In the cooperative solutions, (12) and (13) represent incentive compatibility constraints in the planner problem. However, when $\Phi \neq \Phi^*$ the marginal utility of revenues is distorted across countries. There is an additional distortion to cope with. With sticky prices, the central planner tries to correct this relative distortion by exploiting the covariance between the consumption and the relative prices, departing then from the flexible-price allocation. Indeed, as shown in Obstfeld and Rogoff (1998), covariance

[13] Their result holds also in the case in which all goods are non-tradables because there is no consumption risk to share.

[14] Since the elasticity of substitution among differentiated goods within a country is the same across countries, this implies that the distorting taxes should also be equal across countries.

[15] We are using here the assumption that utility function is isoelastic in both arguments.

terms are crucial for determining the level of variables. This is not possible when either $\theta = 1$ or $\theta = \rho^{-1}$.

If $\theta = 1$ the utility of nominal income is completely risk-shared across countries, i.e.

$$\frac{U_C(C_t)}{P_t} P_{H,t} Y_{H,t} = \frac{U_C(C_t)}{P_t^*} P_{F,t}^* Y_{F,t}^* = U_C(C_t) C_t.$$

Instead, when $\theta = \rho^{-1}$, there is no interdependence between the two countries, from a stabilization's point of view. Conditions (12) and (13) can be written as

$$\begin{aligned} \mathrm{E}_{t-1}\{\Lambda_t\} &= \mathrm{E}_{t-1}\left\{\left[\frac{(1-\Phi)}{\upsilon} Y_{H,t}^{\frac{\theta-1}{\theta}} - V\left(Y_{H,t}, z_t\right)\right]\right\} = 0, \\ \mathrm{E}_{t-1}\{\Lambda_t^*\} &= \mathrm{E}_{t-1}\left\{\left[\frac{(1-\Phi^*)}{\upsilon} Y_{F,t}^{*\frac{\theta-1}{\theta}} - V\left(Y_{F,t}^*, z_t^*\right)\right]\right\} = 0. \end{aligned}$$

In this case, the cooperative problem can be divided in two separate problems in which Y_H and Y_F^* should be optimally chosen. Note also that in this case the utility with respect to the consumption index C becomes separable in C_H and C_F and then in Y_H and Y_F^*. Furthermore, the real marginal costs in each country becomes proportional to the respective output gap. Using the definition of notional price, it can be shown that with isoelastic preferences the home and foreign output gap can be controlled directly by the deviation of the respective notional producer price to the average actual price as in

$$\frac{Y_{H,t}}{Y_{H,t}^n} = \left(\frac{1}{1-\Phi}\right)^{\frac{1}{\rho+\eta}} \left(\frac{p_{H,t}^N}{P_{H,t}}\right)^{\frac{1+\sigma\eta}{\rho+\eta}}, \qquad \frac{Y_{F,t}^*}{Y_{F,t}^{*n}} = \left(\frac{1}{1-\Phi^*}\right)^{\frac{1}{\rho+\eta}} \left(\frac{p_{F,t}^{*N}}{P_{F,t}^*}\right)^{\frac{1+\sigma\eta}{\rho+\eta}}.$$

Our analysis extends to the case in which the weights are arbitrary. When $\theta = 1$ or $\theta = \rho^{-1}$, the flexible price allocation is always efficient independently of the chosen weights (as in Obstfeld and Rogoff, 2001). On the other hand, under more general preferences, there is a pair of weights for which the flexible price allocation is efficient and the determination of these weights depend on the degrees of monopolistic distortion (i.e. when the degrees of monopolistic distortions are equal, the weights correspond to the size of the countries).

What happens when the conditions stated in proposition 5 are not met?

In general the efficient equilibrium might require variable social mark ups. In these cases the optimal allocation under sticky prices improves upon the flexible price allocation which is still feasible but not longer optimal. In general, a policy of state contingent producer price inflation is optimal but we do not quantify its dimension here.

Our result is related to the one obtained by Adao, Correia and Teles (2001) in a closed economy framework. They characterize the conditions under which the flexible price allocation is optimal and show that in general the optimal sticky price allocation dominates the flexible price one. In our open economy framework, departures from price stability arise even without assuming transaction frictions, public expenditure shocks or more general preferences.

How plausible are the parametric restrictions needed for the flexible price allocation to be efficient?

Some recent studies, such as Harrigan (1993) and Trefler and Lai (1999), find that a sensible assumption for θ is 6. Rotemberg and Woodford (1997) in their estimated optimizing model find a value for ρ equal to 0.16. Only in this case, ρ^{-1} will be close to θ. On the other hand, Eichenbaum et al. (1988) suggest that a sensible range for ρ is from 0.5 to 3 making this case less plausible.

5 Price stability as a Nash equilibrium

5.1 Commitment solution

We now move to the analysis of the strategic interaction between the two policymakers. Our objective is to characterize the conditions under which price stability can be implemented in a decentralized setting.

Here, it is crucial to specify the strategy space of each policymaker. We assume that each policymaker can set her policy in terms of the ratio of the notional price with respect to the average actual price. So country H controls the ratio $p^N_{H,t}/P_{H,t}$ while country F controls $p^{*N}_{F,t}/P^*_{F,t}$.[16]

At a first pass, and similarly to Devereux and Engel (2000) and Obstfeld and Rogoff (2001) we analyze the case in which both policymakers commit to the chosen policy. Conditions (12) and (13) act as incentive compatibility constraints.

[16] Using the consumers' Euler equation for each country, one can retrieve the interest rate adjustment needed in order to control the notional price for given strategy on the notional price of the other country.

Proposition 6 *Within the class of preferences of section 2.3, when shocks are symmetric, i.e.* $z_t = z_t^*$ *in all contingencies and at all times, price stability is always a Nash equilibrium, under ex-ante commitment solution. When shocks are asymmetric the strategy of price stability in both countries is a Nash equilibrium under ex-ante commitment if either* $\theta = 1$ *or* $\theta = \rho^{-1}$ *for any given* Φ *and* Φ^**.*

Proof. The proof is in the Appendix. ■

This proposition contrasts the intuition given by Obstfeld and Rogoff (2001). It is no longer true that there is a one to one correspondence between the conditions under which the flexible-price allocation is efficient and those under which is implementable as a Nash equilibrium under ex-ante commitment –within our class of strategies. Comparing proposition (5) and (6), we note that the conditions that characterize price stability in the decentralized setting are a subset of those that hold in a centralized setting. The case in which $\Phi = \Phi^*$ no longer implements the flexible-price allocation.

The intuition for this result is simple: even if $\Phi = \Phi^*$, each policymaker does not internalize the negative externalities on the other country's revenues and will try to exploit the terms of trade effect on its real income, when possible. As in the case before, when $\theta = 1$ and $\theta = \rho^{-1}$ this incentive disappears and there is mutual agreement between the two policymakers on stabilizing the economy at the flexible-price allocation. Furthermore, when $\theta = \rho^{-1}$ the resulting Nash equilibrium is in dominant strategies, within the class of strategies assumed. This depends on the fact that country-specific output gap can be controlled directly by the strategy of the respective policymaker without any link to the strategy of the other policymaker.

Our results point toward the conclusion that Nash and cooperative solutions need not to converge even if financial markets become more integrated (see Obstfeld and Rogoff, 2001, on this). Despite market completeness, the flexible-price allocation is not always constrained efficient. In general as we have discussed, the incentives to internalize the externalities are different whether one looks at the problem from a centralized or a decentralized perspective. Only under special 'knife-edge' conditions, self-oriented policy rules can implement the flexible-price allocation. There are then gains from cooperation even under complete markets.

