

# Is it is or is it Ain't my Obligation? Regional Debt in Argentina\*

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## Abstract

This paper studies the implications of the circulation of small denomination, interest bearing regional debt, such as the Patacones issued by a regional government in Argentina. The question is whether the circulation of this debt, particularly given its small denomination, has the same monetary implications as the printing of money by a regional government. Or are the obligations of this debt simply backed by future taxation with no inflationary consequences?

We argue here that both interpretations can arise in equilibrium. In the model economy we consider there are multiple equilibria which reflect the perceptions of agents regarding the manner in which the debt obligations will be met. In one equilibrium, termed Ricardian, the future obligations are met with taxation by a regional government while in the other, termed Monetization, the central bank is induced to print money to finance the region's obligations. The multiplicity of equilibria reflects a commitment problem of the central bank. A key indicator of the selected equilibrium is the distribution of the holdings of the regional debt.

## 1 Motivation

This paper studies the inflationary implications of regional debt. Consider a federation of regions (states) in which monetary policy is centralized but fiscal policy is determined at the regional level. Individual countries, such as Argentina and the U.S., fit this description as does a coalition of countries which delegate monetary policy to a single central bank, such as the European Monetary Union.

Recent experience in Argentina motivates an analysis of the link between regional debt and inflation. The Argentine province of Buenos Aires circulated about 18 million pesos of small denomination, interest bearing provincial bonds starting in July 2001, during the currency board regime in Argentina. In fact, this debt, called Patacones, were almost the same size and had a design quite similar to the Argentine peso. The initial

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issue of Patacones were paid-off with interest in July 2002. At that time, about 2.65 billion pesos of new regional debt was issued with a maturity of November 13, 2006. Interestingly, the province announced that taxes could be paid with this debt, evidently at face value. In addition, the federal government issued 3.3 billion pesos of small denomination bonds called Lecops. Other provinces have also issued small denomination debt. To create some perspective, nominal GDP in Argentina in the fourth quarter of 2002 was 342 billion pesos and the money supply (M1) was 42 billion pesos in January 2003. <sup>1</sup>

This experience in Argentina with regional debt leads to a number of general questions:

- Is the circulation of small denominated debt by a regional government equivalent to the creation of money by the central bank and thus inflationary?
- Or, does the issuance of such debt lead to the postponement of taxation without any money creation and thus without any inflation?
- What types of government interventions, such as limitations on regional debt, are required?

Addressing these questions is important both for assessing the current situation in Argentina and, more generally, for understanding the interplay between different levels of governments within either a country or a monetary union. Moreover, policy actions have been taken to place limits upon regional debt. In Argentina, the recent negotiations with the IMF have focused on the lack of fiscal discipline at the regional level. The imposition of borrowing restrictions within a monetary union, such as those currently imposed within the EMU, is another example.

We address these questions in an abstract monetary model and show the existence of two types of equilibria. The multiplicity reflects a commitment problem of the central government. We thus argue that the answer to these questions must be determined by the interactions of the market participants and cannot be answered *a priori* by theory. From this perspective, policy interventions may be useful to coordinate on a socially preferred outcome.

In one equilibrium, which we term *Ricardian*, the regional debt is simply a bond. These bonds finance a transfer to young agents in a region and these agents save in anticipation of future taxes. Given these holdings of debt, the central government will not bail-out the region but instead allow it to default. Anticipating this, the regional government will prefer to tax its own citizens to repay the debt. So these bonds are debt and do not lead to any money creation.

In a second equilibrium, which we term *Monetization*, all agents anticipate a bail-out by the central government. Thus they all hold money and debt and indeed the central bank will choose to monetize the debt. Anticipating this, the regional government will not raise taxes to pay the debt but will choose to turn the obligation over to the central bank.<sup>2</sup> Here the bonds are money in that they lead to the printing of

<sup>1</sup>We are extremely grateful to Maria Alzua for supplying us with these data about the regional debt and to George McCandless and Carlos Zarazaga for discussions on this experience. The money supply and nominal GDP figures are from <http://www.mecon.gov.ar/progeco/dsbb.htm>.

<sup>2</sup>This theme of a region inducing monetization is present in related papers, including Aizenman (1992), Zarazaga (1995), Chari and Kehoe (1998), Chari and Kehoe (2002), Cooper and Kempf (2001) and Cooper and Kempf (2000). Here that argument is made in a setting with bonds and money. Further, those papers do not characterize the multiplicity of equilibria that may occur.

currency by the central bank. This monetization of the debt will lead a region to run large deficits since the burden of the debt is shared by all agents.

The multiplicity of equilibria reflects a commitment problem by the central government vis-a-vis the regional government. *Ex post*, the central government has an incentive to redistribute consumption across agents in the various regions. If it was feasible, the central government would commit to never bailing out the regional government. Absent this ability to commit, the desire to redistribution may lead the central government to bail-out a region.

Interestingly, this incentive to bail-out a regional government reflects, among other things, the distribution of debt holdings across agents in the economy. As made precise in the model, it is this interaction between the incentives of the central government and the distribution of debt holdings that underlies the multiplicity of equilibria. Throughout, individuals are indifferent with regards to the composition of their portfolios; it is the distribution of holdings that matters for the equilibrium and a single individual has no influence over this distribution.

To make the analysis transparent, the first section of the paper studies a real version of this problem. The regional government issues some debt in period 1 and in period 2 there is an extensive form game played by the regional and central government to determine how the obligation will be met. In one equilibrium, the obligation is met by a tax on all agents set by the central government and in the other it is met by regional taxation. The key distinguishing feature across the two equilibria is whether the debt is widely held in the economy or only held by agents in the single region as this distribution affects the *ex post* incentives of the central government. In the former case, the central government will use its taxing ability to pay-off the debt. In the latter case, the costs of default are isolated and in that case the central government will allow default rather than pay the obligation of the region.

The second section of paper studies these same themes in a monetary economy. This basic commitment problem of the government emerges again though here we study the behavior of a central bank and its use of the inflation tax. We again find multiple equilibria which are differentiated by the holdings of regional debt. If only region 1 agents hold the debt then the equilibrium is Ricardian. If the holdings are sufficiently dispersed, then the Monetization equilibrium occurs.

The paper concludes with a discussion of policy remedies. In the Monetization equilibrium, the level of government debt is excessive and some interventions, such as debt limits or even dollarization may be socially desirable.<sup>3</sup>

## 2 A Real Game

Consider a two-period economy composed of two regions, indexed  $i=1,2$ . There are  $N_i$  agents in region  $i$ . Agents have endowments in youth and old age and have access to a storage technology. There are two levels of government which are active: the government of region 1 and a central government (CG). These governments have different objectives: the region 1 government is only concerned with the welfare of its

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<sup>3</sup>The discussion of dollarization draws upon Cooper and Kempf (2001) and the arguments for constraints is related to the points made in Chari and Kehoe (2002) and Cooper and Kempf (2000).

citizens while the central government considers the welfare of all agents.

The timing of moves is given by.

- Period 1:
  - Young agents in region 1 receive a real transfer, denoted  $g^1$ , from their government.
  - Transfers to region 1 agents are financed by issuing debt,  $B^1 = N_1 g^1$ .
  - All young agents make savings decisions in anticipation of period 2 government policies.
- Period 2:
  - the region 1 government chooses to tax region 1 agents to finance its debt obligation or to pass the obligation to the central government.
  - if the region 1 government does not levy the tax, then the central government can choose to levy an economy-wide tax to finance the debt obligation of region 1.
  - if the central government decides not to levy this tax, then region 1 automatically defaults on its debt and region 1 agents bear a default cost of  $\kappa$ .

