TECHNICAL APPENDIX

7. Agents' Decisions in the Cobb-Douglas Case

A closed form solution for production and trade obtains if we assume that utility is a Cobb-Douglas function of the consumption bundle:

$$u = x_1^{\gamma} x_2^{1-\gamma}.$$

Here, x denotes consumption. With some judicious substitution, we obtain a closed form solution for the quantities in four steps.

First, we obtain a solution for the import relative share. The combination of the efficiency conditions (1.5) for imports and (1.6) for output implies

$$\frac{\pi u_1^G}{(1-\pi)u_1^B} = \frac{p}{a_1/a_2 - p}.$$

For the Cobb-Douglas case this implies

$$\frac{\pi\left(x_2^G/x_1^G\right)^{1-\gamma}}{(1-\pi)\left(x_2^B/x_1^B\right)^{1-\gamma}} = \frac{\pi}{1-\pi}(x_1^B/x_1^G)^{1-\gamma} = \frac{p}{a_1/a_2 - p}.$$

Here, we have used the fact that $x_2 = y_2 + m_2$ in each state. Now note that

$$x_1^B/x_1^G = y_1/(y_1 + m_1).$$

Solving this expression for the import relative share m_1/y_1 we obtain

$$\text{(7.1)} \qquad \frac{m_1}{y_1} = \left[\frac{(1-\pi)p}{\pi\left(a_1/a_2 - p\right)}\right]^{-1/(1-\gamma)} - 1 \equiv f(p,\pi,\alpha,\gamma),$$

where $\alpha = a_1/a_2$. The import relative share is undefined at $\pi=1$, as is appropriate since in that case the classic Ricardian model obtains and production will either be equal to zero or indeterminate. It is defined everywhere else, which means that with Cobb-Douglas preferences, complete specialization is never optimal in the presence of predation.

Second, we obtain the consumption ratio in the two states in terms of the import relative share and the production ratio. We substitute into the ratio of consumption in the two states using $m_2 = -pm_1$ to solve in terms of m_1/y_1 and y_2/y_1.

$$\frac{x_1^B}{x_2^B} = \frac{y_1}{y_2 + m_2} = \frac{1}{y_2/y_1 - pm_1/y_1} \quad \text{and}$$

$$\frac{x_1^G}{x_2^G} = \frac{y_1 + m_1}{y_2 + m_2} = \frac{1 + m_1/y_1}{y_2/y_1 - pm_1/y_1}.$$

Third, we solve for the production ratio. Substituting the preceding expressions for the consumption ratios into the efficiency condition for imports and using $f(p)$ for the import relative share m_1/y_1 we obtain:

$$\frac{\pi u_1^G}{\pi u_2^G + (1-\pi)u_2^B} = p = \frac{\pi\gamma\left[\frac{1+f()}{y_2/y_1 - pf()}\right]^{\gamma-1}}{\pi(1-\gamma)\left[\frac{1+f()}{y_2/y_1 - pf()}\right]^{\gamma} + (1-\pi)(1-\gamma)\left[\frac{1}{y_2/y_1 - pf()}\right]^{\gamma}}.$$

This expression may be solved for y_2/y_1 to yield:

$$\frac{y_2}{y_1} = \frac{p\left\{\pi(1-\gamma)[1+f()]^{\gamma} + (1-\pi)(1-\gamma)\right\}}{\pi\gamma[1+f()]^{\gamma-1}} + pf()$$

$$= p\frac{1-\gamma}{\gamma} + pf\frac{1}{\gamma} + p\frac{1-\pi}{\pi}\frac{1-\gamma}{\gamma}(1+f)^{1-\gamma}$$

$$= p\frac{f}{\gamma} + \alpha\frac{1-\gamma}{\gamma} = \frac{pf+\alpha}{\gamma} - \alpha.$$

Finally, in combination with the full employment constraint $\mathbf{a}'\mathbf{y} \le l^S$ the production ratio yields the closed form solution for y_1, m_1, y_2, m_2 as functions of the exogenous variables p and π and the technology parameter α.

(7.2) $$y_1 = \frac{\gamma l^S / a_2}{pf(p,\pi,\alpha,\gamma) + \alpha}.$$

Then in turn:

(7.3) $$m_1 = \frac{\gamma f(p,\pi,\alpha,\gamma)}{pf(p,\pi,\alpha,\gamma) + \alpha} l^S / a_2$$

(7.4) $$y_2 = l^G / a_2 - \alpha\gamma \frac{l^S / a_2}{pf(p,\pi,\alpha,\gamma) + \alpha}$$

(7.5) $$m_2 = -\gamma pf(p,\pi,\alpha,\gamma)\frac{l^S / a_2}{f(p,\pi,\alpha,\gamma) + \alpha}.$$

Now we are in a position to consider the partial equilibrium comparative statics of system (7.2)-(7.5). It is immediate that a rise in ‘effective size’ l^G/a_2 will raise trade volume, as is intuitive. We anticipate that a rise in π will raise the level of trade m_1 and the degree of specialization measured by y_2. A rise in α should also raise trade as it increases the gap between the autarky price ratio and the price available through trade.

