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Appendix

Proof of Proposition 1: The HQ should (1) request the same level of goodwill effort from all managers of L units, and (2) offer the same high-powered contract to all managers of H units.

(1) Let $\{g(j)\}_{j\in\mathcal{L}}$ be a profile of goodwill effort levels and $g \equiv \int_{\mathcal{L}} g(j)dj/d(\mathcal{L})$, the average of $\{g(j)\}_{j\in\mathcal{L}}$. Since $y(s,G)$ is concave in s, $y(T-g,G)$ is concave is g. Then by Jensen's Inequality,

$$\int_{\mathcal{L}} y(T-g(j),G)dj \leq \int_{\mathcal{L}} y(T-g,G)dj.$$

Therefore, the solution to program $(OP-G)$ is the choose $g(j)$ to be a constant. That is, The HQ should request the same level of goodwill effort from all managers of L units.
(2) Let us consider program $(OP-HQ')$. Assumptions 2 and 3 say that $y(s,G)$ is concave in s and $\lim_{s\to 0} y_s(s,G) = \infty$. Therefore, incentive compatibility constraint $(OP-s)$ can be replaced by

$$\alpha(i)y_s(s(i),G) = c'(s(i)).$$

By part (1) of this proposition, incentive compatibility constraint $(OP-G)$ becomes

$$g = \arg\max_g py(T-\frac{G}{p},G) + \int_{\mathcal{I}-\mathcal{L}} [(1-\alpha(i))y(s(i),G) - \beta(i)]di,$$

where, $G = pg$. It is easy to show that, since $y(s,G)$ is concave in (s,G),

$$py(T-\frac{G}{p},G) + \int_{\mathcal{I}-\mathcal{L}} [(1-\alpha(i))y(s(i),G) - \beta(i)]di$$

is concave in G. Its derivative with respect to G is

$$py_G(T-\frac{G}{p},G) - y_s(T-\frac{G}{p},G) + \int_{\mathcal{I}-\mathcal{L}} (1-\alpha(i))y_G(s(i),G)di,$$

which decreases with G, by Assumption 3, goes to ∞ as $G\to 0$, and goes to $-\infty$ as $G \to pT$. Therefore, $(OP-G)$ can be replaced by

$$py_G(T-\frac{G}{p},G) - y_s(T-\frac{G}{p},G) + \int_{\mathcal{I}-\mathcal{L}} (1-\alpha(i))y_G(s(i),G)di = 0. \qquad (FOC-G)$$

The objective function of program $(OP-HQ')$ is now

$$py(T-g,G) + \int_{\mathcal{I}-\mathcal{L}} [y(s(i),G) - c(s(i)) - \frac{1}{2}r\sigma^2\alpha(i)^2]di,$$

which is also concave in G and the derivative of which with respect to G is positive for G satisfying constraint $(FOC-G)$. Therefore, program $(OP-HQ')$ can be rewritten as

$$\begin{array}{lll} \max & py(T-g,G) + \int_{\mathcal{I}-\mathcal{L}} [y(s(i),G) - c(s(i)) - \frac{1}{2}r\sigma^2\alpha(i)^2]di & (OP-HQ') \\ s.t. & py_G(T-\frac{G}{p},G) - y_s(T-\frac{G}{p},G) + \int_{\mathcal{I}-\mathcal{L}} (1-\alpha(i))y_G(s(i),G)di \geq 0 & (IC-G) \\ & \alpha(i)y_s(s(i),G) = c'(s(i)) & (IC-s) \end{array}$$