We search for a sub-game perfect Nash equilibria of this game. Accordingly, the central government is unable to make threats not to bail-out the regional government which are not credible.

Before proceeding, it is useful to characterize the planner's solution as a benchmark. Assume that the planner chooses an allocation of consumption goods over time and over regions at the start of time given the endowments of agents in each period of time and given a technology that creates  $x$  units of period 2 goods per unit stored in period 1. If the objective function of the planner is the population weighted average of the lifetime utilities of individual agents, then the solution is to equalize consumption of agents across regions and the optimal consumption profile  $(c^{y*}, c^{o*})$  satisfies the Euler equation  $u'(c^{y*}) = xv'(c^{o*})$ .

We term this the **commitment solution** as it corresponds to the outcome if the central government could commit, at the start of period 1 before young agents make their saving decision, not to levy an economy-wide tax to bail-out the regional government.<sup>4</sup> Of course, in the extensive form game outlined above, the central government does not have this commitment ability. We now turn to an analysis of the equilibria for the game without commitment.<sup>5</sup>

## 2.1 Period 1 Optimization

Region  $i$  young agents solve

$$\max_{s^i} u(\omega^y + g^i - s^i) + v(\omega^o + s^i x - \tau - \tau^i) \quad (1)$$

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<sup>4</sup>By committing not to levy the economy-wide tax, the central government can support the planner's solution.

<sup>5</sup>Thus the problem falls within the general class of team incentive problems where the central government is the principal and the regional governments are the agents. The structure of the problem is thus similar to the family incentive problem and the "Rotten Kid Theorem" as formalized in Bergstrom (1989).

where  $s^i$  is real savings,  $g^i$  is a real transfer in youth per capita in region  $i$ ,  $\omega^y$  is the endowment in youth,  $\omega^o$  is the endowment in old age,  $\tau$  is a common tax and  $\tau^i$  is the regional tax. Savings takes two forms: storage ( $k^i$ ) and the holding of region 1 debt ( $b^i$ ). The return on storage is given by  $x$  and, in equilibrium, this is the return on regional debt as well. Assume that the only action is in region 1 so that  $g^2 \equiv \tau^2 \equiv 0$ . Note that taxes are lump-sum. Assume that both  $u(\cdot)$  and  $v(\cdot)$  are strictly increasing,  $u(\cdot)$  is concave and  $v(\cdot)$  is strictly concave.

The first-order condition for this problem is

$$u'(c^{yi}) = xv'(c^{oi}) \quad (2)$$

for  $i = 1, 2$ . Here  $(c^{yi}, c^{oi})$  represents consumption in the two periods of life for an agents in region  $i$ .

The savings decision will depend, in part, on the taxes that young agents anticipate in period 2. Nonetheless, it is straightforward to see that if  $u(c)$  is strictly concave,  $s^1 > s^2$ .<sup>6</sup>

The composition of saving though is indeterminate as government debt must offer the same return as storage in order for there to be an equilibrium in which debt is held. Still it is important for characterizing the set of equilibria to keep track of the distribution of debt holdings.

Let  $\theta = \frac{N_1 b^1}{B^1} = \frac{\Delta b^1}{\bar{B}}$  where  $\bar{B} \equiv \frac{B^1}{N}$  is the level of region 1 debt divided by the population,  $b^1$  is the amount of debt held by a representative region 1 agent and  $\Delta$  is the fraction of the population in region 1. By symmetry,  $(1 - \theta) = \frac{(1 - \Delta)b^2}{\bar{B}}$ .

## 2.2 Bail-out Equilibrium

In a bail-out equilibrium,  $\tau^1 = 0$  and  $\tau = \Delta g^1 x = \bar{B}x$ . Savings by the two regions are given by the first-order conditions anticipating this common tax.

To illustrate, we construct an equilibrium assuming the debt is held equally by all agents  $\theta = \Delta$ . So, differences in savings are made up entirely by differences in storage. We also assume that  $u(c) = c$ . We then provide a more general characterization of the set of equilibria returning to the more general case of a strictly concave  $u(c)$ .

To see if this is an equilibrium, we check incentives of the central and regional governments and the private agents. For the CG, let  $W^b$  and  $W^d$  denotes the payoffs to a bail-out and default respectively. These are given by:

$$W^b = \Delta v(\omega^o + s^1 x - \tau) + (1 - \Delta)v(\omega^o + s^2 x - \tau) \quad (3)$$

and

$$W^d = \Delta v(\omega^o + k^1 x) + (1 - \Delta)v(\omega^o + k^2 x) - \Delta \kappa \quad (4)$$

where  $k^i$  is the real storage by a member of region  $i$ .

The assumption  $u(c^y) = c^y$  implies that saving is the same across regions. So, in equilibrium  $s^1 = s^2 = s$  and, since all agents hold the same debt,  $k^1 = k^2 = k$ . In this case,  $W^b = v(\omega^o + kx)$ . This is clearly in

<sup>6</sup>This reflects two features of the problem:  $g^1 > g^2 = 0$  and  $\tau^1 \geq \tau^2 = 0$ . The result that  $s^1 > s^2$  if  $u(c)$  is strictly concave then follows from (2).

excess of  $W^d$  with  $\kappa > 0$ . Hence in this equilibrium the central government will prefer to bail-out the region rather than allowing default. Consequently the region will prefer to allow the central government to bail-out rather than tax its own citizens.

More generally, one can show that there are many bail-out equilibria as long as  $\theta$ , the share of the debt held by region 1 agents is not too large.<sup>7</sup> The results in the following proposition hold for  $u(c)$  strictly concave.

**Proposition 1** *For  $\kappa \geq 0$ , there exists a bail-out equilibrium for  $\theta \in [0, \Delta]$ .*

**Proof.** We first establish that at  $\kappa = 0$ ,  $W^b = W^d$  if  $\theta = \Delta$ . This see why, consider a version of (4)

$$W^d = \Delta v \left( \omega^o + \left( s^1 - \frac{\theta \bar{B}}{\Delta} \right) x \right) + (1 - \Delta) v \left( \omega^o + \left( s^2 - \frac{(1 - \theta) \bar{B}}{(1 - \Delta)} \right) x \right) - \Delta \kappa. \quad (5)$$

Further,  $W^b$  can be written as

$$W^b = \Delta v(\omega^o + s^1 x - \bar{B}x) + (1 - \Delta) v(\omega^o + s^2 x - \bar{B}x) \quad (6)$$

since  $\tau = \bar{B}x$  in the bail-out equilibrium. Comparing (5) and (6),  $W^b = W^d$  if  $\theta = \Delta$  and  $\kappa = 0$ .

We establish  $k^1 > k^2$  for any  $\theta \leq \Delta$ . At  $\theta = \Delta$ ,  $b^1 = b^2$  and  $s^1 > s^2$  implies  $k^1 > k^2$ . As  $\theta$  decreases,  $b^1$  falls relative to  $b^2$  and so  $k^1$  increases relative to  $k^2$  since  $s^1$  and  $s^2$  are given from (2). Hence,  $k^1 > k^2$  for any  $\theta \leq \Delta$ . A consequence of this is that the derivative of  $W^d$  with respect to  $\theta$ , given by  $-[v'(\omega^o + k^1 x) - v'(\omega^o + k^2 x)]\bar{B}x$ , is positive when  $\theta \leq \Delta$ .

