Aiming for the Bull's Eye: Inflation Targeting under Uncertainty*

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Abstract

We study the implications of uncertainty for inflation targeting. We apply Brainard's static framework which imposes multiplicative uncertainty in the monetary transmission. Brainard's main result is that in the presence of uncertainty, monetary authorities become naturally more cautious. But this also implies that monetary objectives are seldom achieved. We therefore attempt next to find a monetary rule that reaches the objectives set more often, improving therefore the welfare of the Central Bank. Such a rule is the result of a new algorithm that we put forward, in which the inflation target is state contigent. The Central Bank sets (as an auxiliary step) therefore, a variable inflation target that depends optimally, on both the degree of uncertainty as well as on the shocks that occur each time. We show that such a rule helps the CB attain its objectives more often thereby reducing the losses incurred. Moreover, and as a corollary to such an approach, the rule derived is *ex ante* neutral to the degree of uncertainty.

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1 Introduction

The benefits of inflation targeting in the Svensson (1999) sense amount to providing a nominal anchor for the private sector to infer policies with, in order to formulate expectations with greater accuracy. For the Central Bank (CB) on the other hand, inflation targeting provides an implicit commitment mechanism which increases its cost of deviating from announced targets and hence discourages it from doing so. The economy on the whole benefits from greater transparency because it leads to greater credibility and by consequence to effective monetary policies. From a political economy standpoint therefore, the literature associates a forthcoming central bank with more credible and effective policies. This constitutes the starting point of our paper, but we then argue that it is not unequivocally the case that transparent regimes enhance credibility in monetary policy. "It appears that for monetary policy makers, announcements alone are not enough; the only way to gain credibility is to earn it". (Bernanke and Mishkin, 1997). Moreover, inflation targeting[1] is neither *necessary* nor *sufficient* for effective monetary policies. Indeed, credible policies do not necessitate transparency! It is in the absence of credibility, that transparency becomes an issue. Furthermore, in the presence of uncertainty, full transparency by itself is not sufficient to achieve credible outcomes (Dillén and Nilsson,1998). It is this point that we wish to examine further in this paper.

We argue that operating in an uncertain environment redefines somewhat the concept of transparency. In that sense, it is now closer identified with an effective communication of the balance of risks, rather than with the announcement of exact targets. It is this which in combination with one's track record of meeting their objectives over a sustained period of time, eventually establishes the credibility of the Central Bank[2]. We will argue that acknowledging the presence of uncertainty implies that one can achieve a pre-announced inflation target on average at best and not at any point in time. By consequence, the likelihood of actually reaching the target is maximised when one aims *for* the bull's eye on average, and not *at* it all the time.

Similarly to Bernanke and Mishkin (1997) therefore, we view inflation targeting no so much as a rule *per se,* but as a framework for monetary policy within which "constrained discretion" can be exercised. In fact this is not contrary to the experience of inflation targeting countries which found themselves unable at times to sacrifice important elements of flexibility in the name of transparency (Bernanke *et al*, 1999, with reference to the Riksbank). There remains however an important question. It is perhaps understandable that the Central Bank would be keen to reduce the overall level of transparency. Svensson (1999) argues that this allows it to pursue its objectives without jeopardising its reputation. The issue that arises however, is why would the public ever allow for

[1] We use the terms inflation targetting and transparency interchangably.

[2] Mackie (2001) argues "There are probably two factors that determine a central Bank's credibility: first, success in meeting its objectives over a sustained period, and second the communication of a clear and realistic assessment of current conditions and of the near term outlook, with an appropriate analysis of the balance of risks."

less clearly defined monetary objectives and hence agree to receiving less information[3]. Our justification stems from the presence of uncertainty. As argued above, what the public cares about is credible policies and ultimately effective monetary performance (in the sense of achieving its objectives) sustained over time. Uncertainty by definition makes the effects of monetary policy harder to predict. But announcing a target that is unlikely to be achieved is not necessarily increasing one's credibility (Posen, 2002). Instead, we argue that if one allows for a certain degree of flexibility in pursuing the objective, (when events warrant it) and manipulates it in a transparent way (i.e. operates in an optimising framework), then society improves its welfare. If the public can discern advantages from this, it may consent to a what appears to be, less than fully transparent regime.