When we change the equality sign in $(FOC - G)$ to $\geq$ in $(IC - G)$, we expand the feasible region of program $(OP - HQ')$ to the left along the G-direction, as the left hand side of $(FOC - G)$ decreases with G. This does not change the optimum because the objective function of $(OP - HQ')$ decreases with G in the expanded feasible region. Let $\phi(s, G) \equiv c'(s)/y_s(s, G)$. Then $(IC - s)$ implies $\alpha(i) = \phi(s(i), G)$, which by Assumption 4 is convex in $s(i)$. Substitute $\alpha(i) = \phi(s(i), G)$ into the objective function and constraint $(IC - G)$. Then the integrand in the objective function,

$$y(s(i), G) - c(s(i)) - \frac{1}{2} r\sigma^2 \alpha(i)^2,$$

becomes a concave function of $s(i)$. The integrand in constraint $(IC - G)$,

$$(1 - \alpha(i)) y_G(s(i), G),$$

is also concave in $s(i)$ in the convex range $\{s(i) : c'(s(i))/y_s(s(i), G) \leq 1\}$, because $(1 - \alpha(i))$ is non-negative, concave and decreasing in $s(i)$, and $y_G(s(i), G)$ is, by Assumption 5, positive, concave and increasing in $s(i)$; the product of two non-negative concave functions is concave if one of them is increasing and the other decreasing. Given a profile of sales effort levels,

$\{s(i)\}_{i \in \mathcal{I}-\mathcal{L}}$, let $s \equiv \int_{\mathcal{I}-\mathcal{L}} s(i) di / d(\mathcal{I} - \mathcal{L})$, the average of $\{s(i)\}_{i \in \mathcal{I}-\mathcal{L}}$. Then, for any given G, Jensen's inequality implies

$$\int_{\mathcal{I}-\mathcal{L}} [y(s(i), G) - c(s(i)) - \frac{1}{2} r\sigma^2 \alpha(i)^2] di \leq \int_{\mathcal{I}-\mathcal{L}} y(s, G) - c(s) - \frac{1}{2} r\sigma^2 \alpha^2 di,$$

and

$$\begin{aligned} & p y_G(T - \tfrac{G}{p}, G) - y_s(T - \tfrac{G}{p}, G) + \textstyle\int_{\mathcal{I}-\mathcal{L}} (1 - \alpha(i)) y_G(s(i), G) di \\ \leq \ & p y_G(T - \tfrac{G}{p}, G) - y_s(T - \tfrac{G}{p}, G) + \textstyle\int_{\mathcal{I}-\mathcal{L}} (1 - \alpha) y_G(s, G) di, \end{aligned}$$

where $\alpha = \phi(s, G)$. Therefore, choosing the same α and s for all managers of H units is better than choosing different ones. That is, the HQ should offer the same high-powered contract to all managers of H units. ■

Proof of Lemma 4: (1) s and G are continuous at $\alpha = 0$. (2) When $p = 0$, $s(\alpha, 0)$ is uniquely determined by $(FOC - s)$ and is continuous in α.

(1) We first prove that $\lim_{(\alpha, p') \to (0, p)} s(\alpha, p') = s(0, p) = T$. By $(FOC - s)$,

$$\alpha y_s(s(\alpha, p'), G) = c'(s(\alpha, p')).$$

Therefore,

$$0 \leq c'(s(\alpha, p')) - c'(s(0, p)) = \alpha y_s(s(\alpha, p'), G) \leq \alpha y_s(T, T) \to 0$$

as $\alpha \to 0$. The last inequality holds because $s(\alpha, p') \geq T$, $G \leq T$, and $y_s(s, G)$ increases with G and decreases with s. Since $c'' > 0$ as $t > T$, $(c')^{-1}$ is continuous in $[0, \infty)$ with

$(c')^{-1}(0) = T$. Therefore, $c'(s(\alpha,p')) - c'(s(0,p)) \to 0$ implies $\lim_{(\alpha,p')\to(0,p)} s(\alpha,p') = s(0,p) = T$, i.e., s is continuous at $\alpha = 0$.

When $p > 0$,

$$\frac{\partial^2 \Pi}{\partial G^2} = p y^L_{GG} - 2 y^L_{sG} + \frac{1}{p} y^L_{ss} + (1-p)(1-\alpha) y^H_{GG} < 0$$

by $(FOC - G)$ and the concavity of y, where a function with a superscript H (L, resp.) means that it is evaluated at (s, G) ($(T - \frac{G}{p}, G)$, resp.). Therefore $(FOC - G)$ implies G is a differentiable function of (s, α, p) when $p > 0$, by Implicit Function Theorem.