In particular in our context the strategies are specified in terms of notional producer prices, without any reference to the shocks affecting the economy, be them from the Home or Foreign side. On the other hand there are different strategies that can implement the efficient allocation as a Nash equilibrium, which, however, might require responses of each monetary authorities to the shocks of the other country. In their analysis, Devereux and Engel (2000) and Obstfeld and Rogoff (2001) use money rules, that react to domestic and foreign shocks, while Corsetti and Pesenti (2001b) adopt strategies in terms of nominal spending. Devereux and Engel (2000) and Corsetti and Pesenti (2001b) have further shown that with local currency pricing there are gains from cooperation.

5.2 Discretion

As in the closed-economy model, ex-ante commitment solutions assume that the policymakers are able to bind themselves to the chosen rules. However, policymakers that act under discretion, re-optimize in each period taking as given the constraint implied by the optimal price-setting choice. As in the closed-economy model, the set of discretionary equilibria, with rational expectations, has measure zero. There is only one of them which belongs to the set of Nash equilibria with ex-ante commitment. In the other cases, as we have shown in the previous section, and consistently with Betts and Devereux (2000), there is no finite discretionary equilibrium inflation rate. One has to assume an arbitrary ad hoc costs in terms of the actual inflation rate. Differently from the closed-economy case, the open-economy discretionary equilibrium involves a positive degree of monopolistic distortions.

Proposition 7 *Within the class of preferences of section 2.3., in the case $\theta = 1$ the strategy of price stability is a time-consistent Nash equilibrium if and only if $\Phi = \overline{\Phi}$ and $\Phi^* = \overline{\Phi}^*$ with*

$$\overline{\Phi} = \frac{(1-n)n^{-1\frac{(\rho+\eta)}{(1+\eta)}}}{1+(1-n)n^{-1\frac{(\rho+\eta)}{(1+\eta)}}} \qquad \overline{\Phi}^* = \frac{(1-n)^{-1}n^{\frac{(\rho+\eta)}{(1+\eta)}}}{1+(1-n)^{-1}n^{\frac{(\rho+\eta)}{(1+\eta)}}}.$$

In the case $\theta = \rho^{-1}$, the strategy of price stability is a time-consistent Nash equilibrium if and only if $\Phi = \overline{\overline{\Phi}}$ and $\Phi^* = \overline{\overline{\Phi}}^*$ with

$$\overline{\overline{\Phi}} = 1-n \qquad \overline{\overline{\Phi}}^* = n.$$

Under these assumptions, this result holds also under a Calvo's style price-setting mechanism.

Proof. The proof is in the Appendix. ■

Again a simple intuition explains why the discretionary equilibrium can be only supported at a positive level of monopolistic competition. As in the closed economy case, monopolistic competition is associated with an inflationary-biased policy. However, in an open-economy framework, each policymaker might also face a deflationary-biased policy, due to the incentive to manipulate the terms of trade in her favor. Each policymaker would normally try to generate a surprise deflation. Indeed, a contractionary monetary policy in a country decreases consumption and appreciate the exchange rate. Given the fact that prices are sticky, then the terms of trade appreciate. Thus through the expenditure switching effect, production decreases within the country and increases abroad. It can be the case that the reduction in utility that comes from the decrease in consumption can be more than offset by the reduction in the disutility of producing goods. This deflationary bias can be as well welfare improving. There exists a point, with positive monopolistic distortions, at which the inflationary and deflationary incentives balance exactly. In particular this point is a function of the various elasticity of substitution and most importantly of the economic size of a country.[17] When n goes close to 1, the home country becomes more of a closed-economy and the incentive to deflate by using the terms of trade becomes relatively less important. On the other side, the foreign country becomes more open and more affected by movements in the terms of trade. It follows that the discretionary equilibrium is supported by a low and close to zero level of monopolistic distortions in the Home country, and a higher level in the Foreign country.

It follows that price stability can be implemented in a discretionary equilibrium only if an output tax in each country corrects appropriately the distortion associated with monopoly power in production, without neutralizing it completely as in the closed-economy model.

When $\theta = \rho^{-1}$ price stability is a dominant strategy. In this case, if $\Phi \neq \overline{\overline{\Phi}}$ the policymaker in country H has an incentive to inflate or to deflate depending on Φ being above or below, respectively, of $\overline{\overline{\Phi}}$. Instead, in the case $\theta = 1$, this argument, for an inflationary or deflationary bias, applies provided the equilibrium strategy of the policymaker of country F is taken has given.

[17] It is worth noting that the conditions on Φ and Φ^* are similar to the conditions that characterize the absence of the incentive to strategically use the terms of trade in the perfect foresight models of Corsetti and Pesenti (2001a), Tille (2000) and Benigno (2001a). However, in these frameworks the set of strategies comprise only unexpected and exogenously movements in the money supply.

Once the appropriate output tax eliminates the incentive to inflate or deflate, the monetary policymakers can implement the flexible-price allocation by following a policy of price stability. The exchange rate plays the role of absorbing asymmetric shocks. It is worth stressing that there might exist other strategies that implement the flexible-price allocation in a Nash equilibrium, in a discretionary way. These strategies (e.g. money rules) not necessarily require the same degrees of monopolistic distortions as in our proposition. Nor they imply that conditions (3) and (4) can be taken as given in the strategic game. With log-utility in consumption and linear disutility in labor, Corsetti and Pesenti (2001b), along these lines, have shown that a strategy expressed in terms of nominal spending can implement the flexible-price allocation in a discretionary equilibrium in dominant strategies. With local currency pricing, they have further shown that a policymaker acting under discretion would not find optimal to replicate the flexible-price allocation since this implies excessive variation of the exchange rate.[18]

6 A linear-quadratic special case

In this section we focus our attention to the case in which our equilibrium conditions are log-linearized. We define the conditions under which a quadratic approximation of households' expected utility can be used as a proper welfare metric by relying only on a log-linear approximation to the equilibrium conditions. The attention to this case is motivated by recent developments in monetary economics in which primary focus has been given to dynamic models, with a staggered price-setting mechanism. The use of microfounded models is appealing from a positive and normative point of view. In a log-linear form, the resulting structural equations have been interpreted as an AS-IS-LM microfounded model in works by Clarida, Gali and Gertler (1999), Kerr and King (1996) and Woodford (1996, 2000). While appealing as tools for quantitative analysis, log-linear approximations are less useful when welfare evaluations are performed. Indeed, the errors that are made when taking a log-linear approximation can be relevant for an exact rank of alternative regimes while performing the welfare analysis (see Kim and Kim, 2000, and Woodford, 1999a). It is then important, once we move to a normative analysis, to define the conditions under which these quadratic

[18] Betts and Devereux (2000) examine the problem of international monetary cooperation with local currency pricing with policymakers that act with discretion in a model with perfect foresight

approximations are accurate. Here, in an open-economy framework, we define the conditions for which we can obtain a quadratic loss function at the single country level. Interestingly these conditions are *exactly* the ones that defines our finite discretionary equilibrium.