The paper is organised as follows. Section 2 describes the model used and presents our two benchmark cases: first, that of Certainty Equivalence in the absence of uncertainty and second the Brainard result from his (1967) seminal paper with multiplicative uncertainty. We extend this analysis in section 3, by introducing a two-step inflation targeting procedure and describing the algorithm in detail. This constitutes the main contribution of the paper. Numerical simulations evaluate the benefits of our proposed technique in Section 4. Section 5 concludes.

2 The Model

The economy is described by a simple reduced form of a demand-supply static system as follows:

$$\pi - \pi_{-1} = -ai + \varepsilon \qquad (1)$$

$$y = \pi - \pi^e + \eta \qquad (2)$$

where (1) represents a demand equation, in which deviations of inflation from a given starting point (π_{-1}) are a function of the interest rate (as in Ball 1999)[4] and (2) is a traditional expectations augmented Phillips curve. Let i denote

[3] Lewis (1991) argues that society actually does not aim to minimise secrecy of the deliberations of the Central Bank because by doing so it actually incurs higher costs, than otherwise. This is not what we will be arguing here.

[4] Traditionally equation (1) is written as $\pi - \pi^* = -ai + \varepsilon$ where π^* is the level of inflation that the CB targets (see for example, Faust and Svensson 2001). But this implies that expectations are always tied to the announcements and in the absence of shocks, if monetary policy does nothing ($i = 0$), inflation will reach its target. Our task however, will be to show how uncertainty can prevent an announced target from being credible. In the Barro Gordon (1983) set-up, equation (1) is consistent with a vertical long-run Phillips curve and a negatively sloped short-run curve which the Central Bank wants to shift downwards to a lower level of assumed expectations. Replacing (1) with $\pi - \pi^* = -ai + \varepsilon$ implies that expectations are now fixed and the central bank moves the instrument to deal with shocks that make it move along a given short-run curve.

the policy maker's intended deviation of its instrument from its neutral level. Term a can be either a constant (and positive) parameter if we assume certainty equivalence or it may be stochastic in nature drawn from a normal distribution $a \to N\left(\overline{a}, \sigma_a^2\right)$, in line with Brainard's methodology[5]. Terms ε and η represent a demand and supply shock respectively[6] and are independently normally distributed variables with known properties, $\varepsilon \to N(0, \sigma_\varepsilon^2)$ and $\eta \to N(0, \sigma_\eta^2)$. We consider a static but sequential game between the Central Bank and the private sector. The latter forms expectations at the start of the period about the level of inflation at the end of the period. These expectations form the basis which to base wage negotiations on, such that $w = \pi^e$. A shock occurs next and the CB reacts by choosing that interest rate which optimises the conditional expectation of its loss function, expressed in terms of deviations of inflation and output from their respective targets.

$$\min_i E(L) = \frac{1}{2} E\left[(\pi - \pi^*)^2 + y^2\right] \tag{3}$$

The loss function (3) shows that the Central Bank follows a flexible inflation targeting rule, as defined by Svensson (1999). We have attached for simplicity purposes, equal weights to the two objectives. Furthermore, in the absence of any other policy agent in the economy, the Central Bank's objectives are identified with those of the median voter. At the end of the period, the effects of the Central Bank's policies are revealed and in a rational expectations world, the discretionary outcome occurs. There is symmetric information shared across the agents with respect to both the sequence of events as well as the existence of uncertainty. The only difference in information therefore, (given the timing of the game) is that private agents have no knowledge of the shock, whereas the CB reacts to it in full knowledge of its extent.

Table 1 summarises the results produced under the assumption of Certainty Equivalence (CE), as well as under transmission uncertainty *à la* Brainard (see appendix A and B for detailed derivations).

Table1: Equilibrium Values

Variable	Certainty Equivalence	Brainard Uncertainty
i	$-\frac{1}{a}\left(\pi^* - \pi_{-1}\right) + \frac{1}{2a}\left(2\varepsilon + \eta\right)$	$-\frac{\overline{a}}{(\overline{a}^2 + \sigma_a^2)}\left(\pi^* - \pi_{-1}\right) + \frac{\overline{a}}{2(\overline{a}^2 + \sigma_a^2)}\left(2\varepsilon + \eta\right)$
π^e	$\left(\pi^* - \pi_{-1}\right)$	$\frac{\overline{a}^2}{\overline{a}^2 + 2\sigma_a^2}\left(\pi^* - \pi_{-1}\right)$
π	$\left(\pi^* - \pi_{-1}\right) - \frac{1}{2}\eta$	$\frac{\overline{a}^2}{(\overline{a}^2 + 2\sigma_a^2)}\left(\pi^* - \pi_{-1}\right) + \frac{2\sigma_a^2\varepsilon - \overline{a}^2\eta}{2(\overline{a}^2 + \sigma_a^2)}$
y	$\frac{1}{2}\eta$	$\frac{\sigma_a^2}{(\overline{a}^2 + \sigma_a^2)}\varepsilon + \frac{\overline{a}^2 + 2\sigma_a^2}{2(\overline{a}^2 + \sigma_a^2)}\eta$