When $p = 0$,

$$G(\alpha, p) - G(0, 0) = pg \to 0.$$

(2) Now we prove that $s(\alpha, 0)$ is continuous in α. $s(\alpha, 0)$ is defined by

$$\alpha y_s(s(\alpha, 0), G) = c(s(\alpha, 0)).$$

If $y_s(T, 0) = 0$, then $y_s(s, 0) = 0$ for all $s \geq T$ by the concavity of y, and therefore, $s(\alpha, 0) = T$ for all α.

If $y_s(T, 0) \neq 0$, then for $\alpha > 0$, $s(\alpha, 0) > T$ and thus $c''(s(\alpha, 0)) > 0$. Implicit Function Theorem then implies that $s(\alpha, 0)$ is differentiable with respect to α. The continuity of $s(\alpha, 0)$ at $\alpha = 0$ is implied by $\lim_{\alpha' \to 0} c'(s(\alpha', 0)) = \alpha' y_s(s(\alpha', 0)) = 0$. ■

Proof of Proposition 2: At the optimum, $\alpha \in (0, 1)$.

We first prove the result for the very special case of $p = 1$. In this case, $(FOC - G)$ becomes

$$y_G(T - G, G) - y_s(T - G, G) = 0,$$

which implies that G does not depend on α. Therefore, α is chosen to

$$\begin{array}{rcll} \pi^H & = & \max_{\alpha,s} & y(s,G) - c(s) - \frac{1}{2} r\sigma^2\alpha^2 \\ & & s.t. & \alpha y_s(s,G) - c'(s) = 0 \qquad (IC) \end{array}$$

The Lagrangian of the program is

$$L = y(s, G) - c(s) - \frac{1}{2} r\sigma^2\alpha^2 + \lambda[\alpha y_s(s, G) - c'(s)].$$

Differentiation yields

$$\begin{array}{rcl} \frac{\partial L}{\partial \alpha} & = & -r\sigma^2\alpha + \lambda y_s(s, G), \\ \frac{\partial L}{\partial s} & = & (1-\alpha) y_s(s, G) + \lambda[\alpha y_{ss}(s, G) - c''(s)]. \end{array}$$

By the incentive compatibility constraint, $s \geq T > 0$. Therefore, $\frac{\partial L}{\partial s} = 0$. If $\alpha = 1$, then $\frac{\partial L}{\partial s} = 0$ implies $\lambda = 0$, which in turn implies $\frac{\partial L}{\partial \alpha} < 0$. This is a contradiction. If $\alpha = 0$, the incentive compatibility constraint implies that $s = T$. Therefore, $\frac{\partial L}{\partial s} = y_s(T, G) > 0$. This is again a contradiction. Therefore, $\alpha \in (0, 1)$.

Now, we consider the more general case of $p < 1$. At $\alpha = 1$, $(FOC - \alpha)$ becomes

$$\frac{d\Pi}{d\alpha} = -(1-p)r\sigma^2 + (1-p)y_G^H \frac{dG}{d\alpha}.$$

Apply the implicit function theorem to $(FOC - s)$ and $(FOC - G)$. We have, at $\alpha = 1$,

$$\frac{dG}{d\alpha} = \frac{1}{|J|}(1-p)y_G^H(y_{ss}^H - c''),$$

where, the Jacobian matrix

$$J = \begin{pmatrix} -c''(s) & 0 \\ 0 & py_{GG}^L - 2y_{sG}^L + \frac{1}{p}y_{ss}^L \end{pmatrix} + \begin{pmatrix} \alpha y_{ss}^H & \alpha y_{sG}^H \\ (1-p)(1-\alpha)y_{sG}^H & (1-p)(1-\alpha)y_{GG}^H \end{pmatrix}$$

is positive definite; it is the sum of a positive-definite matrix and a semipositive- definite matrix. Therefore, at $\alpha = 1$, $\frac{dG}{d\alpha} < 0$, which implies that $\frac{d\Pi}{d\alpha} < 0$. Thus the optimal α is not 1.