To develop these ideas it is necessary as a preliminary step to differentiate the import relative share function $f(p,\pi,\alpha)$.

$$f(p,\pi,\alpha,\gamma) \equiv \left[\frac{p(1-\pi)}{(\alpha-p)\pi}\right]^{-1/(1-\gamma)} - 1, \text{ hence}$$

(7.6)
$$f_p = -\frac{1+f}{1-\gamma}\left[1/p + 1/(\alpha-p)\right] < 0$$

$$f_\pi = \frac{1+f}{1-\gamma}\left[1/\pi + 1/(1-\pi)\right] > 0$$

$$f_\alpha = \frac{1+f}{1-\gamma}\left[1/(\alpha-p)\right] > 0.$$

Now we are in a position to analyze the properties of the per capita import demand function $m_1(p,\pi,a)$. Differentiating (7.3) with respect to p:

$$m_{1_p} = m_1\left[\frac{f_p}{f}\left(1-\frac{pf}{pf+\alpha}\right)-\frac{1}{pf+\alpha}\right] < 0. \quad (7.7)$$

The negative sign follows from noting that the square bracket term is negative for positive imports.

As for the response of m_1 to a rise in π, we can show that this is positive and approaches zero as complete specialization is approached:

$$m_{1_\pi} = m_1\left(1-\frac{p}{pf+\alpha}\right)\frac{f_\pi}{f} > 0. \quad (7.8)$$

Deriving the foreign economy's excess demand functions in the Cobb-Douglas case simply replicates the steps above, recognizing that the role of goods 1 and 2 is switched, and recognizing that the relative price of imports for the foreigner is *1/p* and that the marginal rate of transformation relevant to the steps above is that for the import good in terms of the export good , so $\alpha^* \equiv a_2^*/a_1^*$. All properties are the same, *mutatis mutandis*.

8. The Effect of Continuous Defense Choice

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Agents choose whether to commit anything to defense by solving the binary choice problem:

$$\max_{l} \max\{d(l)v^S(l),[1-d(l)]v^A,[1-d(l)]v^B\},$$

where *d(l)* is a dummy variable equal to 1 if *l>0* and equal to zero if *l=0.* This problem serves to sort all agents into specialized producers on the one hand and either bandits or autarkic producers on the other hand.

Now consider the choice of level of defense for a specialized producer. The individual's probability of successful exchange is equal to

$$\phi(l/\bar{l})\pi$$

$$\phi' > 0 \; for \; 0 \le l < \bar{l}, \; \phi' = 0 \; for \; l > \bar{l}$$

$$\phi(1) = 1,$$

$$\phi(0) = 0.$$

Here, π is the social success rate for exchange and φ is the individual success rate for initiating shipment of exports. This specification is rationalized by thinking of the process of exchange in 2 parts. Each shipment must depart the producer's location and then travel to an exchange point. Bandits are diffused in the initial stage, covering all producer locations. Under our assumptions, the producer can guarantee successful initial shipment with a defense commitment equal to $\bar{l}$. In the exchange stage, bandits are more concentrated around the exchange points and the probability of successful exchange is equal to π.

The specialized agent's choice of the interior level of defense must satisfy:

$$\frac{\partial v^S}{\partial l}(l) = 0; \; where \; l < \bar{l} \; and \; \frac{\partial^2 v^S}{\partial l^2} < 0.$$

If this condition cannot be satisfied at the interior, then $l = \bar{l}$ and v^S is decreasing for decreases in defense effort.

8.1. Determination of defense effort

When the derivative is examined in detail, the corner solution $l = \bar{l}$ need not obtain even with the simple case in which $\phi = l/\bar{l}$. Here, the indirect utility function for specialized producers is:

$$v^S = \left[\frac{l}{\bar{l}}\pi\left(\frac{1+f}{pf+\alpha}\right)^{\gamma} + \left(1-\frac{l}{\bar{l}}\pi\right)\left(\frac{1}{pf+\alpha}\right)^{\gamma}\right]\frac{1-l}{a_2},$$

where $f = f(p,\phi\pi,\alpha,\gamma) = \left[\frac{p(1-\phi\pi)}{(\alpha-p)\phi\pi}\right]^{-1/(1-\gamma)} - 1.$

Differentiating with respect to reductions in $l\ l$ at the limit value (using the convention that φ has a left hand derivative at $l=\bar{l}$ and using a minus sign to denote left hand differentiation) and simplifying,

$$\left.\frac{\partial v^S}{\partial l}\right|_{l=\bar{l}}^{-} = -\frac{v^S}{1-\bar{l}} + \frac{1}{\bar{l}}\left[v^G - \left(\frac{1}{pf+\alpha}\right)^{\gamma}\frac{1-\bar{l}}{a_2}\right] + \frac{\partial v^S}{\partial f} f_2 \pi \frac{1}{\bar{l}}, \text{ where}$$

$$\frac{\partial v^S}{\partial f} = \gamma\left(\frac{1}{pf+\alpha}\right)^{\gamma-1}\left[\phi\pi\left(1-p\frac{1+f}{pf+\alpha}\right)(1+f)^{\gamma-1} - (1-\phi\pi)p\frac{1+f}{(pf+\alpha)^2}\right]$$