The closed economy case

In a closed-economy model, Woodford (1999a) has defined the conditions under which an appropriate quadratic approximation of households' expected utility can be correctly evaluated by relying only on a log-linear approximation to the structural equilibrium conditions. In his case[19], the welfare can be approximated by the function

$$W_t = -\Omega \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} [\lambda y_\tau^2 + \pi_\tau^2] \right\} \tag{16}$$

where π and y are the inflation rate and the output gap, respectively, and Ω and λ are combinations of the structural parameters of the model. The crucial assumption in deriving the above welfare criterion is the existence of an appropriate taxation subsidy that offsets the monopolistic distortions. This is needed in order to eliminate the first-order terms in the quadratic expansion of the utility of the consumers. Only in this case, a log-linear approximation to the equilibrium conditions can correctly evaluate (16). If we couple (16) with the appropriate log-linear AS equation which can be derived from the Calvo's style price-setting model

$$\pi_t = k y_t + \beta \mathrm{E}_t \pi_{t+1},$$

we can observe that the optimal policy, both in a commitment or in a discretionary equilibrium, is that of stabilizing inflation (see Woodford, 1999a). Indeed, in the AS equation, there is no trade-off between stabilizing inflation and the output gap. This is consistent with our general findings in section 2.

The open economy case

In open-economy the conditions are more stringent. They coincide with those that enforce price stability as a Nash equilibrium.[20]

[19] We are now implicitly assuming that the price setting mechanism follows a partial adjustment rule a la Calvo (1983) as in Woodford (1999a).

[20] Benigno (2001b) has shown that in a two-country model a quadratic approximation of the centralized welfare can be correctly evaluated if $\Phi = \Phi^* = 0$.

Proposition 8 *Within the class of preferences of section 2.3, under the assumption* $\theta = 1$, *the strategy of zero producer inflation in each country is a Nash equilibrium if* $\Phi = \overline{\Phi}$ *and* $\Phi^* = \overline{\Phi}^*$ *where*

$$\overline{\Phi} = \frac{(1-n)n^{-1}\frac{(\rho+\eta)}{(1+\eta)}}{1+(1-n)n^{-1}\frac{(\rho+\eta)}{(1+\eta)}} \qquad \overline{\Phi}^* = \frac{(1-n)^{-1}n\frac{(\rho+\eta)}{(1+\eta)}}{1+(1-n)^{-1}n\frac{(\rho+\eta)}{(1+\eta)}},$$

Under the assumption $\theta = \rho^{-1}$, *the strategy of zero producer inflation is a Nash equilibrium if* $\Phi = \overline{\overline{\Phi}}$ *and* $\Phi^* = \overline{\overline{\Phi}}^*$ *where*

$$\overline{\overline{\Phi}} = 1-n \qquad \overline{\overline{\Phi}}^* = n.$$

Proof. The proof is in the Appendix. ■

We first discuss the case in which $\theta = \rho^{-1}$. As we have underlined in the previous section, the country-specific real marginal costs are proportional to their respective output gap. Under the assumptions on Φ and Φ^*, the quadratic approximation of the welfare of each country can be written as

$$W_t = -\Lambda \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t}[\varphi y_{H,\tau}^2 + \pi_{H,\tau}^2] \right\} \qquad W_t^* = -\Lambda^* \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t}[\psi^* y_{F,\tau}^2 + \pi_{F,\tau}^{*2}] \right\},$$

where Λ, Λ^*, φ and φ^* depend on the structural parameters of the model. The two AS equations can be written in a log-linear form as

$$\pi_{H,t} = k y_{H,t} + \beta \mathrm{E}_t \pi_{H,t+1} \qquad \pi_{F,t}^* = k^* y_{F,t}^* + \beta \mathrm{E}_t \pi_{F,t+1}^*$$

where k and k^* are functions of the structural parameters of the model. Under the assumption $\theta = \rho^{-1}$, each country can control its own output gap by specifying a path for the inflation rate. Moreover, there is no trade-off between stabilizing the output gap and the producer inflation rate in each country. It is then the case that a strategy of zero producer inflation is a Nash equilibrium in dominant strategies. The conditions on Φ and Φ^* are required to eliminate any first-order incentive to inflate or deflate, that arises independently of the stabilization problem. Differently from Woodford (1999a), monopolistic distortions are not completely offset, since as we have already stressed, in an open economy framework a deflationary bias associated with the strategic use of the terms of trade arises along with the familiar inflationary bias.

If the assumption $\theta = \rho^{-1}$ is not satisfied, we cannot separate completely the maximization problems for the two policymakers.

In general, as shown in Benigno and Benigno (2001b), there is no direct relation between the output gap and the producer inflation rate. Two AS equations can be written as

$$\pi_{H,t} = k_H[(1-n)(1+\eta\theta)q_t + (\rho+\eta)y_t^W] + \beta E_t \pi_{H,t+1} \tag{17}$$

$$\pi_{F,t}^* = k_F[-n(1+\eta\theta)q_t + (\rho+\eta)y_t^W] + \beta E_t \pi_{F,t+1}^* \tag{18}$$

where q is the deviation of the terms of trade from the flexible-price allocation and y^W is the world output gap. However under the assumption $\theta = 1$, we can obtain further interesting results. In this case, *given the strategy of zero inflation rate in one country*, e.g. country F, we can write a proportional relation between the terms of trade gap and the output gap, by using equation (18)

$$q_t = \frac{(\rho+\eta)}{n(1+\eta)} y_t^W. \tag{19}$$

Equation (19) holds as an *exact* condition, under the hypothesis of isoelastic preferences and the assumption $\theta = 1$. Given the strategy of zero producer inflation in country F, we can write the AS equation of country H as

$$\pi_{H,t} = k_H(1-n)n^{-1}[(\rho+\eta)y_t^W] + \beta E_t \pi_{H,t+1}. \tag{20}$$

Under such conditions, it is also possible to write the approximation of the welfare of country H as

$$W_t = -\Sigma \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} [\lambda_W (y_\tau^W)^2 + \pi_{H,\tau}^2] \right\}, \tag{21}$$

where Σ and λ_W are functions of the structural parameters of the model. The result that (19) holds in an exact form is crucial, combined with the assumption on Φ. In fact, if (19) was holding in a first-order approximation, then second-order terms would be crucial in evaluating a second-order expansion of the utility function. A second-order expansion of the structural equilibrium conditions would be needed. Instead in the case analyzed, given the zero producer inflation strategy in country F, there is no trade-off between stabilizing the Home producer inflation and the world output gap. Under the assumption on Φ, a strategy of zero producer inflation in the home country maximizes (21) under (20). Then, under the conditions stated above, the strategy of zero-producer inflation in both countries is a Nash equilibrium.

Along these lines, Clarida et al. (2001) have shown that the stabilization problem in a small open-economy can be reconducted to be isomorphic to the closed-economy case they had analyzed in Clarida et al. (1999). Although appealing as explanation, all these results hinge on special assumptions. Beside the analysis of equilibria with price stability, strategic and stabilization problem in open economy need further tools, as second-order approximations to the structural equilibrium conditions. Progress in this direction has been made by Sims (2000).

7 Conclusion

In this paper we have analyzed the conditions under which price stability arises as an equilibrium outcome in open economies. We show that the flexible price allocation is not efficient unless special conditions are met. In general, the degrees of monopolistic distortion need to be equalized across countries. Otherwise, special values for the intratemporal elasticity of substitution are required.

The analysis of non cooperative solutions suggests that price stability, in the special sense employed in this paper, is unlikely to emerge as an equilibrium, even in the restricted cases where it is efficient. Price stability as a Nash equilibrium under ex-ante commitment relies on a subset of the conditions under which price stability is efficient. In particular, the condition in which the degrees of monopolistic distortions are equalized across countries, is not sufficient in implementing price stability as a decentralized equilibrium. Under discretion, there is even less scope for an equilibrium to exist. There exists only one finite discretionary equilibrium supported by a specific value of monopolistic distortions. At this value, inflationary and deflationary policy biases offset each other conditionally on the equilibrium strategy of the other policymaker.