At $\alpha = 0$, we cannot use the above argument anymore because the Jacobian becomes singular. We need to utilize the interaction between α and p and therefore some results from the next subsection. Suppose $\alpha = 0$, then $s = T$ by $(FOC - s)$. Thus $\frac{\partial s}{\partial p} = 0$ and $(FOC - p)$ becomes

$$\begin{aligned} \tfrac{d\Pi}{dp} &= \pi^L - \pi^H + gy_s^L \\ &= y(T-g,G) - y(T,G) + gy_s(T-g,G) > 0, \end{aligned}$$

as $g(s, G)$ is concave in s. This implies that the optimal $p = 1$. The first part of the proof shows that in this case the optimal α is not 0, a contradiction to our assumption that $\alpha = 0$. Therefore, $\alpha \in (0, 1)$ for the case of $p < 1$ also. ■

Proof of Proposition 3: For any given p, $\pi^L < \pi^H$ at the corresponding HQ's optimal choice of α, denoted by $\alpha^*(p)$. In particular, $\pi^L < \pi^H$ holds in equilibrium.

Note from the discussion preceding Lemma 4 that, when $\alpha = 0$, an H manager chooses $s = T$, and $\pi^H(\alpha = 0) \equiv \Pi^H(\alpha \to 0) = y(T, G)$. Now, we prove the result for two separate cases:

Case 1: $p < 1$

By the definition of π^L,

$$\pi^L(\alpha = \alpha^*(p)) = y(T - g(\alpha^*), pg(\alpha^*)) \leq \max_g y(T - g, pg).$$

Since $y(s, G)$ increases with s and $p < 1$,

$$\max_g y(T-g, pg) < \max_g py(T-g, pg) + (1-p)y(T, pg) = \pi(\alpha = 0).$$

By the definition of $\alpha^*(p)$,

$$\pi(\alpha = 0) \leq \pi(\alpha = \alpha^*(p)) = p\pi^L(\alpha = \alpha^*(p)) + (1-p)\pi^H(\alpha = \alpha^*(p)).$$

Combining the above three inequalities, we have:

$$\pi^L(\alpha = \alpha^*(p)) < p\pi^L(\alpha = \alpha^*(p)) + (1-p)\pi^H(\alpha = \alpha^*(p)),$$

which implies that $\pi^L(\alpha = \alpha^*(p)) < \pi^H(\alpha = \alpha^*(p))$.
Case 2: $p = 1$

In this case, $(FOC - G)$ implies that G, and thus π^L, is independent of α. Therefore, the optimal α maximizes π^H. Then $\pi^H(\alpha = \alpha^*(p)) \geq \pi^H(\alpha = 0) = y(T, G) > y(T - G, G) = \pi^L$. ■

Proof of Lemma 5: $\liminf_{(\alpha',p)\to(\alpha,0)} \Pi(\alpha', p) \geq \Pi(\alpha, 0)$. *Furthermore, if* $y(s, 0) = 0$, *then* $\lim_{(\alpha',p)\to(\alpha,0)} \Pi(\alpha', p) = \Pi(\alpha, 0)$.

At $(\alpha, p = 0)$, $G = 0$ and $s(\alpha, p = 0)$ is determined by

$$\alpha y_s(s, 0) = c'(s).$$

At $(\alpha', p > 0)$, $G > 0$ by $(FOC - G)$ and $s(\alpha', p)$ is determined by

$$\alpha' y_s(s, G) = c'(s). \qquad (A1)$$

Because $y_{sG} > 0$, it is easy to see that $s(\alpha', p) > s(\alpha', p = 0)$, for $p > 0$.