So,  $W^b = W^d$  when  $\theta = \Delta$  and  $\kappa = 0$  implies  $W^b > W^d$  for  $\theta < \Delta$  and  $\kappa = 0$ . Since,  $W^d$  is falling as  $\kappa$  increases,  $W^b > W^d$  for  $\theta \leq \Delta$  and  $\kappa \geq 0$ .

This implies that the CG will bail-out the regional government for  $\theta \leq \Delta$ . Anticipating this, the regional government will always choose not to tax for  $\theta \leq \Delta$ . Thus there exists a bail-out equilibrium for  $\theta \leq \Delta$ . ■

The proof rests upon the basic intuition associated with the *ex post* incentive of the central government to redistribute resources towards a more equal allocation. For  $\theta \leq \Delta$ , consumption levels are more equal across regions under a bail-out than under a default. Consequently, the CG is unable to commit not to redistribute resources. The regional government recognizes this and chooses not to tax it citizens.

At  $\kappa = 0$ , the largest value of  $\theta$  consistent with a bail-out equilibrium is  $\theta = \Delta$ . From a comparison of (5) and (6),  $W^b = W^d$  when  $\theta = \Delta$  and  $\kappa = 0$ . In fact, as suggested by this intuition, there are additional bail-out equilibria even if  $\theta > \Delta$  for  $\kappa > 0$ .

### 2.3 A Ricardian Equilibrium

In this section we construct a Ricardian Equilibrium in which the regional government uses its tax to pay-off the debt issued to finance the transfer,  $g^1$ . Thus  $\tau^1 = xg^1$  and  $\tau = 0$  along the equilibrium path. In equilibrium, the region 1 agents will save the transfer to pay for their future taxes:  $s^1 = s^2 + g^1$ . This is an immediate consequence of (2).

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<sup>7</sup>We are grateful to Marco Bassetto for discussions which led to the development of this section.

To construct this equilibrium, we assume only region 1 agents choose to hold region 1 debt. That is, in the proposed equilibrium,  $s^2 = k^2$  so that  $s^1 = s^2 + b^1$ . Hence  $\theta = 1$ . At the individual level, this is without loss of generality as debt and storage have the same return of  $x$ . But this has consequences for the outcome as it influences the incentives of the central government. Given this conjectured equilibrium, we check the incentives of the central and regional governments as well as the private agents.

Since the CG moves last, we check its incentives first. If the regional government deviates from the candidate equilibrium and chooses not to raise taxes to pay the debt obligation, will the CG allow default? If  $\kappa$ , the cost of default, is sufficiently small, this is the optimal choice of the CG. We see this by noting that social welfare if the central government bails-out the region, denoted  $W^b$ , is

$$W^b = \Delta v(\omega^o + s^1 x - \tau) + (1 - \Delta)v(\omega^o + s^2 x - \tau) \quad (7)$$

where  $\Delta$  is the population (and welfare) weight for region 1.<sup>8</sup> From the central government's budget constraint,  $\tau = \Delta g^1 x$ . From the curvature in  $v(\cdot)$ ,

$$W^b < v(\omega^o + (\Delta s^1 + (1 - \Delta)s^2)x - \tau) = v(\omega^o + s^2 x) \quad (8)$$

using  $s^1 = s^2 + g^1$  and  $\tau = \Delta g^1 x$ .

If the central government allows default then social welfare, denoted  $W^d$ , is given by

$$W^d = \Delta v(\omega^o + (s^1 - g^1)x) + (1 - \Delta)v(\omega^o + s^2 x) - \Delta \kappa = v(\omega^o + s^2 x) - \Delta \kappa \quad (9)$$

where the last equality again uses  $s^1 = s^2 + g^1$ . In (9), the consumption of region 1 agents reflects the default on the debt they hold and the absence of taxation. Hence, from (8) and (9), for  $\kappa$  sufficiently small,  $W^d > W^b$ .

It is important to understand more intuitively this choice by the CG. From (8) and (9), the central government is interested in the *ex post* distribution of consumption across agents in the different agents. Since  $v(\cdot)$  is strictly concave, the CG prefers a more equitable allocation. Thus the result that  $W^d > W^b$  reflects the fact that default delivers more equitable consumption across the regions than would a bail-out. Thus if the regional government does not tax its citizens, it realizes that the central government will allow a default.

What about the region? Will it prefer to tax or allow default? If it taxes, then its agents get  $c^{1o} = \omega^o + s^2 x$ .<sup>9</sup> This is better than default because with default region 1 agents have the same consumption but lose  $\kappa$ .

Finally, from (2), it is easy to check that region 1 agents will simply save the entire transfer in order to pay their tax obligations to the regional government: i.e.  $s^1 = s^2 + g^1$ . Given the construction that only region 1 agents hold region 1 debt, all agents have the same real storage: i.e.  $s^1 = s^2 + g^1$  and  $s^2 = k^2$  implies that  $k^1 = k^2$ .

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<sup>8</sup>Since first period consumption is predetermined, welfare simply depends on the consumption in old age of agents in the two regions. As usual, it is precisely this feature that "bygones are bygones" which leads to the "commitment problem".

<sup>9</sup>Note that this calculation is given the savings of the young since the choice of the regional government occurs **after** the savings decision.

So private agents are optimizing and neither the regional nor the central government has an incentive to deviate. Hence there is an equilibrium in which regional debt is paid-off by regional taxation. This is a standard Ricardian allocation.

In the construction of this equilibrium, we assumed that all debt was held by region 1 agents. In fact, we can relax this restriction and uncover additional Ricardian equilibria.

Let  $\theta^R(\kappa)$  solve

$$v\left(\omega^o + \left(s^1 - \frac{\bar{B}}{\Delta}\right)x\right) = v\left(\omega^o + \left(s^1 - \frac{\theta\bar{B}}{\Delta}\right)x\right) - \kappa. \quad (10)$$

The left-side is the welfare of region 1 agents if the regional government chooses to tax its citizens. This expression uses  $\tau^1 = \frac{x\bar{B}}{\Delta}$ , the tax rate on region 1 agents needed to pay-off the outstanding debt. The right-side is the welfare of a region 1 agent if the regional government chooses not to tax and the CG allows the default. From this expression,  $\theta^R(\kappa)$  is decreasing in  $\kappa$ .

**Proposition 2** *For  $\kappa$  near zero, there exists a Ricardian equilibrium for  $\theta \in [\theta^R(\kappa), 1]$ .*

**Proof.** By construction of  $\theta^R(\kappa)$ , for  $\theta \in [\theta^R(\kappa), 1]$  the regional government prefers to tax its agents rather than allow default by the CG. This is an immediate consequence of the fact that the payoff for default, the right-side of (10) is decreasing in  $\theta$ .

As (10) was constructed assuming the CG would allow a default, it is necessary that the CG will allow a default for all  $\theta \in [\theta^R(\kappa), 1]$ . From the proof of Proposition 1, at  $\kappa = 0$ , the CG is indifferent between allowing a default and bailing out the regional government when  $\theta = \Delta$ . Further, at  $\kappa = 0$ , (10) implies  $\theta^R(\kappa) = 1$ . Thus at  $\kappa = 0$ , the CG will not bail-out the regional government for  $\theta \geq \theta^R(\kappa)$ . By continuity, this is true for  $\kappa$  near zero. ■

In intuitively, for sufficiently large values of  $\theta$ , the central government will not bail-out the regional government. In this case, the regional government may prefer to tax its citizens rather than default. The gain to regional taxation is avoiding the payment of  $\kappa$ . The cost of taxation is that for  $\theta < 1$ , the proceeds of the tax on region 1 agents will flow to region 2 agents. This intuition underlies the existence of a critical value of  $\theta$ , which balances these costs and benefits.