By focusing on price stability, the important lesson from our paper is that non-cooperative Nash equilibria converge to cooperative ones only under *special* circumstances. There are of course other ways to achieve the optimal allocation. GIven this restricted focus, one conclusion is that gains from international cooperation may be possible, even if markets are complete and producer currency pricing holds. However, we have not attempted to quantify these gains, which may be small, or difficult to achieve, in practice.

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Appendix

Cashless Economy In this appendix, we discuss the meaning of a cashless-limiting economy that applies to our case. We consider a generic utility function for the representative household of country H that includes also utility derived from real money balance in an additive way.

$$U_t^j = \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(C_\tau) + \chi L(M_\tau/P_\tau) - V(Y_{H,\tau}, z_\tau) \right] \right\},$$

where χ indicates the importance of the utility derived from the liquidity service of holding money with respect to the other terms in the utility function; M denotes the money holding, while $L(.)$ is an increasing concave function of the real money balances which displays satiation at a determined level of real money balance. We interpret a cashless-limiting economy as the case in which χ goes to zero. To counteract this interpretation, we show that the monetary policymaker in each country can control her notional price level by moving the money supply and that as χ goes to zero the utility derived from the real money balances becomes small with respect to the other terms in the utility function.

Recalling the equations that implicitly define the notional price levels in both countries

$$(1-\Phi) U_C(C_t) \frac{p_{H,t}^N}{P_t} = V_y \left(\left(\frac{p_{H,t}^N}{P_{H,t}} \right)^{-\sigma} \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t, z_t \right),$$

$$(1-\Phi^*) U_C(C_t) \frac{p_{F,t}^{*N}}{P_t^*} = V_y \left(\left(\frac{p_{F,t}^{*N}}{P_{F,t}^*} \right)^{-\sigma} \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\theta} C_t, z_t^* \right),$$

and the restriction on the relative prices implied by the consumption-based price indexes

$$1 = n \left(\frac{P_{H,t}}{P_t} \right)^{1-\theta} + (1-n) \left(\frac{P_{F,t}^*}{P_t^*} \right)^{1-\theta},$$

we can write

$$\begin{aligned}
C_t &= \digamma_{1,t} \left(\frac{p_{H,t}^N}{P_{H,t}}, \frac{p_{F,t}^{*N}}{P_{F,t}^*}, z_t, z_t^* \right), \\
\frac{P_{H,t}}{P_t} &= \digamma_{2,t} \left(\frac{p_{H,t}^N}{P_{H,t}}, \frac{p_{F,t}^{*N}}{P_{F,t}^*}, z_t, z_t^* \right), \\
\frac{P_{F,t}^*}{P_t^*} &= \digamma_{3,t} \left(\frac{p_{H,t}^N}{P_{H,t}}, \frac{p_{F,t}^{*N}}{P_{F,t}^*}, z_t, z_t^* \right).
\end{aligned}$$

Substituting the above conditions into the equation that defines the optimal price for country H we obtain

$$\mathrm{E}_{t-1} \left\{ \left[(1-\Phi) U_C(\digamma_{1,t}(.)) \digamma_{2,t}(.) - V_y \left(\digamma_{2,t}(.)^{-\theta} \digamma_{1,t}(.), z_t \right) \right] \digamma_{2,t}(.)^{-\theta} \digamma_{1,t}(.) \right\} = 0,$$

we can see that once the strategy of the other policymaker in terms of notional price with respect to the respective average actual price is taken as given, the average actual price in country H for time t is a function of the joint distribution of the notional prices expected for time t and the shocks z and z^*, with the information set of time $t-1$. Moreover $P_{H,t}$ is a homogeneous function of degree 1 in $p^N_{H,t}$.

Considering the Euler equation in the home economy

$$\frac{U_C(C_t)}{P_{H,t}}\frac{P_{H,t}}{P_t} = \beta(1+i_t)\mathrm{E}_t\left\{\frac{U_C(C_{t+1})}{P_{H,t+1}}\frac{P_{H,t+1}}{P_{t+1}}\right\}$$

we can write it as

$$\frac{U_C(\digamma_{1,t}(.))}{P_{H,t}}\digamma_{2,t}(.) = \beta(1+i_t)\mathrm{E}_t\left\{\frac{U_C(\digamma_{1,t}(.))}{P_{H,t+1}}\digamma_{2,t}(.)\right\} \tag{A.1}$$

where one can see how the interest rate should be brought about to obtain the desired path of the notional price, once the strategy of the other policymaker is taken as given. We can also derive the money demand equation associated with the utility function above as

$$\frac{\chi L_M(M_t/P_t)}{U_C(C_t)} = \frac{i_t}{1+i_t}$$

which can be rewritten in a more familiar form as

$$\frac{M_t}{P_t} = \chi\Gamma(C_t, i_t)$$

where Γ is an increasing function of C and decreasing in i. The above equation can be also rewritten in the form

$$\frac{M_t}{P_{H,t}} = \frac{P_t}{P_{H,t}}\chi\Gamma(C_t, i_t), \tag{A.2}$$

in which we can substitute for C_t, $P_{H,t}/P_t$ and i_t the respective functions of the home notional price, the strategy of the policymaker of country F and the shocks z and z^*. By using (A.2) at time t and for the subsequent periods, we can derive the path of money supply needed in order to control the home notional price, for given strategy of the other policymaker. Once the policymaker has chosen a desired path of notional prices, as χ goes to zero the path of money needed in order to sustain the desired path of notional prices varies. The level of money decreases in all periods. Instead, the paths of C and Y_H do not change. It follows that as χ goes to zero, the utility derived form the liquidity services given by money decreases and becomes small with respect to the other terms in the utility function, for the desired path of notional prices. This is the interpretation we refer to, when, in the text, we neglect the additional terms given by the utility derived from real money balances.

Proof of Propositions In what follows we define:

$$V(H_t) \equiv V\left(\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t, z_t\right) \qquad V(F_t) \equiv V\left(\left(\frac{P_{F,t}}{P_t}\right)^{-\theta} C_t, z_t^*\right),$$

$$\Pi_{H,t} \equiv \frac{P_{H,t}}{P_t} \qquad \Pi_{F,t} \equiv \frac{P_{F,t}}{P_t} = \frac{P_{F,t}^*}{P_t^*}.$$

Proof of Proposition 5.

In the efficient allocation, the central planner is maximizing the welfare

$$\mathrm{E}_{t-1}\left\{\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left[U(C_\tau) - nV(H_t) - (1-n)V(F_t)\right]\right\}, \tag{A.3}$$

under the constraints

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi)U_C(C_t)\Pi_{H,t} - V_y(H_t)\right]\Pi_{H,t}^{-\theta}C_t\right\} = 0, \tag{A.4}$$

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi^*)U_C(C_t)\Pi_{F,t} - V_y(F_t)\right]\Pi_{F,t}^{-\theta}C_t\right\} = 0 \tag{A.5}$$

for each time t and the constraints

$$1 = n\Pi_{H,t}^{1-\theta} + (1-n)\Pi_{F,t}^{1-\theta} \tag{A.6}$$

for each contingencies at each time t. Since there are no intertemporal linkages, we can simplify the analysis and look at the Lagrangian problem at a generic time t. We denote with $n \cdot \Gamma$ the Lagrangian multiplier associated with the constraint (A.4); $(1-n)\cdot\Omega$ is the lagrangian multiplier associated with the constraint (A.5) and μ_t is the state-contingent lagrangian multiplier associated with the constraint (A.6).