As the table above demonstrates, increasing uncertainty confirms Brainard's observations on optimal monetary policy. In particular, the presence of uncertainty has the following effects:

[5] We assume for simplicity purposes that a, ε and η are all independent of each other.

[6] ε is an implementation or control error.

- The use of the instrument is constrained. In Brainard's terminology, the policy maker becomes naturally more cautious and at the limit abandons it altogether.(i.e. $\lim_{\sigma_a^2 \to \infty} i = 0$) [7].
- The inflation target itself becomes less important in the formulation of expectations about future inflation. The private sector discounts the ability of the central bank to achieve the announced inflationary target, in proportion to the level of uncertainty. At the same time, this implies that on average inflation will never reach its target.
- Both supply and demand shocks have an effect on output in the presence of uncertainty because monetary policy is unable to insulate the real side from demand shocks.

Comparing the two different scenarios, the presence of uncertainty reduces the efficiency of monetary policy making, which is thus moved away from its first best.

3 Inflation Targeting in Two Steps

The direct corollary of a less effective monetary policy is that the objectives of the Central Bank are also seldom achieved. What we investigate next is whether there exists a policy rule that can help monetary authorities achieve their assumed targets more often and thus reduce their welfare costs. We do so, by introducing an implicit target $(\pi^* + \theta)$[8], θ being the implicit component subject to discretionary change, still to be determined. It acquires therefore, the role of a choice variable and π^* assumes now the role of a *declaration of intent* or *deep* target. Furthermore, and for simplicity, we normalise $\pi_{-1} = 0$ which requires that $\pi^* \neq 0$ such that there is a reason for monetary policy to be active, even in the absence of shocks. This is assumed in all the discussion that follows. Equation 1 now becomes,

$$\pi = -ai + \varepsilon \tag{4}$$

and the results in Table 1 imply that the term $(\pi^* - \pi_{-1})$ is replaced by π^*. The timing of the game is described as follows:

[7] It is important to note that the Brainard result that monetary policy is less active, depends on the source of uncertainty assumed. Brainard himself was aware that uncertainty some times calls for a more agressive response depending on the covariances between model parameters and the error terms. Furthermore, Craine (1979) shows that the timing is of crucial importance. Increases in future uncertainty thus raise the current level of the average policy response, whereas increases in current uncertainty lower it.

[8] Geraats (2001) draws π^* from a distribution to reflect uncertainty in the CB's preferences. We do not allow for such asymmetry here and justify therefore any deviations from π^* as a means of dealing with uncertainty. Indeed, parameter θ will vary according to the level of uncertainty.

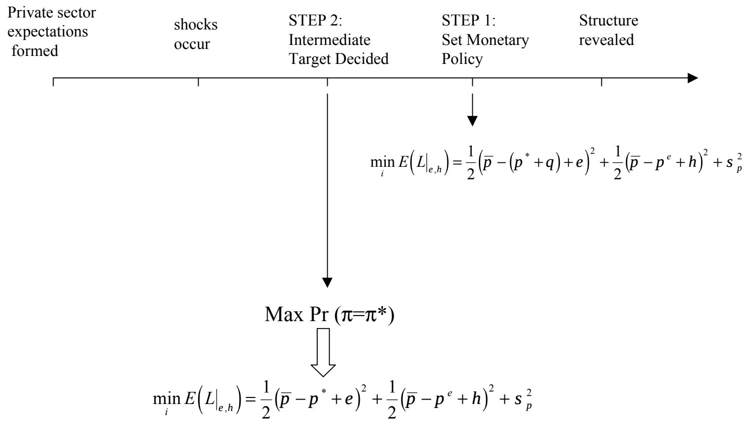

Figure 1: Timing of Events

The algorithm that we put forward implies therefore that the CB optimises its actions in two steps[9]:

3.1 Step 1

The Central Bank identifies the optimal rule as a function of θ. In other words it optimises the expected value of the following auxiliary objective function:

$$\min_i E(L) = \frac{1}{2} E\left\{ [\pi - (\pi^* + \theta)]^2 + y^2 \right\} \tag{5}$$

or the conditional expectation, given (2) and (4)

$$\min_i E(L \mid_{\varepsilon,\eta}) = \frac{1}{2}\left\{ [-\overline{a}i - (\pi^* + \theta) + \varepsilon]^2 + (-\overline{a}i - \pi^e + \varepsilon + \eta)^2 \right\} + i^2\sigma_a^2 \tag{6}$$

following Brainard's methodology (see Appendix B). Optimising (6) gives the following monetary policy reaction function and the resulting inflation, for given private sector expectations.

$$i = -\frac{\overline{a}}{2\left(\overline{a}^2 + \sigma_a^2\right)} [(\pi^* + \theta) + \pi^e] + \frac{\overline{a}}{2\left(\overline{a}^2 + \sigma_a^2\right)} (2\varepsilon + \eta) \tag{7}$$

$$\pi = \frac{\overline{a}^2}{2\left(\overline{a}^2 + \sigma_a^2\right)} [(\pi^* + \theta) + \pi^e] + \frac{2\sigma_a^2\varepsilon - \overline{a}^2\eta}{2\left(\overline{a}^2 + \sigma_a^2\right)} \tag{8}$$

Based on (8), the private sector anticipates the following rate of inflation:

[9] Solved backwards.

$$\pi^e = \frac{\overline{a}^2}{\overline{a}^2 + 2\sigma_a^2}(\pi^* + \bar{\theta}) \tag{9}$$

where $\bar{\theta}$ is the *exante* average departure from the target anticipated by the private sector. As we will show further down, $\bar{\theta}$ is always positive, a feature specific to our model since inflationary expectations achieved under Brainard uncertainty fall always short of π^*[10]. The respective Rational Expectations solutions are then:

$$\begin{aligned}
i_{RE} &= -\frac{\overline{a}}{\overline{a}^2 + 2\sigma_a^2}\left[\pi^* + \frac{\overline{a}^2}{2\left(\overline{a}^2 + \sigma_a^2\right)}\bar{\theta}\right] + \frac{\overline{a}}{2\left(\overline{a}^2 + \sigma_a^2\right)}(2\varepsilon + \eta - \theta) && (10)\\
\pi_{RE} &= \frac{2\overline{a}^2\left(\sigma_a^2 + \overline{a}^2\right)\pi^* + 4\varepsilon\sigma_a^2 + 2\overline{a}^2\sigma_a^2(\varepsilon - \eta + \theta) + \overline{a}^4\left(\theta + \bar{\theta} - \eta\right)}{2\left(\overline{a}^2 + \sigma_a^2\right)\left(\overline{a}^2 + 2\sigma_a^2\right)} && (11)\\
y_{RE} &= \frac{\sigma_a^2}{\left(\overline{a}^2 + \sigma_a^2\right)}\varepsilon + \frac{\left(\overline{a}^2 + 2\sigma_a^2\right)}{2\left(\overline{a}^2 + \sigma_a^2\right)}\eta + \frac{\overline{a}^2(\theta - \bar{\theta})}{2\left(\overline{a}^2 + \sigma_a^2\right)} && (12)
\end{aligned}$$

The above three equation rules imply that for a given level of uncertainty, the CB will choose to deviate from its ultimate target π^* by a given θ (and on average by $\bar{\theta}$).

3.2 Step 2

But the degree of deviation θ is chosen optimally. In other words, the CB applies θ to maximise the probability of achieving its true objectives. The derived rules from Step 1 for π, (11) and y, (12) are thus substituted into the objective function of the Central Bank:

$$\min_{\theta} E(L) = \frac{1}{2}E\left[(\pi - \pi^*)^2 + y^2\right] \tag{13}$$

to produce

$$\min_{\theta} E(L\mid_{\varepsilon,\eta}) = f(\theta, \bar{\theta}, \sigma_a^2, \varepsilon, \eta) \tag{14}$$

Given the rules, the aim of the CB is to find the optimal inflation target, contingent on the shock hitting the economy and the perceived uncertainty of the transmission of policies, i.e.:

$$\theta(\sigma_a^2, \varepsilon, \eta) = \arg\min_{\theta} E(L\mid_{\varepsilon,\eta})$$

which in its analytical form is

[10] This is because inflation is zero to start with.The Central Bank needs therefore to implement some action in order to get to π^*, even in the absence of shocks.