### 3 A Monetary Economy with Regional Debt

This analysis of the real game serves two purposes. First it highlights the commitment problem of the central authority within a federation. Second, it indicates that a central government will *ex post* use its tax and transfer power to redistribute resources across regions. This incentive will be relevant even in a monetary economy.

Yet the discussion of regional debt occurs in a monetary setting where the connection between regional debt and inflation arises from the use of the inflation tax by a central government. Thus understanding the interactions between the regional and central government in a monetary setting is important. Here we construct an overlapping generations model. Instead of assuming there exists a central government which can tax all agents, there is a central bank (CB) which can print money and transfer it to the regional government to pay its obligations.

In the overlapping generations model, agents live for two periods. Lifetime utility is given by  $u(c^y) + v(c^o)$  and we assume that  $u(\cdot)$  and  $v(\cdot)$  are strictly increasing and strictly concave. All agents are endowed with  $\omega^y$  units of the consumption good in youth and  $\omega^o$  in old age. Agents have access to a storage technology that yields  $x > 1$  units of the consumption good in period  $t+1$  for each unit stored in period  $t$ . In addition, agents may save by holding debt issued by the region 1 government. Finally, there is a legal restriction that requires money to be held in proportion to the level of real storage, as in Smith (1994). One interpretation is that access to the storage technology requires an intermediary which must hold money as a reserve requirement.<sup>10</sup>

Agents live in one of two regions. As in the previous section, the key is the game between the region 1 government and the central bank (CB). There is a second group of agents living in region 2, whose government does not issue debt.<sup>11</sup> Nonetheless region 2 agents are important as their welfare is reflected in the decisions of the CB.

Each young agent of generation  $t$  born in region 1 receives a real transfer of  $g^*$  from the regional government and that government sells debt of  $B^*$ .<sup>12</sup> Governments are associated with a generation not a time period; the regional government elected in period  $t$  sells its debt and then in period  $t+1$  decides either to tax the consumption of the old within its region or to turn the obligation over to the CB.<sup>13</sup>

Formally, we consider an extensive form game, played each period, which is similar to that in the previous section. Here though the move of the central government has been replaced by a choice of the central bank.

- regional government either raises taxes to pay its obligation (**pay**) or not (**no pay**) and passes it to the CB
- if the regional government chooses (**no pay**), the CB either pays the obligation, financed by printing money, or denies it
- if the CB denies the obligation, then the region defaults and its citizens suffer a penalty/loss of  $\kappa$ .

This game is played in period  $t+1$  by the regional government representing region 1, generation  $t$  agents. Importantly, the taxation decisions associated with generation  $t$  agents are made in period  $t+1$  after savings decisions have been made by that generation.

We construct two types of steady states for this economy. In the first, which is an extension of the bail-out equilibrium, the CB monetizes the debt of the regional government. In the second, akin to the Ricardian outcome, the CB refuses to monetize and, in anticipation, the regional government taxes its citizens.

The co-existence of these equilibria again reflects the commitment problem faced by the CB as in the real game. In the extensive form game, the CB has the weighted utilities of all old agents as its objective.

<sup>10</sup>We are grateful to Todd Keister for discussions on this point. Alternatively, we could assume there is a reserve requirement on all savings, including the holding of government debt. Given the small denomination of the regional debt, there is no need for it to be intermediated and thus no basis for a reserve requirement.

<sup>11</sup>In the context of Argentina, region 1 is intended to represent the province of Buenos Aires and region 2 representing the citizens outside of this region. This simplification clearly misses the fact that other regions have issued small denomination debt. But the province of Buenos Aires accounts for about 60 % of the regional debt outstanding.

<sup>12</sup>Hereafter, variables with an \* are steady state equilibrium values.

<sup>13</sup>In fact, this is apparently without loss of generality since a regional government in period  $t$  has no influence over any state variables that matter for future generations.

Accordingly, it is ultimately interested in equalizing the real consumptions of these agents. Whether or not it allows default on the debt depends on the holdings of this debt across agents. In the monetization equilibrium, the debt is widely held and allowing default is undesirable due to the penalty. But, if the debt is held by agents in region 1, then default leads to more equitable consumption and this supports the Ricardian equilibrium.

Thus which equilibrium will prevail depends on who holds the regional debt. Since, in equilibrium, agents get the same return from holding the regional debt as from storage (i.e. default never actually occurs in equilibrium), agents are indifferent with respect to their portfolios.<sup>14</sup> Despite this indifference at the level of the individual, the outcomes in the two equilibria may be quite different.

### 3.1 Equilibrium with Monetization

Here we construct a stationary equilibrium in which the CB monetizes the obligation rather than allowing default. The agents anticipate this and adjust their saving accordingly. Further, the region chooses **no pay** and sends the obligation to the CB. In equilibrium, the CB prefers monetization over default.

There is an interpretation of the patacones in this equilibrium. Their creation is ultimately inflationary as the CB is unable to stop itself from monetizing the regional debt.

Along the equilibrium path, each region 1 government transfers  $g^*$  to young agents. These transfers are financed by issuing debt each period of  $B^*$  per capita.<sup>15</sup> In equilibrium, all young agents hold the same amount of real debt regardless of their region:  $b^{i*} \equiv b^* = B^*$  for  $i = 1, 2$ . By the regional government's budget constraint,  $B^* = b^* = \Delta g^*$  where  $\Delta$  is the population size of region 1.

We begin with the optimization problem of a representative young agent in region  $i$ , period  $t$ . That agent solves

$$\max_{k^i, b^i, m^i} u(\omega^y + g^i - k^i - b^i - m^i) + v(\omega^o + k^i x + b^i R + m^i \tilde{\pi}) \quad (11)$$

where  $g^i$  is the real transfer to each region 1 young agent. There are three types of savings:  $k^i$  is real storage with return  $x$ ,  $b^i$  is the holding of real debt with a real return of  $R$  and  $m^i$  is the holding of real money. We impose by a legal restriction;  $m^i \geq \lambda k^i$ . In this expression,  $\tilde{\pi}$  is the inverse of (one plus) the inflation rate and is the return on the holding of money. Along the equilibrium path, the inflation is anticipated by young agents.

Since the return on holding of money will, in equilibrium, be less than the return on storage, the reserve requirement will bind:  $m^i = \lambda k^i$ . Thus the return on storage, given the reserve requirement, is  $\frac{x + \lambda \tilde{\pi}}{1 + \lambda}$  per unit placed in storage.<sup>16</sup> In equilibrium, this must be the same as the return on regional debt,  $R$ .

With this in mind, the optimization problem simplifies to

$$\max_s u(\omega^y + g^i - s^i) + v(\omega^o + s^i R) \quad (12)$$

<sup>14</sup>Interestingly, the Patacones have traded at less than face value perhaps indicating that private agents place positive probability on default.

<sup>15</sup>So here these variables are divided by total population, normalized at 1, and not region 1 population.

