9. Appendix:CALCULATION OF MCF IN CGE MODELS

Not for publication

The calculation of the Marginal Cost of Funds (MCF) in Computable General Equilibrium (CGE) models is done in a three step procedure, since as it is a 'compensated equilibrium' concept, it does not fall out of a CGE model in a single step. (See Anderson and Martin, 1995 for more details, including reasons for preferring the compensated to the uncompensated version of MCF).

9.1. Calculation

The first step is to run the CGE experiment which calculates the rate of change of money metric utility with respect to an external transfer offset by a change in domestic distortionary taxes. In the first step, perform the following operations:

- Transfer an additional exogenous amount $d\beta$ into the economy,[16]
- matched by an endogenous proportionate change in the tax vector of interest; e.g., $(p-p^*)d\alpha$, where $d\alpha$ is the endogenous scalar; and
- calculate the change in money metric utility which arises from this experiment ($E_u du$ and $d\alpha$ are endogenous, the government budget constraint and the private budget constraint are the two equations which determine them). This is denoted $E_u du/d\beta(1)$.

The second step is to run the CGE experiment which calculates the shadow price of foreign exchange, also called the fiscal multiplier by Anderson and Martin (1995).

- Transfer another exogenous amount $d\beta$ into the economy.
- Calculate the rate of change in money metric utility which results. This value is $\mu(2)$, the shadow price of foreign exchange calculated in experiment 2.

The third step is to calculate MCF^p using the results of the first two steps. A complicating factor is that some CGE models apply a tax rate to external transfers, call this rate τ. Also, some CGE models have savings as a part of intertemporal structure. Let the marginal propensity to save be denoted s. The MCF^p formula works out as:

$$MCF^p = \frac{E_u \dfrac{du}{d\beta}(1) - (1-\tau)(1-s)}{\tau + E_u \dfrac{du}{d\beta}(1)\left(\dfrac{1}{1-s} - \dfrac{1}{\mu(2)}\right)}.$$

The money metric version of MCF must be obtained in a further step omitted here. It is approximately equal to $MCF^p\mu(2)$.

[16] In practice, it is necessary to experiment a bit with the size of the external transfer needed to get the model to calculate the MCF. Too small, and the model will not move away from its initial equilibrium. Too large and approximation error will grow large.

9.2. Derivation

The derivation of the MMCF, MCF and μ functions is based on the 2 equation system of the government and private sector budget constraints:

(A.1) $\tau\beta + (p-p^*)'E_p + (q-q^*)'E_q - \pi G - R = 0$ government

(A.2) $E - (1-s)(1-\tau)\beta - (1-s)R = 0$ private,

where E(p,q,G,u) is the private net expenditure on private goods, G is the government good obtained at external price π, R is the transfer from the government to the private sector, p is a domestic price vector for the class of goods we are interested in for MCF purposes and q is a domestic price vector for some other class of goods subject to distortions. For a model with savings, there is also a macroeconomic balance equation $s[(1-\tau)\beta + R]$ = Investment, where Investment causes demand links to the general equilibrium structure which need not be detailed here.

9.2.1. MCF Experiment

The domestic price vector p will change according to $dp = (p-p^*)d\alpha$, where α is a scalar. dβ is an exogenous shift, and dα and du are endogenous changes which satisfy the two constraints in changes. R, G, q and τ are constant. The manipulations are simplest to follow if we solve first from the government budet constraint for dα/dβ:

(A.3) $$\frac{d\alpha}{d\beta} = \frac{-\tau + \{(p-p^*)'E_{pu} + (q-q^*)'E_{qu}\}du/d\beta}{(p-p^*)'E_p + (p-p^*)'E_{pp}(p-p^*) + (q-q^*)'E_{qp}(p-p^*)}.$$

Substituting into the differential of the private budget constraint we obtain:

(A.4) $$\{1 - MCF^p[(p-p^*)'E_{pu}/E_u + (q-q^*)'E_{qu}/E_u]\}E_u du = (1-\tau)(1-s) + \tau MCF^p$$

where

$$MCF^p = \frac{E_p'(p-p^*)}{E_p'(p-p^*) + (p-p^*)'E_{pp}(p-p^*) + (q-q^*)'E_{qp}(p-p^*)}.$$

Solving for the money metric utility rate of change:

(A.5) $$E_u\frac{du}{d\beta}(1) = \mu(1-\tau)(1-s) + \mu\tau MCF^p,$$

where μ is the inverse of the coefficient multiplying $E_u du$ on the left hand side of (A.4):

(A.6) $$\mu = \frac{1}{\{1 - MCF^p[(p-p^*)'E_{pu}/E_u + (q-q^*)'E_{qu}/E_u]\}}.$$

9.2.2. Shadow price of foreign exchange experiment

The redistribution R changes endogenously along with u in response to an exogenous change in the external transfer β, to satisfy the two constraints in changes. The variables p,G and q are constant. Solving the government budget constraint for dR/dβ:

$$\frac{dR}{d\beta} = \tau + [(p-p^*)'E_{pu} + (q-q^*)'E_{qu}]du/d\beta.$$

Substituting into the differential of the private budget constraint:

$$\{1-(1-s)[(p-p^*)'E_p + (q-q^*)'E_q]\}E_u du/d\beta = (1-s)[(1-\tau)+\tau] = 1-s.$$

Therefore, solving for the rate of change in money metric utility:

(A.7) $$E_u \frac{du}{d\beta}(2) = \frac{1-s}{\{1-(1-s)[(p-p^*)' E_p + (q-q^*)' E_q]\}} = \mu(2).$$

Note that the shadow price of foreign exchange in this experiment is not the same as that for the MCF experiment, μ(2) is not equal to μ. The relation between them is:

(A.8) $$\frac{1}{\mu} = 1 + \left(\frac{1}{\mu(2)} - \frac{1}{1-s}\right) MCF^p.$$

9.2.3. Solving for MCF

Divide equation (A.5) through by μ:

$$E_u du/d\beta(1)/\mu \quad = \quad (1\text{-}s)(1\text{-}\tau) + \tau MCF^p.$$

Then use equation (A.8) to substitute for 1/μ and solve for MCF^p:

(A.9) $$MCF^p = \frac{E_u \frac{du}{d\beta}(1) - (1-\tau)(1-s)}{\tau + E_u \frac{du}{d\beta}(1)\left(\frac{1}{1-s} - \frac{1}{\mu(2)}\right)}.||$$