Taking the first-order condition with respect to C at a generic contingency at time t, we obtain

$$\begin{aligned}0 = {} & U_C(C_t) - n\Pi_{H,t}^{-\theta}V_y(H_t) - (1-n)\Pi_{F,t}^{-\theta}V_y(F_t) - n(1-\Phi)\Gamma U_C(C_t)\Pi_{H,t}^{1-\theta} + \\ & n\Gamma\Pi_{H,t}^{-\theta}V_y(H_t) - n(1-\Phi)\Gamma U_{CC}(C_t)\Pi_{H,t}^{1-\theta}C_t + n\Gamma\Pi_{H,t}^{-\theta}V_{yy}(H_t)\Pi_{H,t}^{-\theta}C_t + \\ & -(1-n)(1-\Phi^*)\Omega U_C(C_t)\Pi_{F,t}^{1-\theta} + (1-n)\Omega\Pi_{F,t}^{-\theta}V_y(F_t) + \\ & -(1-n)(1-\Phi^*)\Omega U_{CC}(C_t)\Pi_{F,t}^{1-\theta}C_t + (1-n)\Omega\Pi_{F,t}^{-\theta}V_{yy}(F_t)\Pi_{F,t}^{-\theta}C_t,\end{aligned} \tag{A.7}$$

where V_y is the derivative of the function V with respect to the first argument.

Taking the first-order condition with respect to Π_H at a generic contingency at time t, we obtain

$$\begin{aligned}0 = {} & \theta V_y(H_t)\Pi_{H,t}^{-1}C_t - (1-\theta)(1-\Phi)\Gamma U_C(C_t)C_t + \\ & -\theta\Gamma V_y(H_t)\Pi_{H,t}^{-1}C_t + -\theta\Gamma V_{yy}(H_t)C_t\Pi_{H,t}^{-\theta-1}C_t - \mu_t(1-\theta).\end{aligned} \tag{A.8}$$

Taking the derivative with respect to Π_F at a generic contingency at time t, we obtain

$$0 = \theta V_y(F_t)\Pi_{F,t}^{-1}C_t - (1-\theta)(1-\Phi^*)\Omega U_C(C_t)C_t + \\ -\theta\Omega V_y(F_t)\Pi_{F,t}^{-1}C_t + -\theta\Omega V_{yy}(F_t)C_t\Pi_{F,t}^{-\theta-1}C_t - \mu_t(1-\theta). \tag{A.9}$$

Combining conditions (A.8) and (A.9), we get

$$V_y(H_t)\Pi_{H,t}^{-1}[\theta - \theta\Gamma(1+\eta)] - (1-\Phi)(1-\theta)\Gamma U_C(C_t) \tag{A.10}$$
$$= V_y(F_t)\Pi_{F,t}^{-1}[\theta - \theta\Omega(1+\eta)] - (1-\Phi^*)(1-\theta)\Omega U_C(C_t),$$

while condition (A.7) can be written

$$U_C(C_t)[1 - n(1-\Phi)(1-\rho)\Gamma\Pi_{H,t}^{1-\theta} - (1-n)(1-\Phi^*)(1-\rho)\Omega\Pi_{F,t}^{1-\theta}] \\ = n\Pi_{H,t}^{-\theta}V_y(H_t)[1-\Gamma(1+\eta)] + (1-n)\Pi_{F,t}^{-\theta}V_y(F_t)[1-\Omega(1+\eta)]. \tag{A.11}$$

We can then rewrite condition (A.10) and (A.11) as

$$\{1 - (1-\Phi)(1-\rho)\Omega + n\Theta\Pi_{H,t}^{1-\theta}\}U_C(C_t)\Pi_{F,t} \\ = [1-\Omega(1+\eta)]V_y(F_t) \tag{A.12}$$

$$\{1 - (1-\Phi^*)(1-\rho)\Gamma - (1-n)\Theta\Pi_{F,t}^{1-\theta}\}U_C(C_t)\Pi_{H,t} \\ = [1-\Gamma(1+\eta)]V_y(H_t). \tag{A.13}$$

where

$$\Theta \equiv [(1-\rho) + (1-\theta)\theta^{-1}][(1-\Phi)\Omega - (1-\Phi^*)\Gamma].$$

Taking the ratio of (A.12) and (A.13), using the assumption of isoelastic preferences, it can be shown that if the shocks are symmetric, i.e. $z_t = z_t^*$ at all times and contingencies, then $\Pi_{H,t}$ and $\Pi_{F,t}$ are time-invariant. This implies in (A.12) and (A.13) that the flexible-price allocation is the optimal response to symmetric shocks. When the shocks are asymmetric, the flexible price allocation is optimal under certain conditions. Either $(1-\Phi)\Omega = (1-\Phi^*)\Gamma$ which is only possible if $\Phi = \Phi^*$, or $\theta = 1$ for any Φ, Φ^* or $\theta = \rho^{-1}$ for any Φ, Φ^*.

We now extend our proposition to the case in which the weights do not coincide with country size. The central planner is maximizing the welfare

$$\mathrm{E}_{t-1}\left\{\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left[U(C_\tau) - \gamma V(H_t) - (1-\gamma)V(F_t)\right]\right\}, \tag{A.14}$$

under the constraints given by (A.4), (A.5), (A.6). Following the same steps as before it is possible to rewrite conditions (A.12) and (A.13) as

$$\begin{aligned}\{1-(1-\Phi)(1-\rho)\Omega+n\Theta\Pi_{H,t}^{1-\theta}\}U_C(C_t)\Pi_{F,t}\\ =\left[\frac{1-\gamma}{1-n}-\Omega(1+\eta)\right]V_y(F_t)\end{aligned}\tag{A.15}$$

$$\begin{aligned}\{1-(1-\Phi^*)(1-\rho)\Gamma-(1-n)\Theta\Pi_{F,t}^{1-\theta}\}U_C(C_t)\Pi_{H,t}\\ =\left[\frac{n}{\gamma}-\Gamma(1+\eta)\right]V_y(H_t).\end{aligned}\tag{A.16}$$

where

$$\Theta\equiv[(1-\rho)+(1-\theta)\theta^{-1}][(1-\Phi)\Omega-(1-\Phi^*)\Gamma].$$

As before, when the shocks are asymmetric, the flexible price allocation is optimal under certain conditions. Either $(1-\Phi)\Omega=(1-\Phi^*)\Gamma$, or $\theta=1$ for any Φ, Φ^* or $\theta=\rho^{-1}$ for any Φ, Φ^*. Now the condition $(1-\Phi)\Omega=(1-\Phi^*)\Gamma$ does not imply $\Phi=\Phi^*$. In particular we have that the relation between the degrees of monopolistic competition and the weights is given by

$$(1-\Phi)\Omega=(1-\Phi^*)\Omega+\left[\frac{\gamma-n}{n(1-n)}\right]\frac{(1-\Phi^*)}{1+\eta}$$

Proof of Proposition 6

Under ex-ante commitment, we show that given the strategy of price stability for the Foreign policymaker, the optimal strategy for the Home policymaker is price stability when appropriate conditions are satisfied. Under ex-ante commitment, the Home policymaker maximizes domestic agents' expected utility

$$\mathrm{E}_{t-1}\left\{\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left[U(C_\tau)-V\left(H_\tau\right)\right]\right\},\tag{A.17}$$

under the sequence of incentive compatibility constraints given by the price-setting condition in the Home country

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi)U_C(C_t)\Pi_{H,t}-V_y\left(H_t\right)\right]\Pi_{H,t}^{-\theta}C_t\right\}=0,\tag{A.18}$$

one for each date t, taking into account that the price-stability strategy of the Foreign policymaker implies

$$(1-\Phi^*)U_C(C_t)\Pi_{F,t}=V_y\left(F_t\right),\tag{A.19}$$

in all contingencies and at all times and the usual constraint on price indexes

$$n\Pi_{H,t}^{1-\theta}+(1-n)\Pi_{F,t}^{1-\theta}=1,\tag{A.20}$$

in all contingencies and at all times. Since there are no intertemporal linkages, we can focus on the optimal condition at a generic time t. First, we analyze the Ramsey problem in which it can be possible to choose freely C_t, $\Pi_{H,t}$, $\Pi_{F,t}$. Γ is the Lagrangian multiplier associated with the constraint (A.18), λ_t is the state contingent lagrangian multiplier associated with the constraint (A.19) and μ_t is the lagrangian multiplier associated with the constraint (A.20).