<sup>16</sup>Put differently, it costs  $1 + \lambda$  units of consumption today to get  $x + \lambda \tilde{\pi}$  units of consumption tomorrow. Hence the return per unit stored is the ratio.

where  $s^i \equiv k^i(1+\lambda) + b^i$  represents total saving and  $R = \frac{x+\lambda\tilde{\pi}}{1+\lambda}$ . The optimal savings decision, which depends on  $R$ , is denoted by  $s^{*i}(R)$ . This satisfies

$$u'(\omega^y + g^i - s^{*i}) = Rv'(\omega^o + s^{*i}R) \quad (13)$$

for  $i = 1, 2$ .

Since only region 1 has transfers, set  $g^1 > 0$  and  $g^2 = 0$ . This implies that  $s^{*1} > s^{*2}$ . If, to the contrary,  $s^{*1} \leq s^{*2}$ , then the left-side of (13) would be lower for region 1 agents and the right-side would be higher. This would violate (13). We assume throughout that  $s^i$  is strictly positive, i.e.  $w^y$  is sufficiently large relative to  $w^o$ . Further, in the equilibria we construct, the constraint that  $k^i \geq 0$  does not bind, i.e.  $B^*$  is not too large.

The rate of inflation is determined from market clearing and the activity of the central bank. Monetization of the debt  $B^*$  by the central bank implies

$$\frac{M' - M}{p'} = RB^*. \quad (14)$$

Here unprimed variables are current ones and primed ones are future variables. So, for any generation,  $M$  is the current money supply and  $M'$  is the future stock of money. Likewise,  $p$  is the current prices of goods in terms of money and  $p'$  is the future price.<sup>17</sup>

Goods market clearing requires that the demand for goods by the old equals the supply by the young who sell a part of their endowment to meet their reserve requirement. So goods market clearing is

$$\frac{M}{p} = \lambda \bar{k}^*(R). \quad (15)$$

Here  $\bar{k}^*(R) \equiv \Delta k^{*1}(R) + (1 - \Delta)k^{*2}(R)$  and represents total storage. Using (15) in (14) yields

$$\frac{M'}{p'} - \frac{M}{p} \frac{p}{p'} = \lambda \bar{k}^*(R)(1 - \tilde{\pi}) = RB^* \quad (16)$$

where  $\tilde{\pi} = \frac{p}{p'}$ .

These conditions for market clearing, along with the choices of young agents will characterize a steady state equilibrium with monetization. This equilibrium is comprised of a vector of choices by agents in each region and a rate of return on money:  $(k^{*1}, s^{*1}, k^{*2}, s^{*2}, \tilde{\pi}^*)$ . Along the equilibrium path, given the constant level of government debt  $B^*$ , there will be constant growth of the money supply, constant inflation and thus a constant real return on money,  $\tilde{\pi}^*$ . This gets factored into the return on savings so that  $R^* = \frac{x+\lambda\tilde{\pi}^*}{1+\lambda}$  is the return on savings along the equilibrium path and determines  $s^{i*}$ . As storage and regional debt have the same return, we can freely construct agents' portfolios as part of the equilibrium.

In characterizing the individual decisions and market clearing, we have assumed an equilibrium with CB monetization. There are no regional taxes assumed in (13) and all financing was through money creation, as in (14). We need to check that this is an equilibrium by evaluating the incentives of the regional government and the central bank. The following intuition underlies the proof of Proposition 3.

First, consider the incentives of the CB. Its choice about monetizing the regional debt influences the current nominal money supply and thus may redistribute purchasing power across old agents. But this

<sup>17</sup>As we focus on steady states, we have ignored all the  $t$  subscripts.

choice has no effect on future generations since the inherited stock of fiat money is completely neutral.<sup>18</sup> So the CB looks only at the welfare of the current old. If it bails-out, then social welfare,  $W^b$ , is given by

$$W^b = \Delta v(\omega^o + R^* s^{1*}) + (1 - \Delta)v(\omega^o + R^* s^{2*}) \quad (17)$$

in the steady state. If the CB allows default, then the welfare of the current old is given by

$$W^d = \Delta v(\omega^o + k^{1*}(x + \lambda)) + (1 - \Delta)v(\omega^o + k^{2*}(x + \lambda)) - \Delta\kappa \quad (18)$$

since, under default, there is no return on the holding of government debt and no inflation for this generation of old agents.<sup>19</sup>

To compare  $W^d$  against  $W^b$  we have to compare the *ex post* consumption levels. As argued in the proof of Proposition 3, the consumption allocation under bail-out is closer to the social optimum of equal consumption and hence  $W^b > W^d$ .

Finally, we need to be sure that the region will pass the obligation to the CB given that it recognizes the CB will choose to monetize, i.e.  $W^b > W^d$ . Again this is intuitively true: why pay a tax which can in part be passed to other agents? This is formalized in the proof of Proposition 3.

**Proposition 3** *There exists a steady state for a given level of region 1 debt,  $B^*$ , characterized by  $(k^{*1}, s^{*1}, k^{*2}, s^{*2}, \tilde{\pi}^*)$  with  $b^{*i} = B^*$ , in which the central bank monetizes the regional debt obligation each period.*

**Proof.** First, we argue that there exists a  $(k^{*1}, s^{*1}, k^{*2}, s^{*2}, \tilde{\pi}^*)$  which solves the conditions for a stationary monetary equilibrium. Second, we check the incentives of the regional and central governments.

The existence proof relies on two equilibrium conditions: (16) and  $R(\tilde{\pi}) = \frac{x + \lambda\tilde{\pi}}{1 + \lambda}$ . Substitution of  $R(\tilde{\pi})$  into (16) yields:

$$\lambda\bar{k}(\tilde{\pi})(1 - \tilde{\pi}) = B^* \left[ \frac{x + \lambda\tilde{\pi}}{1 + \lambda} \right]. \quad (19)$$

Here  $\bar{k}(\tilde{\pi})$  reflects the dependence of aggregate savings and thus aggregate storage (given  $B^*$ ) on  $R(\tilde{\pi})$ . Denote the left-side of (19) by  $H(\tilde{\pi})$  and the right-side by  $G(\tilde{\pi})$ . Clearly  $G(\tilde{\pi})$  is linear with a positive intercept. With  $B^* \geq 0$ ,  $\pi \in [0, 1]$ .  $H(1) = 0$  and thus  $H(1) < G(1)$ . Since both functions are continuous, if  $H(0) > G(0)$ , there will exist a  $\pi$  which solves (19). We assume positive saving at  $\tilde{\pi} = 0$ . Hence for  $B^*$  sufficiently low,  $H(0) > G(0)$  and so there will exist a  $\tilde{\pi}$ , denoted  $\tilde{\pi}^*$ , solving (19). Given  $\tilde{\pi}^*$ ,  $R^*$  is determined and thus so is total savings and storage.

To see the incentive of the CB to monetize, write the consumption of old agents in region  $i$  under monetization as

$$c^{io*} = \omega^o + b^*R + k^{i*}(x + \lambda\tilde{\pi}^*) = \omega^o + k^{i*}(x + \lambda) + R^*b^* \left(1 - \frac{k^{i*}}{\bar{k}^*}\right) \quad (20)$$

where we substituted for  $\tilde{\pi}^*$  using (16) and used  $b^{*1} = b^{*2} = b^* = B^*$ . Write the consumption of old agents in region  $i$  under default as

$$\tilde{c}^{oi} = \omega^o + k^{i*}(x + \lambda). \quad (21)$$

<sup>18</sup>So while the stock of fiat money is changing over time, it has not influence on the set of feasible consumption allocations.