By the definition of Π,

$$\Pi(\alpha', p) = py(T - g, G) + (1-p)[y(s(\alpha', p), G) - c(s(\alpha', p)) - \frac{1}{2}r\sigma^2\alpha'^2], \qquad (A2)$$

and

$$\Pi(\alpha, 0) = y(s(\alpha, 0), 0) - c(s(\alpha, 0)) - \frac{1}{2}r\sigma^2\alpha^2]. \qquad (A3)$$

By (A2) and (A3) and rearrangement, we have,

$$\begin{aligned}\Pi(\alpha', p) - \Pi(\alpha, 0) &= p[y(T - g, G) - \Pi(\alpha, 0)] + \tfrac{1}{2}(1-p)r\sigma^2(\alpha^2 - \alpha'^2) \\ &+(1-p)[y(s(\alpha', 0), 0) - c(s(\alpha', 0)) - y(s(\alpha, 0), 0) + c(s(\alpha, 0))] \\ &+(1-p)[y(s(\alpha', p), G) - c(s(\alpha', p)) - y(s(\alpha', 0), 0) + c(s(\alpha', 0))].\end{aligned}$$

In the above equation, as $p \to 0$ and $\alpha' \to \alpha$, the first two terms go to 0. By Lemma 1(2), $s(\alpha, 0)$ is continuous in α and thus the third term goes to 0 as $\alpha' \to \alpha$. Therefore, the last term is crucial in determining the sign of $\Pi(\alpha', p) - \Pi(\alpha, 0)$. We want to show that the last term is non-negative.

$$\begin{aligned} & y(s(\alpha', p), G) - c(s(\alpha', p)) - y(s(\alpha', 0), 0) + c(s(\alpha', 0)) \\ = \; & y(0, G) + \textstyle\int_0^{s(\alpha',p)}[y_s(s, G) - c'(s)]ds - y(0, 0) - \int_0^{s(\alpha',0)}[y_s(s, 0) - c'(s)]ds \\ \geq \; & \textstyle\int_{s(\alpha',0)}^{s(\alpha',p)}[y_s(s, G) - c'(s)]ds \qquad (A4) \\ \geq \; & \textstyle\int_{s(\alpha',0)}^{s(\alpha',p)}[y_s(s(\alpha', p), G) - c'(s(\alpha', p))]ds \qquad (A5) \\ = \; & [s(\alpha', p) - s(\alpha', 0)](1 - \alpha')y_s(s(\alpha', p), G) \geq 0. \qquad (A6)\end{aligned}$$

Inequality (A4) is because $y(0,G) \geq y(0,0)$. Inequality (A5) holds because $y_s(s,G) - c'(s)$ decreases in s. Equation (A6) is by (A1). Therefore, $\liminf_{(\alpha',p)\to(\alpha,0)} \Pi(\alpha',p) \geq \Pi(\alpha,0)$.

When $y(s,0) = 0$, by (A2),

$$\begin{aligned}
& \limsup\nolimits_{(\alpha',p)\to(\alpha,0)} \Pi(p,\alpha') \\
\leq\ & \limsup\nolimits_{(\alpha',p)\to(\alpha,0)} py(T-g,G) + (1-p)y(s(\alpha',p),G) - \tfrac{1}{2}(1-p)r\sigma^2\alpha'^2 \\
\leq\ & -\tfrac{1}{2}r\sigma^2\alpha^2 \\
\leq\ & \Pi(0,\alpha).
\end{aligned}$$

Combining this with the above result, we have $\lim_{(\alpha',p)\to(\alpha,0)} \Pi(p,\alpha') = \Pi(0,\alpha)$. ■

Proof of Proposition 4: It is optimal for the company to have some L units.

If $y(s,0) = 0$ for all s, we have argued in the text why the optimal p is positive. If $y(s,0)$ is not always zero, then the concavity, the monotonicity, and the non- negativity of y implies that $y(s,0) > 0$ for all $s > 0$.