Taking the first-order condition with respect to C_t, we obtain

$$\begin{aligned} 0 = U_C(C_t) - \Pi_{H,t}^{-\theta} V_y(H_t) - \Gamma(1-\Phi) U_{CC}(C_t) C_t \Pi_{H,t}^{1-\theta} + \\ \Gamma \Pi_{H,t}^{-\theta} V_{yy}(H_t) \Pi_{H,t}^{-\theta} C_t - \Gamma(1-\Phi) U_C(C_t) \Pi_{H,t}^{1-\theta} + \Gamma V_y(H_t) \Pi_{H,t}^{-\theta} + \\ -\lambda_t(1-\Phi^*) U_{CC}(C_t) \Pi_{F,t} + \lambda_t V_{yy}(F_t) \Pi_{F,t}^{1-\theta}. \end{aligned} \tag{A.21}$$

Taking the derivative with respect to $\Pi_{H,t}$ we obtain

$$\begin{aligned} 0 = \theta \Pi_{H,t}^{-\theta-1} C_t V_y(H_t) - \Gamma(1-\Phi)(1-\theta) U_C(C_t) C_t \Pi_{H,t}^{-\theta} + \\ -\theta\Gamma V_{yy}(H_t) C_t \Pi_{H,t}^{-\theta} C_t \Pi_{H,t}^{-\theta-1} - \theta\Gamma V_y(H_t) C_t \Pi_{H,t}^{-\theta-1} - (1-\theta) n \mu_t \Pi_{H,t}^{-\theta}. \end{aligned} \tag{A.22}$$

Taking the derivative with respect to $\Pi_{F,t}$ we obtain

$$\lambda_t(1-\Phi^*) U_C(C_t) + \lambda_t \theta V_{yy}(F_t) C_t \Pi_{F,t}^{-\theta-1} + (1-n)(1-\theta) \mu_t \Pi_{F,t}^{-\theta} = 0. \tag{A.23}$$

We can combine conditions (A.22) and (A.23), obtaining

$$\begin{aligned} (\theta - \theta\Gamma - \theta\Gamma\eta) V_y(H_t) C_t \Pi_{H,t}^{-1} - \Gamma(1-\Phi)(1-\theta) U_C(C_t) C_t = \\ -\frac{n}{1-n}(1+\theta\eta)\lambda_t \Pi_{F,t}^{\theta-1} V_y(F_t), \end{aligned} \tag{A.24}$$

where η is the inverse of the elasticity of substitution in the disutility of providing the goods. We can instead rewrite (A.21) as

$$[1+\Gamma(1-\Phi)(\rho-1)\Pi_{H,t}^{1-\theta}] U_C(C_t) = \Pi_{H,t}^{-\theta} V_y(H_t)[1-(1+\eta)\Gamma] - \lambda_t C_t^{-1} V_y(F_t)[\rho+\eta]. \tag{A.25}$$

Combining equations (A.24) and (A.25), we finally obtain

$$\frac{U_C(C_t)\Pi_{H,t}}{V_y(H_t)} = \frac{[1-\Gamma(1+\eta)](\Pi_{H,t}^{1-\theta} + \frac{1-n}{n}\theta\frac{\rho+\eta}{1+\theta\eta}\Pi_{F,t}^{1-\theta}]}{1+\Gamma(1-\Phi)[(\rho-1)\Pi_{H,t}^{1-\theta} + \frac{1-n}{n}\frac{\rho+\eta}{1+\theta\eta}(1-\theta)\Pi_{F,t}^{1-\theta}]} \tag{A.26}$$

A similar condition can be obtained for the other country. Combining both conditions, it is possible to show that when the shocks are symmetric, i.e. $z_t = z_t^*$, then the Nash-equilibrium response to the shocks, when each country follows the strategy of price stability, coincides with the response that arises under flexible-price. In the case the shocks are asymmetric, we remind that the flexible-price allocation in the Home country implies that

$$(1-\Phi) U_C(C_t) \Pi_{H,t} = V_y(H_t). \tag{A.27}$$

Comparing condition (A.26) with (A.27), it can be shown that they coincide when either $\theta = 1$ or $\theta = \rho^{-1}$. Under these conditions, given that (A.27) is implied by a strategy of price stability in the Home country, then price stability is a Nash equilibrium in a solution with ex-ante commitment.

Proof of Proposition 7

Prices fixed one-period in advance

Under discretion, we show that given the strategy of price stability for the Foreign policymaker, the optimal strategy for the Home policymaker is price stability when appropriate conditions are satisfied.

In the discretionary equilibrium, at time t, the domestic policymaker re-optimizes without taking into account the constraint (A.18). Once prices are fixed, the policymaker maximizes the utility at a generic time t

$$U(C_t) - V(H_t) + E_t \left\{ \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \left[U(C_\tau) - V(H_\tau) \right] \right\}$$

under the constraints (A.19) and (A.20) in all contingencies and at each time t. Again we focus on a generic time t. λ_t is the state contingent lagrangian multiplier associated with the constraint (A.19) and μ_t is the state-contingent lagrangian multiplier associated with the constraint (A.20).

First, we formulate the Ramsey problem. Taking the derivative of the lagrangian with respect to C_t we obtain

$$U_C(C_t) = V_y(H_t)\Pi_{H,t}^{-\theta} + \lambda_t(1 - \Phi^*)U_{CC}(C_t)\Pi_{F,t} - \lambda_t V_{yy}(F_t)\Pi_{F,t}^{-\theta}, \tag{A.28}$$

Taking the derivative with respect to $\Pi_{H,t}$ we obtain

$$\theta \Pi_{H,t}^{-\theta-1} C_t V_y(H_t) - (1-\theta)\mu_t n \Pi_{H,t}^{-\theta} = 0, \tag{A.29}$$

Taking the derivative with respect to $\Pi_{F,t}$ we obtain

$$-\lambda_t(1 - \Phi^*)U_C(C_t) - \theta\lambda_t V_{yy}(F_t) C_t \Pi_{F,t}^{-\theta-1} - (1-n)\mu_t(1-\theta)\Pi_{F,t}^{-\theta} = 0. \tag{A.30}$$

Combining conditions (A.29) and (A.30), we get

$$\lambda V_y(F_t)\Pi_{F,t}^{-1}(1+\theta\eta) = -\frac{1-n}{n}\theta\Pi_{H,t}^{-1} C_t V_y(H_t)\Pi_{F,t}^{-\theta},$$

that can be used into (A.28) to get

$$U_C(C_t)\Pi_{H,t} = V_y(H_t)\left[\Pi_{H,t}^{1-\theta} + \frac{1-n}{n}\theta\left(\frac{\rho+\eta}{1+\theta\eta}\right)\Pi_{F,t}^{1-\theta}\right]. \tag{A.31}$$