<sup>19</sup>Importantly, we are considering a one-time deviation from a candidate equilibrium.

Aggregate consumption by the old is the same regardless of monetization or default:  $\Delta c^{o1*} + (1-\Delta)c^{o2*} = \Delta \tilde{c}^{o1} + (1-\Delta)\tilde{c}^{o2}$ . Thus the comparison of  $W^b$  and  $W^d$  depends on the distribution of consumption across the regions. Since  $s^{*1} > s^{*2}$  and  $b^{*1} = b^{*2}$ ,  $k^{1*} > \bar{k}^* > k^{2*}$ . So, inflation redistributes consumption from region 1 agents to region 2 agents. Thus, relative to the allocation under default, consumption is more equal under monetization. Further, the consumption of region 1 agents exceeds that of region 2 agents in the monetization equilibrium since  $s^{*1} > s^{*2}$ . Thus, from the strict concavity of  $v(\cdot)$ ,  $W^b > W^d$  so that the CB will prefer to monetize rather than allow default.

Finally, we inspect the incentives of the region 1 government. Let  $\hat{c}^1$  denote the consumption of region 1 agents in the event that the region 1 government taxes the consumption of these agents at a rate of  $\tau^1$ . From the budget constraint of the regional 1 government,  $\Delta\tau^1 = B^*R^*$ . Using this constraint to determine taxes,

$$\hat{c}^1 = \omega^o + xk^{1*}(x + \lambda) + b^*R^* - \frac{B^*R^*}{\Delta}. \quad (22)$$

Note that in this expression the savings choices of the private agents are  $(k^{1*}, b^{1*})$  since this proposed deviation from the equilibrium occurs **after** the private agents choose their savings.

There are two differences between this and old consumption in the steady state, given in (20) for  $i = 1$ . First, the inflation is zero and second there is a tax to be paid. The difference between  $c^{1o*}$  and  $\hat{c}^1$  is

$$b^*R^* \left[ \frac{1}{\Delta} - \frac{k^{1*}}{\bar{k}^*} \right] = \frac{b^*R^*}{\Delta \bar{k}^*} (\bar{k}^* - \Delta k^{1*}) > 0. \quad (23)$$

Thus the regional government prefers to allow the CB to monetize the debt rather than tax its agents directly.

■

This result indicates the obligations of the regional government will be assumed by the CB. As a consequence, the regional transfers are financed by an economy-wide inflation tax, partially borne by agents in region 2.

### 3.2 Ricardian Equilibrium

Here we characterize a second equilibrium in which the regional government prefers to tax its agents. In this equilibrium the CB, given the opportunity to act, would not choose to monetize. Rather it would allow default. In anticipation of this, the region will tax. Given this, the agents in region 1 save more and thus pay taxes from this extra savings. In the equilibrium this extra savings is in the form of holding of debt. This is essential for the construction of this equilibrium. It makes clear that the distribution of debt holdings matters for the equilibrium outcome.

This is a Ricardian equilibrium. In such an equilibrium, the patacones issued by the regional government in Argentina are not money in the traditional sense as their creation is not associated with increases in prices. Instead, they simply represent debt, backed by future taxes.<sup>20</sup>

To characterize this outcome we return to the basic optimization problems and equilibrium conditions.

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<sup>20</sup>In the model there is no rollover option. But, if there is a limit on debt then ultimately the game we outline between the regional government and the central bank will occur.

The representative young agent in region  $i$ , period  $t$  solves

$$\max_s u(\omega^y + g^i - s^i) + v(\omega^o + s^i R - \tau^i). \quad (24)$$

This optimization yields

$$u'(\omega^y + g^i - s^{*i}) = Rv'(\omega^o + s^{*i}R - \tau^i) \quad (25)$$

where  $\tau^2 = g^2 \equiv 0$ . Along this equilibrium path there will be no inflation so  $R = \frac{x+\lambda}{1+\lambda}$ .

In the construction of this equilibrium, we assume that only region 1 agents hold regional debt:  $b^{1*} = B^*/\Delta, b^{*2} = 0$ . Further, we conjecture (and prove in Proposition 4) that  $s^{1*} = s^{2*} + b^{1*}$  so that  $k^{1*} = k^{2*}$ .

There is a goods market clearing condition that is analogous to (15). This condition will determine the price level given the fixed money supply and the storage decisions of the agents.

To argue that there is an equilibrium with regional taxation and no monetization by the CB, we need to check the incentives for the levels of the government and private agents. This is done formally in Proposition 4; we bring out the intuition here.

We start in the sub-game where the regional government has decided not to tax and the CB must choose to monetize the debt or allow default. This is a deviation from the equilibrium we are trying to construct. Social welfare under a CB bail-out is

$$W^b = \Delta v(\omega^o + k^{*1}(x + \lambda\tilde{\pi}) + b^{*1}R) + (1 - \Delta)v(\omega^o + k^{*2}(x + \lambda\tilde{\pi})) \quad (26)$$

where  $(k^{*i}, b^{*i})$  are obtained from optimal savings along an equilibrium path from (25).<sup>21</sup> Here  $\tilde{\pi}$  is again the inverse of the inflation rate and inflation is caused by the monetization of the debt by the CB.

If the central bank does not bail-out and there is a default, then social welfare is given by

$$W^d = \Delta v(\omega^o + k^{*1}(x + \lambda)) + (1 - \Delta)v(\omega^o + k^{*2}(x + \lambda)) - \Delta\kappa \quad (27)$$

so agents avoid the inflation tax but only get a return on their storage plus money holdings.

The proof of Proposition 4 shows that  $W^d > W^b$ . This reflects two factors which were present in the proof of Proposition 3 as well. First, the actual resources available to distribute to the old agents is the same regardless of the action of the central bank. Second, the central bank wishes to obtain the most equitable distribution of consumption across the old agents since  $v(\cdot)$  is strictly concave. This is achieved under default given that  $k^{1*} = k^{2*}$ . Thus the central bank will have an incentive to allow default rather than to monetize the debt of region 1 for sufficiently small  $\kappa$ .

Anticipating this, region 1 will prefer to raise taxes rather allowing default. Interestingly, in both cases, the consumption of region 1 old agents is the same. Intuitively, the taxes they pay to the regional government are just used to pay-off the debt which they hold. But, by taxing, the regional government can avoid the  $\kappa$  penalty.

**Proposition 4** *If  $\kappa$  is sufficiently low, there exists a steady state equilibrium given  $B^*$  in which the regional debt is held only by region 1 agents and the regional government chooses to raise taxes to pay its obligations.*

<sup>21</sup>Recall that we assume  $b^{*2} = 0$  along the equilibrium path.

**Proof.** First, we argue that there exists a  $(k^{*1}, s^{*1}, k^{*2}, s^{*2}, \tilde{\pi}^*)$  which solves the conditions for a stationary monetary equilibrium. Second, we check the incentives of the regional and central governments.

In the steady state, the level of region 1 transfers to each young agent is  $g^*$ , the total per capita debt outstanding is  $B^*$ . By the budget constraint of region 1,  $B^* = \Delta g^*$  and taxes in old age are given by  $\Delta\tau^1 = R^*B^*$  so that  $\tau^1 = R^*g^*$ . The debt held by each agent in region 1 is  $b^{*1}$  where  $\Delta b^{*1} = B^*$  and region 2 agents do not hold debt.