$$\theta = \frac{\sigma_a^2\left[-2\sigma_a^2(2\varepsilon+\eta-\pi^*)+\overline{a}^2(-2\varepsilon-\eta+\bar{\theta}+2\pi^*)\right]}{\left(\overline{a}^4+2\overline{a}^2\sigma_a^2\right)} \quad (15)$$

$$E(\theta) = \frac{2\sigma_a^2\pi^*}{\overline{a}^2} \quad (16)$$

For positive π^*, $E(\theta)$ (or $\bar{\theta}$) > 0, such that $\pi^e \rightarrow \pi^*$. Substituting $E(\theta)$ into (15) gives a solution for θ:

$$\theta = -\frac{\sigma_a^2\left(2\varepsilon+\eta-2\pi^*\right)}{\overline{a}^2} \quad (17)$$

As uncertainty decreases, the deviations from π^* decrease as well, such that at the limit they become zero, i.e.

$$\lim_{\sigma_a^2\rightarrow 0}(\theta) = 0$$

Proposition 1 *Applying a two-step procedure in which θ is contingent on the shocks that hit the economy, the existing uncertainty and the inflation target, neutralises the ex ante effects of uncertainty on the policy rules.*

Proof 1: Substituting the analytical solutions for θ, (17) and $E(\theta)$, (16) into (9) - (12) produces the *two-step* target rules that a Central Bank needs to apply under uncertainty.

$$\pi^e = \pi^* \quad (18)$$

$$i_{RE} = -\frac{1}{\bar{a}}\pi^* + \frac{1}{2\bar{a}}(2\varepsilon+\eta) \quad (19)$$

$$\pi_{RE} = \pi^* + \frac{1}{2}\eta \quad (20)$$

$$y_{RE} = \frac{1}{2}\eta \quad (21)$$

The rules achieved are similar to those attained under CE (with a replaced by $\bar{a}$). This demonstrates that by varying the target optimally, the uncertainty in the transmission process is neutralised. This result is a direct consequence of the way the inflation target becomes contingent on the shock incurred and the level of uncertainty. Parameter θ is then chosen to maximise the probability of hitting the explicit target π^*(or perhaps more accurately a prespecified area around it). *Ex ante* therefore, we achieve as good a result as possible under certainty equivalence. We explain next, how the algorithm produces this result.[11]

[11] Our approach is in fact equivalent to introducing an extra instrument while the number of targets has remained the same. As Hughes Hallett 1989 mentions "...all the instruments will be needed to combat uncertainty even when there are only a few targets compared to the number of instruments".

3.3 Neutralising Ex Ante Uncertainty

As shown in Appendix B, optimising under uncertainty implies that the objective function[12] is captured by $E(L) = \frac{1}{2}(\bar{\pi} - \pi^*)^2 + \frac{1}{2}\sigma_\pi^2$. The resulting solution in turn implies that the instrument is used less, compared to the solution derived with Certainty Equivalence. This is due to the fact that the loss function is now composed of two terms, each respectively a function of the first and second moment of the inflation distribution. The first is thus the square deviation of the average policy from the target, and is minimised for the same value of i that optimises the loss function under Certainty Equivalence. The second term on the other hand, is a function of the variation of the policy instrument and is minimised when the instrument is not used at all (i.e. $i = 0$). The optimal instrument value that minimises both terms together, depends on the value of the coefficient of variation, $(\frac{\sigma_\pi}{\bar{\pi}})$. In other words, the relative importance of the two first moments will decide where the optimal i will be with respect to the two limit values (Onatski 2000).
We can now rewrite the derived policy rule in terms of the coefficient of variation. From Table 1 (when $\pi_{-1} = 0$), Brainard's interest rate rule is:

$$i_{RE,BR} = -\frac{\overline{a}}{\overline{a}^2 + \sigma_a^2}\pi^* + \frac{\overline{a}\,(2\varepsilon + \eta)}{2\left(\overline{a}^2 + \sigma_a^2\right)} \tag{22}$$

$$\text{or} \quad i_{RE,BR} = -\frac{1}{\overline{a}(1 + V^2)}\pi^* + \frac{(2\varepsilon + \eta)}{2(1 + V^2)} \tag{23}$$

where $V = \frac{\sigma_a}{\overline{a}}$ is the coefficient of variation of a. In the *two-step* targeting case, substituting the explicit forms for θ and $\bar{\theta}$ in the solution for i, (9) the rule is re-written as:

$$i_{RE,TS} = -\frac{\overline{a}}{\overline{a}^2 + \sigma_a^2}\pi^* + \frac{\sigma_a^2\pi^*}{\left(\overline{a}^2 + \sigma_a^2\right)} + \frac{\overline{a}\,(2\varepsilon + \eta)}{2\left(\overline{a}^2 + \sigma_a^2\right)} * \frac{\left(\overline{a}^2 + \sigma_a^2\right)}{\overline{a}^2}$$

$$\text{or} \quad i_{RE,TS} = -\frac{1}{\overline{a}\,(1 + V^2)}\pi^*\left(1 + V^2\right) + \frac{\frac{1}{2\overline{a}}\,(2\varepsilon + \eta) * \left(1 + V^2\right)}{(1 + V^2)} \tag{24}$$

which reduces to

$$i_{RE,TS} = -\frac{1}{\overline{a}}\pi^* + \frac{1}{2\overline{a}}\,(2\varepsilon + \eta) \tag{25}$$

This is analogous to the CE rule, this time with the average policy effect $\overline{a}$ included, instead of parameter a. The intuition behind this result lies in the fact that if the effects of monetary policy are symmetrically random around a specified value $\overline{a}$, then a "prudent" monetary policy à la Brainard biases the effects on one side of this value. The optimal policy can only occur at the

[12] Ignoring for a minute the output objective.

point where the probability of a given monetary effect is maximised, and that is around its mean value. This will on average produce the explicit inflation target π^* aimed at.

4 Numerical Simulations

The results of the previous section are the *ex-ante* expected values of the variables of interest. Naturally, policy makers are mainly interested in *ex-post* distribution properties of the results once a certain policy rule is implemented. To analyse this issue we perform a Monte Carlo simulation of the system in equations 1 and 2, under the two alternative policy rules (23) and (25).

Table 2 shows the results of 100,000 stochastic simulations of our model under the two regimes of fixed, à la Brainard, and *two-step* inflation targeting. Random shock ε is drawn from a $N(0,1)$ distribution, while parameter a is drawn from a $N\left(1, 0.5^2\right)$ distribution. We choose these particular values for the moments of a, in order to have a sufficiently small coefficient of variation which in turn reduces the likelihood of having negative values for a[13]. The inflation target π^* is assumed to be 2 and $\pi_{-1} = 0$. We present the average, standard deviation and the maximum and minimum values of i, π and y derived. The first three columns show the results under a fixed inflation targeting, *à la* Brainard and the last three, those under the *two-step* inflation targeting procedure.

TABLE 2 : Monte Carlo Simulations:

	i_{BR}	π_{BR}	y_{BR}	i_{TS}	π_{TS}	y_{TS}
Mean	−1.59	1.59	0.59	−1.99	1.992	−0.007
St. Dev.	0.8	0.92	0.92	1.00	1.12	1.12
Max	1.65	6.71	5.71	2.01	9.3	7.28
Min.	−4.99	−5.14	−6.14	−6.24	−5.85	−7.85

Following a *two-step* inflation target brings the authority much closer to its objectives but at the cost of greater variability. Nevertheless, in 64 *per cent* of our realizations there is a welfare gain by doing so. The reason can be seen from the table. As argued previously, what the *two-step* inflation target algorithm actually does, is to re-activate monetary policy in a way that, given uncertainty, the probability of hitting the target π^* is maximised. As expected, the average realisation of inflation is much closer to the inflation target of 2 with a *two-step* inflation target (where an optimal value for θ brings the instrument closer to its optimal value).

Nevertheless, the effectiveness of a *two-step* inflation target strategy is a function of the level of uncertainty that the central bank faces, which is succinctly encapsulated in the coefficient of variation ($\sigma_\alpha/\overline{\alpha}$). Brainard concluded in his

[13]Negative values for a imply a perverse monetary policy effect. For the assumed coefficient of variation, $Pr(a < 0)$ is less than 3 *per cent*.

1967 paper that the Central Banker is more cautious for *any* level of uncertainty. We argue that this is not necessarily true for *any* level of uncertainty and that the Central Bank can actually do better *ex post* as well, for manageable levels of uncertainty. We show the relation between the coefficient of variation (CV) and average losses in the two regimes (L_{BR} for the fixed target regime and L_{TS} for the *two-step* one) in table 3. We also include in the last column the frequency with which the *two-step* inflation target is welfare improving.