Now, the flexible-price condition would instead requires

$$(1-\Phi)U_C(C_t)\Pi_{H,t} = V_y(H_t). \tag{A.32}$$

Comparing conditions (A.31) and (A.32), one can see that they will coincide, if $\theta = 1$ when Φ is equal to $\overline{\Phi}$ where

$$\overline{\Phi} = \frac{(1-n)n^{-1}\frac{(\rho+\eta)}{(1+\eta)}}{1+(1-n)n^{-1}\frac{(\rho+\eta)}{(1+\eta)}},$$

instead, when $\theta\rho = 1$, $\overline{\Phi}$ should be such that

$$\overline{\Phi} = 1-n.$$

Now a strategy of price stability, i.e. notional prices equal to the average actual price in all contingencies, implies (A.32). It is then the optimal strategy given the price-stability strategy of the policymaker in countries F. Doing the same steps for country F, one can see that the following conditions are required. When $\theta = 1$, Φ^* should be equal to $\overline{\Phi}^*$ where

$$\overline{\Phi}^* = \frac{(1-n)^{-1}n\frac{(\rho+\eta)}{(1+\eta)}}{1+(1-n)^{-1}n\frac{(\rho+\eta)}{(1+\eta)}},$$

instead, when $\theta\rho = 1$, $\overline{\Phi}^*$ should be such that

$$\overline{\Phi}^* = n.$$

Calvo-style price-setting mechanism

We now show that this proposition can be extended under a more general price setting mechanism. We consider a partial adjustment mechanism *á la* Calvo, in which each seller faces a fixed probability $1-\alpha$ of changing its price at a certain date t independently of the time that has elapsed since its last adjustment. In this case the optimal pricing decision of a home firm that is able to change its price $\widetilde{p}_t(h)$ at a generic time t is

$$\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left\{ \left[(1-\Phi)U_C(C_{t+k}) \frac{\widetilde{p}_t(h)}{P_{H,t+k}} \left(\frac{P_{H,t+k}}{P_{t+k}} \right) - V_y(\widetilde{y}^d_{t,t+k}(h), z_{t+k}) \right] \widetilde{y}^d_{t,t+k}(h) \right\} = 0, \tag{A.33}$$

where

$$\widetilde{y}^d_{t,t+k}(h) = \left(\frac{\widetilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} \left(\frac{P_{H,t+k}}{P_{t+k}} \right)^{-\theta} C_{t+k}.$$

is the total demand for the domestic firm which produces the good h conditional on $\widetilde{p}_t(h)$ being applied at period $t+k$. Note that condition (A.33) holds in all contingencies and at all times t. We specify the strategy space in terms of actual inflation rate, showing that the strategy of zero actual inflation in both countries is a Nash equilibrium if the above conditions on Φ and Φ^* hold. If country F is following the strategy of zero producer inflation, it follows that

$$(1-\Phi^*)U_C(C_t) = \frac{P_{F,t}}{P_t} V_y \left(\left[\frac{P_{F,t}}{P_t} \right]^{-\theta} C_t, z_t^F \right). \tag{A.34}$$

in all states of nature at date t. In a discretionary equilibrium, the Home policymaker chooses the sequence $\{\Pi_\tau\}_{\tau=t}^{\infty}$ with $\Pi_t = P_{H,t}/P_{H,t-1}$ in order to maximize the welfare criterion

$$\begin{aligned} W_t &\equiv \mathrm{E}_t\left\{\sum_{\tau=t}^{\infty}\beta^{\tau-t}w_\tau\right\}, \\ w_\tau &= U(C_\tau) - \frac{\int_0^n V(y_\tau(h), z_\tau)dh}{n} \end{aligned}$$

under the constraints given by (A.34) and

$$P_{H,t}^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha^H)p_t(h)^{1-\sigma}, \tag{A.35}$$

that represents the state equation for the pice index P_H under the Calvo's model. We identify this maximization problem as problem (A). First, we consider the general problem, problem (B) in which the Home policymaker can freely controls the sequences $\{\Pi_\tau, C_\tau, \Pi_{H,\tau}, \Pi_{F,\tau}\}_{\tau=t}^{\infty}$. By enlarging the set of controls to all the variables involved in problem (B), it is possible to obtain its first-best. Moreover, the maximum value of the welfare attainable in problem (B) is always at least as good as the maximum value in problem (A), because the latter is nested in the former. Given the convexity of the disutility function in supplying labor and the fact that $\sigma > 1$, for any path of C, Π_H and Π_F, a necessary condition for a plan in the problem (B) to be optimal is to avoid dispersion of prices across the goods produced in the same country, $\Pi_t = 1$ at all time t. It follows that, in problem (B), it is optimal to stabilize the producer price level. Instead, the sequences of consumption and relative price satisfy the same conditions as in the problem with prices fixed one-period in advance. These conditions can be arranged to get

$$U_C(C_t)\Pi_{H,t} = V_y(H_t)\left[\Pi_{H,t}^{1-\theta} + \frac{1-n}{n}\theta\left(\frac{\rho+\eta}{1+\theta\eta}\right)\Pi_{F,t}^{1-\theta}\right]. \tag{A.36}$$

which again requires the same restriction on Φ in order to be satisfied by the condition

$$(1-\Phi)U_C(C_t)\Pi_{H,t} = V_y(H_t)$$

Looking back at the problem (A), the strategy of zero producer inflation can replicate the optimal path of problem (B), if either $\theta = 1$ or $\theta = \rho^{-1}$ under the appropriate restrictions on Φ. It further satisfies the constraints (A.33) at all dates t. It is then the optimal strategy in problem (A). The strategy of zero inflation rate is then a time-consistent Nash equilibrium.

Proof of Proposition 8

First we show the proposition for the case in which $\theta = 1$. A shown in Benigno (2001), the second-order approximation of the utility flows in the welfare functions

(6) can be written as

$$\begin{aligned} w_t &= U_C\overline{C}[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2 - (1-\Phi)\cdot \widehat{Y}_{H,t} - \frac{(1-\Phi)}{2}\cdot [\widehat{Y}_{H,t}]^2 + \\ & -(1-\Phi)\frac{\eta}{2}\cdot [\widehat{Y}_{H,t}]^2 - \frac{(1-\Phi)}{2}(\sigma^{-1}+\eta)\cdot \mathrm{var}_h \widehat{y}_t(h) + \\ & (1-\Phi)\eta \cdot \widehat{Y}_{H,t}\overline{Y}_t] + \text{t.i.p.} + o(\|\xi\|^3), \end{aligned} \tag{A.37}$$

for country H, while

$$\begin{aligned} w_t^* &= U_C\overline{C}[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2 - (1-\Phi^*)\cdot \widehat{Y}_{F,t} - \frac{(1-\Phi^*)}{2}\cdot [\widehat{Y}_{F,t}]^2 + \\ & -(1-\Phi^*)\frac{\eta}{2}\cdot [\widehat{Y}_{F,t}]^2 - \frac{(1-\Phi^*)}{2}(\sigma^{-1}+\eta)\cdot \mathrm{var}_f \widehat{y}_t(f) + \\ & +(1-\Phi^*)\eta \cdot \widehat{Y}_{F,t}\overline{Y}_t^*] + \text{t.i.p.} + o(\|\xi\|^3), \end{aligned} \tag{A.38}$$

for country F. We have defined as an hat variable the log deviation of a variable from the steady state value; $Y_{H,t}$, $Y_{F,t}$, $y_t(h)$ and $y_t(f)$ are defined as

$$\begin{array}{cc} Y_{H,t} = T_t^{1-n}C_t, & Y_{F,t} = T_t^{-n}C_t, \\ y_t(h) = \left(\frac{p(h)}{P_{H,t}}\right)^{-\sigma} T_t^{1-n}C_t, & y_t(f) = \left(\frac{p(f)}{P_{F,t}}\right)^{-\sigma} T_t^{-n}C_t, \end{array}$$

where $T \equiv P_F/P_H$. Moreover var is the operator variance, t.i.p. includes terms that are independent of the policy and $o(\|\xi\|^3)$ includes terms that are of order higher than the second in the bound $\|\xi\|$ on the amplitude of the shocks considered in the approximation. Furthermore we have defined $V_{yz}(z_t - \overline{z}) \equiv -V_{yy}\overline{Y}_{H,t}\overline{Y}_t$ and $V_{yz}(z_t^* - \overline{z}) \equiv -V_{yy}\overline{Y}_{F,t}\overline{Y}_t^*$. $\overline{C}$ is the steady-state level of consumption.