In equilibrium, the savings decisions of the agents are given by

$$u'(\omega^y + g^i - k^{*i}(1 + \lambda) - b^{*i}) = R^*v'(\omega^o + k^{*i}(x + \lambda) + R^*b^{*i} - \tau^i) \quad (28)$$

where  $R^* = \frac{x+\lambda}{1+\lambda}$ . With  $\tau^1 = R^*g^*$ ,  $k^{1*} = k^{2*}$  and  $b^{*1} = B^*$  clearly satisfies the first order conditions. Thus the equilibrium level of per capita storage ( $k^*$ ) satisfies

$$u'(\omega^y + g^i - k^*(1 + \lambda) - B^*) = R^*v'(\omega^o + k^*(x + \lambda)). \quad (29)$$

Given the strict concavity of  $u(\cdot)$  and  $v(\cdot)$ , a  $k^*$  which solves this condition will exist. We assume that the endowments and preferences are such that  $k^* \geq 0$ .

We now turn to the incentives of the central bank. We argue that if the region does not set taxes to pay its debt obligation, then the central bank will not monetize. To see why, from (27), the consumption levels of agents are equal if the CB allows a default. However, the allocation under monetization provides greater consumption for region 1 agents since they bear only a fraction of the inflation tax and receive full repayment of their debt.

Yet, the total consumption of the old is the same, regardless of default or monetization. Under default, total consumption of the old agents is

$$\omega^o + k^*(x + \lambda). \quad (30)$$

Under monetization, total consumption is

$$\omega^o + k^*(x + \lambda) + \lambda(\tilde{\pi} - 1)\bar{k}^* + B^*R^*. \quad (31)$$

where the rate of inflation is determined from the money creation needed to finance the bail-out. From (16), which would hold if the monetary authority deviated and bailed-out the region, the final two terms in (31) cancel. Hence total consumption is the same regardless of default or bail-out.

Thus for sufficiently small values of  $\kappa$  the CB will prefer default to monetization. A bail-out would yield a less equal distribution of consumption and would not increase total consumption.

Given that the CB will not monetize the debt, the regional government will tax rather than default. This allows them to avoid the penalty of  $\kappa$ . In both cases, the consumption of the region 1 old is given by  $\omega^o + k^1(x + \lambda)$ . ■

### 3.3 Other equilibria

As in the analysis of the real game, there are other equilibria. Proposition 3 characterizes a monetary equilibrium in which all agents have equal holdings of the regional debt. Following Proposition 1, there are

also bail-out equilibrium if region 1 agents hold relatively less of the regional debt. Here  $\theta_i$  is defined from  $b^{*i} = \frac{\theta_i B^*}{\Delta_i}$  and  $\Delta_i$  is the fraction of the population in region  $i$

**Proposition 5** *For  $\kappa \geq 0$  and  $\theta_1 \leq \Delta_1$ , there exists a steady state for a given level of region 1 debt,  $B^*$ , characterized by  $(k^{*1}, s^{*1}, k^{*2}, s^{*2}, \tilde{\pi}^*)$ , in which the central bank monetizes the regional debt obligation each period.*

**Proof.** The existence proof is provided in the proof of Proposition 3 since that proof did not utilize the restriction that all agents have equal debt holdings.

To see the incentive of the CB to monetize, recall from the proof of Proposition 3 that total consumption available to all agents in a given period was independent of whether the CB chose to bail-out the region or allow a default. Thus, if consumption allocations under the bail-out were more equal than consumption allocations under default (assuming that in both cases region 1 old agents had higher consumption), then the CB would chose bail-out.

Extending the proof of Proposition 3, write the consumption of old agents in region  $i$  under monetization as

$$c^{io*} = \omega^o + b^{*i}R + k^{i*}(x + \lambda\tilde{\pi}^*) = \omega^o + k^{i*}(x + \lambda) + R^*B^*\left(\frac{\theta_i}{\Delta_i} - \frac{k^{i*}}{\bar{k}^*}\right) \quad (32)$$

Write the consumption of old agents in region  $i$  under default as

$$\tilde{c}^{oi} = \omega^o + k^{i*}(x + \lambda). \quad (33)$$

The proof of Proposition 3 established that at  $\theta_i = \Delta_i$ ,  $s^{*1} > s^{*2}$  implied  $k^{1*} > \bar{k}^* > k^{2*}$ . Since  $s^{*1} > s^{*2}$  is true for all  $\theta_i$ ,  $k^{1*} > \bar{k}^* > k^{2*}$  for  $\theta_i \leq \Delta_i$ . Further, the consumption of region 1 agents exceeds that of region 2 agents in the monetization equilibrium since  $s^{*1} > s^{*2}$ . So, inflation redistributes consumption from region 1 agents to region 2 agents. Thus, relative to the allocation under default, consumption is more equal under monetization. From the strict concavity of  $v(\cdot)$ ,  $W^b > W^d$  the CB will prefer to monetize rather than allow default.

Finally, we inspect the incentives of the region 1 government. It is straightforward to check that (23) is valid for  $\theta_i \leq \Delta_i$ . Hence, following the logic of the proof of Proposition 3, the regional government will prefer that the CB monetize the debt. ■

## 4 Choice of $B^*$

The equilibria described in the previous section take the steady state level of region 1 debt,  $B^*$ , as given. We now explore the determination of this level of debt.

Let  $V(B)$  be the welfare of a region 1 agent if the stock of debt is  $B$ . In the Ricardian equilibrium, the choice of  $B$  is, by construction, irrelevant for the welfare of region 1 agents. But in the monetization equilibrium, this is not the case. If one takes, for example, the perspective that the equilibrium will be determined by a sunspot process, then  $V(B)$  places some weight on the monetization equilibrium and the

remaining probability on the Ricardian equilibrium.<sup>22</sup> Since welfare of region 1 agents is independent of  $B$  in the Ricardian equilibrium, the only effect of  $B$  occurs when the monetization equilibrium is selected. Thus we focus our discussion of the choice of  $B$  assuming the selection of the monetization equilibrium.

So consider

$$V(B) = u(\omega^y + \frac{B}{\Delta} - s) + v(\omega^o + sR(B)). \quad (34)$$

This is the level of lifetime expected utility for a representative region 1 agent in an equilibrium with monetization. Here  $B$  is the level of debt per capita so that  $\frac{B}{\Delta}$  is the level of debt, and thus the transfer in youth, per young region 1 agent. The function  $R(B)$  is the return on savings if the stock of debt is  $B$  from the equilibrium with monetization. Our main result is that the regional government will prefer a positive level of transfers given the positive probability that the central bank will bail-out this obligation.

**Proposition 6** *The solution to (34) entails  $B > 0$ .*

**Proof.** Using the envelope condition, the optimal choice of  $B$  by the region 1 government satisfies

$$V'(B) = v'(c^o) \left[ \frac{R(B)}{\Delta} - sR'(B) \right] = 0. \quad (35)$$

To show  $V'(0) > 0$ , use  $R(\pi) = \frac{x+\lambda\pi}{1+\lambda}$  and

$$\lambda \bar{k}(R(\pi))(1 - \pi) = R(\pi)B. \quad (36)$$

Taking derivatives to calculate  $R'(B)$  at  $B = 0$  yields

$$R'(0) = -\frac{R}{(1 + \lambda)\bar{k}} \quad (37)$$

Substituting this into (35), yields

$$V'(0) = v'(c^o)R(0) \left[ \frac{1}{\Delta} - \frac{k^1}{\bar{k}} \right] \quad (38)$$

Since  $k^1 = \bar{k}$  in a symmetric steady state with  $B = 0$ ,  $V'(0)$  is positive using  $\Delta < 1$ . Thus the optimal policy of the region 1 government will entail a positive level of  $B$ . ■

## 5 Policy Implications

The two steady state equilibrium characterized above have very different welfare implications for agents in the two regions. Agents in region 1 strictly prefer the monetization equilibrium while those in region 2 prefer the equilibrium with regional taxation. Thus, as indicated by Proposition 6, the region 1 will increase the level of  $B$  above zero and will try to support the equilibrium with monetization. In contrast, agents in region 2 would act to limit region 1 and eliminate the monetization equilibrium. We consider proposed policy measures from the perspective of these two groups of agents.