We consider the limit of $\frac{d\Pi}{dp}$ as $p \to 0$. By $(FOC - p)$,

$$\frac{d\Pi}{dp} = (\pi^L - \pi^H) + gy_s^L + (1-p)\alpha y_G^H \frac{\partial G}{\partial p} + (1-p)(1-\alpha)y_s^H \frac{\partial s}{\partial p}.$$

$(FOC - G)$ implies that,

$$y_s(T-g,G) \geq (1-p)(1-\alpha)y_G(s,G).$$

Since $s \geq T$ and $y_{sG} > 0$,

$$(1-p)(1-\alpha)y_G(s,G) \geq (1-p)(1-\alpha)y_G(T,G),$$

the right hand side of which $\to \infty$ because $G = pg \leq pT$, Assumption 3 says that $\lim_{G\to 0} y_G = \infty$, and Proposition 2 says that $\alpha < 1$. Therefore, $y_s(T-g,G) \to \infty$, which implies $g \to T$. Then, by Assumption 6, the substitution effect in $(FOC - p)$, $gy_s^L \to \infty$;

$$\lim_{p\to 0} gy_s^L = T \lim_{p\to 0} y_s(T-g,G) = T \lim_{(s,G)\to(0,0)} y_s(s,G) = \infty.$$

In $(FOC - p)$, $\pi^L - \pi^H$ is bounded. Then to determine the sign of $\frac{d\Pi}{dp}$ as $p \to 0$, it is sufficient to show that $\frac{\partial G}{\partial p} > 0$ and $\frac{\partial s}{\partial p} > 0$ as $p \to 0$. By Lemma 3, it suffices to show that $\frac{\partial^2\Pi}{\partial p \partial G} > 0$. Substitute $(FOC - G)$ into $\frac{\partial^2\Pi}{\partial p \partial G}$ and rearrange. Then

$$(1-p)\frac{\partial^2\Pi}{\partial p \partial G} = y_G^L + (1-p)gy_{sG}^L - \frac{(1-p)}{p}gy_{ss}^L - y_s^L, \tag{A7}$$

in which only the last term is negative. By Assumption 6, y_s is weakly convex. Then

$$y_s(T,G) - y_s(T-g,G) \geq gy_{ss}(T-g,G),$$

in which $y_s(T-g,G) \to \infty$. Therefore,

$$-gy^L_{ss} = -gy_{ss}(T-g,G) \to \infty.$$

Rearranging $(A7)$ yields

$$\begin{array}{rcl} (1-p)\frac{\partial^2 \Pi}{\partial p \partial G} & = & y^L_G + (1-p)gy^L_{sG} - \frac{(1-2p)}{p} g y^L_{ss} - gy_{ss}(T-g,G) - y^L_s \\ & \geq & y^L_G + (1-p)gy^L_{sG} - \frac{(1-2p)}{p} g y^L_{ss} - y_s(T,G) \to \infty. \end{array}$$

In summary, we have shown that $\frac{d\Pi}{dp} \to \infty$ as $p \to 0$. Therefore, the optimal p is positive unless the value of Π at $p=0$ is higher than $\lim_{p\to 0}\Pi$, which Lemma 5 excludes. This completes the proof of the Proposition. ■

Proof of Lemma 6: When $p=1$, *the optimal* $s > T$. $\frac{d\pi^H}{dr} = -\frac{1}{2}\sigma^2\alpha^2 < 0$, $\frac{d\pi^H}{d\sigma} = -r\sigma\alpha^2 < 0$ *and* $\frac{d\pi^H}{dK} = -c(s) - \lambda c'(s) < 0$, *where* λ *is the Lagrange multiplier of the constraint and is positive.*

The Lagrangian of the program that chooses the optimal α and s is

$$L = y(s,G) - Kc(s) - \frac{1}{2}r\sigma^2\alpha^2 + \lambda[\alpha y_s(s,G) - Kc'(s)].$$

In the proof of Proposition 2, we showed that the optimal $\alpha \in (0,1)$ and $\frac{\partial L}{\partial \alpha} = 0$. Therefore, $\lambda > 0$ and the optimal $s > T$.