TABLE 3 : Analysis of Losses

$C.V.$	L_{BR}	L_{TS}	$\%\,(L_{TS} < L_{BR})$
1	2.022	3.521	44%
0.5	2.203	2.505	64%
0.25	0.864	0.616	78%

Table (3) is important because it identifies the limits of our results. For a relatively high coefficient of variation, the probability of having "perverse" policies is increasing, and thus a policy *à la* Brainard is convenient (in the sense that it correctly immobilises the policy instrument). On the other hand, an activist monetary policy can prove welfare improving so long as the probability of extreme values for a (like $a < 0$, in our simple model) is low. In theoretical terms the *two-step* target case is superior to Brainard, only when the coefficient of variation has relatively small values such that the benefits of hitting the target on average are not overcompensated by the increase in the variance.

5 Conclusions

- Our main conclusion is that the effectiveness of a fixed inflation target, like any other policy rule, depends on the particular system it is applied to. If monetary policy operates in an uncertain environment of the form described above, using a fixed inflation target rule can be inefficient, in the sense of missing its objective. This occurs because under uncertainty the monetary instrument is used less, when this is not necessarily what is warranted by events.

- On the other hand, we argue that the policy authority can bring the system closer to the desired position, if it ties its objectives to the shocks realised each time. This may not achieve the implied targets all the time but it will maximise the probability of hitting them (i.e. it will hit them on average). This is very likely the case when the probability of having extreme values for a is very low. When on the other hand, the variation of a is relatively high, Brainard's advice of greater caution in the use of the instrument is justified.

- Furthermore, we argue that a *two-step* inflation targeting regime does not diminish the degree of transparency. The traditional literature on the subject (see Faust and Svensson, 2001) claims that the private sector would always prefer a fully transparent policy maker and would penalise those who are not. What we assert here instead is that transparency is now a second order problem, ancillary only to the wider issue of uncertainty. Without effective management of the latter, increasing transparency may only cause for private sector expectations and policy objectives to diverge.

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APPENDICES

A Inflation Targeting under Certainty Equivalence

Assuming Certainty Equivalence implies that the first two unconditional moments of the distribution of inflation can be represented by $E(\pi) = -ai$ and $var\,(\pi) = \sigma_{\varepsilon}^2$. Maximising (3) subject to (1) and (2) gives the familiar monetary policy reaction function for the interest rate and inflation:

$$i = -\frac{1}{2a}\left[(\pi^* - \pi_{-1}) + \pi^e\right] + \frac{1}{2a}\left(2\varepsilon + \eta\right) \quad \text{(A1)}$$

$$\pi = \frac{1}{2}\left[(\pi^* - \pi_{-1}) + \pi^e\right] + \frac{1}{2}\eta \quad \text{(A2)}$$

From (A2) expected inflation is thus equal to the target:

$$\pi^e = \pi^* - \pi_{-1} \quad \text{(A3)}$$

Substituting for inflationary expectations, the Rational Expectations rules are summarised as follows:

$$i_{RE} = -\frac{1}{a}\left(\pi^* - \pi_{-1}\right) + \frac{1}{2a}\left(2\varepsilon + \eta\right) \quad \text{(A4)}$$

$$\pi_{RE} = \left(\pi^* - \pi_{-1}\right) - \frac{1}{2}\eta \quad \text{(A5)}$$

$$y_{RE} = \frac{1}{2}\eta \quad \text{(A6)}$$

Note that monetary policy in this context is able to counteract demand shocks in their entirety but only partially offset supply shocks. The Central Bank attains thus its first best. These are summarised in the second column of Table 1 in the main text.

B Inflation Targeting with Brainard Uncertainty

We attempt to proceed here in a similar fashion to Brainard (1967), by introducing uncertainty in parameter a in equation (1). This type of multiplicative uncertainty is thus associated with uncertainty in the transmission process. The CB has thus only limited knowledge of the effects of its policies, as parameter a is stochastic in nature drawn from the following distribution:

$$a \rightarrow N\left(\overline{a}, \sigma_a^2\right)$$

For simplicity, we assume that a is independent of the two shocks. This time, the first two unconditional moments of the distribution of inflation are $E(\pi) =$

$-\overline{a}i + \pi_{-1}$ and $var\,(\pi) = i^2\sigma_a^2 + \sigma_\varepsilon^2$. We assume also that its coefficient of variation $\left(\frac{\sigma_a}{\overline{a}}\right)$ is sufficiently small to reduce the likelihood of having negative values for variable a. Given the stochastic nature of the policy problem, the CB formulates its policies based on the expected structure of the economy[14]. Formally it will be minimising the expected value of L.