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<sup>22</sup>The timing might be as follows. The regional government chooses  $B$  and then a sunspot occurs which selects from the set of equilibria insofar as young agents condition their portfolio choice on the sunspot. This timing may occur each period or just at the start of time.

In this discussion it is also useful to recall the benchmark planner's solution. As in the discussion of the real game, the **ex ante** optimal allocation entails equal consumption across regions in all periods of agent's lives. This reflects the symmetry of the economy, the strict concavity of  $u(\cdot)$  and  $v(\cdot)$  and the use of population weights in the planner's objective function. The planner's solution can be decentralized either by the selection of the Ricardian equilibrium or when  $B^* = 0$ . Thus the policy remedies can also be viewed as devices for supporting the planner's solution.

## 5.1 Restrictions on debt

We consider two types of restrictions. The first is on the total size of the regional debt and the second is on the holdings of that debt.

The first restriction is a debt limit. If  $B^*$  is forced to be zero, then clearly there is no monetization. Clearly a restriction of this form would be favored by region 2 agents.

Within Argentina, there have been numerous attempts to place limits on regional debt. But, not surprisingly, not all regions are in favor of these limits. Interestingly, recent negotiations with the International Monetary Fund have included a discussion of the regional fiscal situation.<sup>23</sup>

This type of restriction is certainly in the same spirit as the limits on debt within the EMU. But, to the extent that deficit spending is desirable as a basis for tax smoothing, such restrictions are costly.

The second restriction is on the holding of debt. Suppose there is a capital control which makes it prohibitively expensive for a private agent in region 2 or a financial intermediary intervening on his behalf to hold region 1 debt. Then this intervention implies that monetization is no longer a steady state and makes the Ricardian equilibrium the only steady state equilibrium. This restriction is in the interest of region 2 agents. Restrictions on the holding of debt represent a commitment device ruling out monetization.

In Argentina, the small-denomination debt of the Buenos Aires region, the so-called Patacones, issued in July 2002 allow for the repayment of debt using these notes. But, no other regions appear willing to accept this currency for payment of taxes. While this is not a policy that prohibits Patacones outside of the Buenos Aires region, neither does this policy encourage their use.

## 5.2 Dollarization

As noted earlier, a commitment by the central bank not to bail-out any regional government would of course eliminate the monetization equilibrium. This is precisely the provision included in the Maastricht Treaty in the case of the EMU. But of course, this begs the question: what is the basis of this commitment power?

There is a more drastic measure, which has been recently widely discussed both by policymakers and economists: dollarization. This entails the complete surrender of any monetary sovereignty, and not just restrictions on the supply of money. Cooper and Kempf (2001) explores the benefits of dollarization as a substitute for the commitment power of a central bank in a multi-region economy (such as Argentina) without

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<sup>23</sup>Details are available in the recent agreement between the IMF and Argentina, [http://www.mecon.gov.ar/finance/sfinan/mep\\_jan.16.2003.pdf](http://www.mecon.gov.ar/finance/sfinan/mep_jan.16.2003.pdf).

regional debt. That argument easily extends so that dollarization will surely eliminate the equilibrium with monetization. But there are two important caveats.

First, while dollarization eliminates bail-out by the central bank, there is still the possibility that the central government bails out by means of central taxes. This corresponds to the real game analysis in section 2. From this analysis, recall there are still multiple equilibria. Hence dollarization does not eliminate the multiplicity nor does it eliminate the pressure by the regional government on the central government.

Second, under dollarization, a version of the multi-region monetary economy reappears at the world level. Suppose that it is Argentina that dollarizes. Assume that prior to dollarization, the US had succeeded in eliminating pressures by the states on the federal government so that the US is in essence one large integrated region.<sup>24</sup>

Thus in a dollarization regime, the US is like region 1 in the model of section 3 and Argentina now behaves as the region 2 passive government and does not issue debt. But, there is one important difference though: the US central government does not include the welfare of Argentina in its objective. Clearly there is now a gain to monetization of the debt by the US central bank since part of this tax will be paid by citizens of Argentina. In fact, there is no longer a Ricardian equilibrium since the central bank will always choose to monetize US debt since this provides a basis for taxation of Argentine citizens. Cooper and Kempf (2001) discuss the implications of a treaty between the US and Argentina as an incentive device on the US central bank to limit the inflation tax.

Of course, the different regions within Argentina would have conflicting views about the value of dollarization. Region 1 agents would not favor this policy while region 2 agents would be in favor.

### 5.3 Bankruptcy Costs

Finally, we can consider the effects of variations in  $\kappa$  as a policy instrument through provisions in the legal system. It may seem that an increase in  $\kappa$  would be beneficial to region 2 agents as it increases the cost of default and thus would make the regional government “behave”. Yet, from the discussion above, we see that the effects of an increase in  $\kappa$  is more complicated.<sup>25</sup> In fact, we have argued that as  $\kappa$  increases, the set of Ricardian equilibria and the set of monetization equilibria both increase. Thus it is not clear that higher values of  $\kappa$  favor region 2 agents relative to region 1 agents.

## 6 Conclusion

The goal of this paper was to determine the impact of issuing small denomination debt by a regional government, such as the patacones circulating in Argentina. Two leading views are relevant: (i) the debt is just a claim on future tax revenues and (ii) the debt was “like” money and hence printing it was tantamount to the printing of fiat money and thus was inflationary.

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<sup>24</sup>Exactly how this is done within the US is an open question but for now we assume that the central government has adequate commitment relative to the states.

<sup>25</sup>assuming this is in the propositions for the monetary case

Our analysis indicates that both interpretations are consistent with an equilibrium of our monetary model. The multiplicity reflects a commitment problem on the part of the central government. Without commitment, the central government will *ex post* always redistribute consumption in the direction of equality of consumption across agents in different regions. Depending on the distribution of the holding of the regional government debt, this desire for redistribution may lead the central government to bail-out a region or it may lead the central government to allow default. In equilibrium, the distribution of the holdings of government debt has powerful effects on the incentives for the central government. The more equal is this distribution, the more likely it is that the central government will prefer a bail-out to a costly default.

The commitment problem of the central government has some important incentive effects on the regions. A bail-out creates a free-rider problem in that regional governments will have an incentive to run inefficiently large deficits in anticipation of a government bail-out.<sup>26</sup> Not surprisingly, other agents in the economy will have an incentive to erect impediments to this free-rider problem including: debt restrictions, limits on the holding of debt by other regions and even dollarization.

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<sup>26</sup>This point is brought out in the context of monetary unions in Chari and Kehoe (1998), Chari and Kehoe (2002) and Cooper and Kempf (2000).