By the envelope theorem, we have, $\frac{d\pi^H}{dr} = -\frac{1}{2}\sigma^2\alpha^2 < 0$, $\frac{d\pi^H}{d\sigma} = -r\sigma\alpha^2 < 0$ and $\frac{d\pi^H}{dK} = -c(s) - \lambda c'(s) < 0$. ■

Proof of Lemma 7: When $p=1$, *if* $y(s,0) > 0$ *for* $s > 0$ *and* $\lim_{(s,G)\to(0,0)} Gy_G(s,G) = 0$, *then* $\frac{d\pi}{dp}\mid_{p=1} < 0$ *for sufficiently small* T.

As $T \to 0$, $\pi^L = y(T-g,T) \to y(0,0)$. When $p=1$, $gy^L_s = Gy_s(T-G,G)$. By (FOC-G), This is $Gy_G(T-G,G)$, which, by the assumption of the Lemma, approaches 0 as both G and T go to 0.

$$\begin{array}{rcl} \pi^H & = & \max_\alpha \quad y(s,G) - c(s-T) - \frac{1}{2}r\sigma^2\alpha^2 \\ & & \quad s.t. \quad \alpha y_s(s,G) - c'(s-T) = 0. \end{array}$$

As $T \to 0$, π^H approaches,

$$\begin{array}{rcl} \pi^H(T=0) & = & \max_\alpha \quad y(s,0) - c(s) - \frac{1}{2}r\sigma^2\alpha^2 \\ & & \quad s.t. \quad \alpha y_s(s,0) - c'(s) = 0, \end{array}$$

which is independent of T and greater than $y(0,0)$. Therefore,

$$\lim_{T\to 0} \frac{d\pi}{dp}\mid_{p=1} = y(0,0) - \pi^H(T=0) < 0,$$

that is, $\frac{d\pi}{dp}\mid_{p=1} < 0$ for sufficiently small T. ■

Proof of Proposition 9: For k_1 and k_2 sufficiently small, the high-powered incentive contract is renegotiation-proof if and and if the manager owns the unit's physical asset.

Consider the HQ's optimization problem

$$\begin{array}{lll} \max & py(T - \frac{G}{p}, G) + (1-p)[y(s,G) - c(s) - \frac{1}{2}r\sigma^2\alpha^2] & (OP-HQ) \\ s.t. & \alpha y_s(s,G) - c'(s) = 0 & (FOC-s) \\ & py_G(T - \frac{G}{p}, G) - y_s(T - \frac{G}{p}, G) + (1-p)(1-\alpha)y_G(s,G) = 0 & (FOC-G) \end{array}$$

where $y(s,G) = z(s,G) + k_1\mu(s) + k_2\nu(G)$. Let the solution to program $(OP-HQ)$ be denoted with a superscript *. We want to show that, for sufficiently small k_1 and k_2,

$$\alpha^* y(s^*, G^*) > y(s^*, 0) = k_1\mu(s^*). \qquad (A8)$$

The proof for the second inequality above Proposition 8 is similar.

If the solution to $(OP-HQ)$, $(\alpha^*, p^*, s^*, G^*)$, is continuous in (k_1, k_2), then, as $(k_1,k_2) \to (0,0)$, $\alpha^* y(s^*, G^*) \to \alpha^0 y(s^0, G^0)$, where (α^0, s^0, G^0) is the equilibrium at $(k_1,k_2) = (0,0)$. By Proposition 2, $\alpha^0 y(s^0, G^0) > 0$. As $(k_1,k_2) \to (0,0)$, $k_1\mu(s^*) \to 0$. Therefore, $(A8)$ holds for sufficiently small k_1 and k_2. Unfortunately, it is not easy to show the continuity of the equilibrium because program $(OP-HQ)$ is in general not concave.