$$\begin{aligned} L &= \frac{1}{2}\left[(\pi - \pi^*)^2 + y^2\right] \qquad \text{or} \\ E(L) &= \frac{1}{2}\left\{[\overline{\pi} - \pi^*]^2 + \sigma_\pi^2 + y^2\right\} \end{aligned}$$

For given shocks, the conditional expectation of the objective function is:

$$E(L\mid_{\varepsilon,\eta}) = \frac{1}{2}\left(\overline{\pi} - \pi^* + \varepsilon\right)^2 + \frac{1}{2}\sigma_\pi^2 + \frac{1}{2}\left(\overline{\pi} - \pi^e + \eta\right)^2 + \frac{1}{2}\sigma_\pi^2 \tag{B1}$$

We define an increase in structural uncertainty as an increase in the variance σ_a^2. Note how there is no σ_ε^2 in the objective function, since the CB reacts to a *given* shock ε and/or η. The first two conditional moments are given now by (B2) and (B3)

$$E(\pi\mid_{\varepsilon,\eta}) = -\overline{a}i + \varepsilon + \pi_{-1} \tag{B2}$$

$$var\,(\pi\mid_{\varepsilon,\eta}) = i^2\sigma_a^2 \tag{B3}$$

The loss function in (B1) can be rewritten as:

$$E(L\mid_{\varepsilon,\eta}) = \frac{1}{2}\left(-\overline{a}i - (\pi^* - \pi_{-1}) + \varepsilon\right)^2 + \frac{1}{2}\left(-\overline{a}i - \pi^e + \varepsilon + \eta\right)^2 + i^2\sigma_a^2 \tag{B4}$$

Optimising (B4) with respect to i gives the following policy reaction function for the instrument and inflation:

$$i = -\frac{\overline{a}}{2\left(\overline{a}^2 + \sigma_a^2\right)}\left[(\pi^* - \pi_{-1}) + \pi^e\right] + \frac{\overline{a}}{2\left(\overline{a}^2 + \sigma_a^2\right)}\left(2\varepsilon + \eta\right) \tag{B5}$$

$$\pi = \frac{\overline{a}^2}{2\left(\overline{a}^2 + \sigma_a^2\right)}\left[(\pi^* - \pi_{-1}) + \pi^e\right] + \frac{2\sigma_a^2\varepsilon - \overline{a}^2\eta}{2\left(\overline{a}^2 + \sigma_a^2\right)} \tag{B6}$$

Taking rational expectations of (B6) and simplifying we have:

$$\pi^e = \frac{\overline{a}^2}{\overline{a}^2 + 2\sigma_a^2}\left(\pi^* - \pi_{-1}\right) \tag{B7}$$

[14] Our work is very similar to what Dillén and Nilsson (1998) examine, except that our optimising framework allows us to carry out a normative analysis.

Substituting (B7)[15] in (B5) and (B6) gives us the optimal equilibrium values for the interest rate, inflation and output consistent with rational expectations:

$$i_{RE} = -\frac{\overline{a}}{\left(\overline{a}^2 + \sigma_a^2\right)} \left(\pi^* - \pi_{-1}\right) + \frac{\overline{a}}{2\left(\overline{a}^2 + \sigma_a^2\right)} \left(2\varepsilon + \eta\right) \quad \text{(B8)}$$

$$\pi_{RE} = \frac{\overline{a}^2}{\left(\overline{a}^2 + 2\sigma_a^2\right)} \left(\pi^* - \pi_{-1}\right) + \frac{2\sigma_a^2\varepsilon - \overline{a}^2\eta}{2\left(\overline{a}^2 + \sigma_a^2\right)} \quad \text{(B9)}$$

$$y_{RE} = \frac{\sigma_a^2}{\left(\overline{a}^2 + \sigma_a^2\right)}\varepsilon + \frac{\overline{a}^2 + 2\sigma_a^2}{2\left(\overline{a}^2 + \sigma_a^2\right)}\eta \quad \text{(B10)}$$

The results are summarised in the third column of Table 1 in the main text.

[15] For positive variation, the coefficient of $\pi^*$is less than one. This implies that the private sector expects the CB to get to something less than π^*. This is a model specific feature which has inflation at the starting point equal to zero. Even in the absence of shocks, the CB needs to undertake action in order to bring inflation to its desired position π^*.