We show that given that one country is following a strategy of zero producer inflation, then the strategy of zero producer inflation is also optimal for the other policymaker and viceversa. If the policymaker in country F is following the strategy of zero producer inflation, then, with isoelastic preferences, condition (A.19) can be written as

$$(1-\Phi^*)C_t^{-\rho} = T_t^{-n}(T_t^{-n}C_t)^{\eta}z_t^*,$$

at each date t, which in a log-linear *exact* form implies that

$$\widehat{T}_t = \frac{(\rho+\eta)}{n(1+\eta)}\widehat{C}_t - \frac{\eta}{n(1+\eta)}\overline{Y}_t^*. \tag{A.39}$$

Condition (A.39) in (A.37), combined with the value of

$$\Phi = \overline{\Phi} = \frac{D}{1+D} \qquad \text{with} \qquad D \equiv (1-n)n^{-1}\frac{(\rho+\eta)}{(1+\eta)}$$

implies that the linear term $\widehat{C}_t - (1-\overline{\Phi})\cdot \widehat{Y}_{H,t}$ disappears.

Furthermore we can write

$$\begin{aligned}
(\widehat{Y}_{H,t})^2 &= (1+D)^2 \cdot \widehat{C}_t^2 - 2(1+D) \cdot \frac{1-n}{n}\frac{\eta}{1+\eta} \cdot \widehat{C}_t\overline{Y}_t^* + \text{t.i.p},\\
\widehat{Y}_{H,t}\overline{Y}_t &= (1+D) \cdot \widehat{C}_t\overline{Y}_t + \text{t.i.p.}
\end{aligned}$$

From which we can simplify w_t to

$$\begin{aligned}
w_t &= U_C\overline{C}[\frac{1}{2}(1-\rho)\widehat{C}_t^2 - (1+\eta)\frac{(1-\overline{\Phi})}{2} \cdot (1+D)^2 \cdot \widehat{C}_t^2 + \\
&\quad +(1-\overline{\Phi})\frac{(1-n)}{n}\eta \cdot (1+D) \cdot \widehat{C}_t\overline{Y}_t^* + \\
&\quad +(1-\overline{\Phi})\eta \cdot (1+D) \cdot \widehat{C}_t\overline{Y}_t + \\
&\quad -\frac{(1-\overline{\Phi})}{2}(\sigma^{-1}+\eta) \cdot \text{var}_h\widehat{y}_t(h)] + \text{t.i.p.} + o(\|\xi\|^3),
\end{aligned}$$

Noting that $(1-\overline{\Phi}) \cdot (1+D) = 1$, we can further simplify to

$$\begin{aligned}
w_t &= U_C\overline{C}[-\frac{(\rho+\eta)}{2n}\widehat{C}_t^2 + \frac{\eta}{n}[n\overline{Y}_t + (1-n)\overline{Y}_t^*] \cdot \widehat{C}_t \\
&\quad -\frac{(1-\overline{\Phi})}{2}(\sigma^{-1}+\eta^H) \cdot \text{var}_h\widehat{y}_t(h)] + \text{t.i.p.} + o(\|\xi\|^3),
\end{aligned}$$

and to

$$w_t = U_C\overline{C}[-\frac{1}{2n}(\rho+\eta)(\widehat{C}_t - \widetilde{C}_t)^2 - \frac{(1-\overline{\Phi})}{2}(\sigma^{-1}+\eta^H) \cdot \text{var}_h\widehat{y}_t(h)] + \text{t.i.p.} + o(\|\xi\|^3),$$

where we have used the definition of $\widetilde{C}$

$$\widetilde{C} \equiv \frac{\eta}{\rho+\eta}[n\overline{Y}_t + (1-n)\overline{Y}_t^*].$$

Following Woodford (1999a) for deriving the term $\text{var}_h\widehat{y}_t(h)$, we can write the welfare criterion W as

$$W_t = -\Sigma \text{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t}[\lambda_W(y_\tau^W)^2 + \pi_{H,\tau}^2] \right\}, \tag{A.40}$$

which corresponds to equation (21) in the main text. Σ and λ_W are functions of the structural parameters of the model. One can further show that under condition (A.39), the AS equation for country H can be written as

$$\pi_{H,t} = k_H(1-n)n^{-1}[(\rho+\eta)y_t^W] + \beta E_t\pi_{H,t+1}. \tag{A.41}$$

Given the zero producer inflation strategy of the Foreign policymaker, the optimal policy for the Home policymaker is to stabilize its producer price inflation at all dates t, if $\Phi = \overline{\Phi}$. The other side of the construction of the Nash equilibrium follows specularly.

Here we outline the proof for the case in which $\theta = \rho^{-1}$. Note that in this case we can write

$$U(C_t) = \frac{C_t^{1-\rho}}{1-\rho} = n^{\frac{1}{\theta}} \frac{C_{H,t}^{1-\rho}}{1-\rho} + (1-n)^{\frac{1}{\theta}} \frac{C_{F,t}^{1-\rho}}{1-\rho}.$$

Remember that

$$C_{H,t} = n \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C$$

which can be rewritten as

$$C_{H,t} = nY_{H,t},$$

using the definition of Y_H. We can then write the utility flow for country H as

$$w_t = U(C_t) - \frac{\int_0^n V(y_t(h), z_t)dh}{n} = n\frac{Y_{H,t}^{1-\rho}}{1-\rho} + (1-n)\frac{Y_{F,t}^{1-\rho}}{1-\rho} - \frac{\int_0^n V(y_t(h), z_t)dh}{n},$$

where

$$y_t(h) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} Y_{H,t}.$$

We can then decompose w_t in

$$w_t = \left[nU(Y_{H,t}) - \frac{\int_0^n V(y_t(h), z_t)dh}{n} \right] + (1-n)U(Y_{F,t}). \qquad \text{(A.42)}$$

Note that the terms in square bracket can be expanded following directly Woodford (1999a) into

$$W = -\Lambda \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\varphi y_{H,t}^2 + \pi_{H,t}^2] \right\} + \text{t.i.p.} + o(\|\xi\|^3) \qquad \text{(A.43)}$$

where the other terms in (A.42), of order lower than the third, can be collapsed in $t.i.p$ for appropriate class of strategies (including the equilibrium class). In fact, with the specification of the strategy space in terms of actual GDP inflation, the Home policymaker cannot control Y_F, while she can control directly Y_H (under the assumption $\theta\rho = 1$). Note that in deriving (A.43), it should be assumed that $\overline{\Phi} = 1 - n$. Indeed in the terms in the square bracket the utility of $Y_{H,t}$ is weighted by n. The expansion for country F follows specularly.