To prove the inequality, we first perform an exercise similar to the proof of Proposition 1(2). Let $\phi(s,G) \equiv c'(s)/y_s(s,G)$. Then $(FOC-s)$ becomes $\alpha = \phi(s,G)$. By Assumption 4, ϕ is convex in s. Substitute $\alpha = \phi(s,G)$ into the objective function and constraint $(FOC-G)$ in $(OP-HQ)$. Then $(OP-HQ)$ becomes

$$\begin{array}{lll} \max & py(T - \frac{G}{p}, G) + (1-p)[y(s,G) - c(s) - \frac{1}{2}r\sigma^2\phi(s,G)^2] & (OP-HQ) \\ s.t. & py_G(T - \frac{G}{p}, G) - y_s(T - \frac{G}{p}, G) + (1-p)(1-\phi(s,G))y_G(s,G) \geq 0 & (FOC-G) \end{array}$$

The reason why we can change the equality in $(FOC-G)$ to inequality is the same as that offered in the proof of Proposition 1(2). Now, given (p,G), $(OP-HQ)$ is a concave program that chooses the optimal s. The solution $s = s(p,G,k_1,k_2)$ is differentiable. Substitute the solution into the objective function. We have an unconstrained optimization problem[25]

$$\max_{p,G} f(p,G,k_1,k_2), \qquad (A9)$$

where f is differentiable. Again, we don't know whether or not the solution to $(A9)$ is continuous in (k_1,k_2).

Define

$$\mathcal{S} \equiv \{(p,G,k_1,k_2) : (p,G) = \arg\max_{p,G} f(p,G,k_1,k_2)\}.$$

We claim that $\mathcal{S}$ is a closed set. Suppose this is not true. Then there exists a sequence $(p_n, G_n, k_{1n}, k_{2n}) \in \mathcal{S}$ such that $\lim_{n\to\infty}(p_n, G_n, k_{1n}, k_{2n}) = (p_0, G_0, k_{10}, k_{20})$ but $(p_0, G_0, k_{10}, k_{20})$ is not in $\mathcal{S}$. There exists (p', G') such that

$$f(p_0, G_0, k_{10}, k_{20}) < f(p', G', k_{10}, k_{20}). \qquad (A10)$$

[25] The constraint that $p \in [0,1]$ does not affect the argument and is thus omitted.

Let $\epsilon \in (0, \frac{1}{2}[f(p', G', k_{10}, k_{20}) - f(p_0, G_0, k_{10}, k_{20})])$. Since f is continuous, for sufficiently large n,

$$|f(p_n, G_n, k_{1n}, k_{2n}) - f(p_0, G_0, k_{10}, k_{20})| < \epsilon,$$

and

$$|f(p', G', k_{1n}, k_{2n}) - f(p', G', k_{10}, k_{20})| < \epsilon.$$

$(A10)$ then implies that

$$f(p_n, G_n, k_{1n}, k_{2n}) < f(p', G', k_{1n}, k_{2n}),$$

which contradicts with the fact that $(p_n, G_n, k_{1n}, k_{2n}) \in \mathcal{S}$. Therefore, $\mathcal{S}$ is a closed set.

Now, we want to show that

$$\liminf_{(k_1,k_2)\to(0,0)} \alpha^* y(s^*, G^*) > 0. \tag{A11}$$

Suppose, on the contrary, $\liminf_{(k_1,k_2)\to(0,0)} \alpha^* y(s^*, G^*) = 0$. Then, there exists a sequence $(p_n, G_n, k_{1n}, k_{2n}) \in \mathcal{S}$ such that $\lim_{n\to\infty}(p_n, G_n, k_{1n}, k_{2n}) = (p_0, G_0, 0, 0)$ and $\alpha_0 y(s_0, G_0) = 0$, where $s_0 = s(p_0, G_0, 0, 0)$ and $\alpha_0 = \phi(s_0, G_0)$. Since $\mathcal{S}$ is a closed set, $(p_0, G_0, 0, 0) \in \mathcal{S}$ and thus $(\alpha_0, p_0, s_0, G_0)$ is an equilibrium for the case of $(k_1, k_2) = (0, 0)$. Proposition 2 implies that $\alpha_0 y(s_0, G_0) \neq 0$. This is a contradiction. Therefore, $(A11)$ holds. ■