Multiplying both sides by $\bar{l}/v^S$ and using φ=1 at the limit value of defense, we obtain:

$$(8.1)\qquad \frac{\bar{l}}{v^S}\left.\frac{\partial v^S}{\partial l}\right|_{l=\bar{l}}^{-} = -\frac{\bar{l}}{1-\bar{l}} + 1 - \frac{1}{\pi(1+f)^{\gamma}+(1-\pi)} + \left[\frac{f}{v^S}\frac{\partial v^S}{\partial f}\right]\left[\frac{f_2\pi}{f}\right].$$

Now we are ready to sign this derivative. Ignoring the effect of changing f, the net effect of the first three terms on the right hand side will be positive for sufficiently large f, but negative at f close to zero. This is simply the effect of gains from trade too small to outweigh the fixed cost of defense. As for the effect of changing f, the elasticity of f with respect to the success rate is greater than one, while $\partial v^S/\partial f$ can be shown to have the sign of $(p-1)f+\alpha-1$.[15] For α<1, since p<α, we know the effect via changes in f must be negative. For α>1 and small f, or for p>1, the effect via changing f is positive.

From these considerations, there is some presumption that the left hand elasticity of utility with respect to defense effort is positive: i.e., utility will fall with a decrease in defense effort. This shows that the binary form of choice used in the text is somewhat more general than it at first appears. However, the analysis also shows that in some cases utility will rise with a cut in defense effort, meaning that an interior solution will be optimal.

For interior solutions to the defense effort problem, l must satisfy:

$$(8.2)\qquad \frac{\partial v^S}{\partial l} = -\frac{v^S}{1-l} + \frac{1}{l}\left[v^S - \left(\frac{1}{pf+\alpha}\right)^{\gamma}\frac{1-l}{a_2}\right] + \frac{\partial v^S}{\partial f} f_2\pi\frac{1}{l} = 0.$$

This does not have a closed form solution.

8.2. The general equilibrium model with continuous defense effort choice

When defense effort is at an interior solution, our general equilibrium model contains 4 simultaneously determined variables: $L^D, L^B, p and \pi$. Their values are determined by (8.2), the labor market entry conditions, the

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[15] Note that from the definition of f we can write

$$(1+f)^{\gamma-1} = \frac{(1-\pi)p}{\pi(\alpha-p)}.$$

Substituting into the expression for $\partial v^G/\partial f$ and simplifying results in a series of positive factors multiplying the term above.

international market clearance condition and the requirement of rational expectations.

The model we have used in the text avoids the complexity of interior defense effort choice, and we maintain that our results are robust with respect to this simplification. Our key results are in two groups.

Our first set of results are that (i) autarky is likely in a wide range of the parameter space, and (ii) that this likelihood is positively affected by relative effectiveness of predation (θ) and negatively affected by the effectiveness of defense (β). We furthermore found that (iii) within some range of parameter values for θ and β, the likelihood of a trading equilibrium was first increased by increases in $\bar{l}$ and then decreased. As for the robustness of finding (i), since we impose limit levels of personal defense effort we may have somewhat widened the range of parameter values for which trade can be supported relative to the more realistic specification in which personal defense is continuously variable. Thus our finding (i) that autarky is widely prevalent is all the more sharp by being biased upward. There is no reason to expect finding (ii) to be affected by the inclusion of continuous defense effort. As for the robustness of finding (iii), for corner solutions even when defense choice is continuous the analysis is identical whereas for interior solutions for defense choice in a trading equilibrium it will no longer necessarily be true that increases in $\bar{l}$ smoothly reduce the net gains from trade and shift the economy from a trading equilibrium into autarky. However, a sufficiently large increase in $\bar{l}$ must once again return the economy to autarky. Thus finding (iii) is also basically robust to continuous defense choice.

The second key group of results concerns immiserizing security. The mechanisms we identify which deliver this outcome --- larger countries tend suffer terms of trade deterioration and poorer countries tend to suffer terms of trade deterioration when security improves --- are still present in a model of continuous defense effort choice. However, they are augmented by a supply side effect as labor flows between production and defense in interior solutions of defense effort. If both countries' defense efforts always move in the same direction as security parameters change, this has ambiguous effects on the terms of trade, but implies that the larger country's import demand and export supply functions shift by more (as a given percent of defensive labor is multiplied by a larger base). Thus there is a presumption that common rises in defense effort will cause a terms of trade deterioration of the larger country, both directly through the labor supply effect and indirectly through the improvement in security. Essentially this effect is picked up in our simulations which vary $\bar{l}$. The final possibility, however, is that defense efforts may move in opposite directions in the two countries. This opens up a richer menu of possible labor supply effects on the terms of trade, which are missing from our model of discrete defense effort choice. Nevertheless, our findings are robust because seems unlikely for this mechanism to be able to rule out immiserizing security.