Lecture Notes
in
Computational Economic Dynamics

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Preface

Many interesting economic models cannot be solved analytically using the standard mathematical techniques of Algebra and Calculus. This is often true of dynamic economic models that attempt to capture the behavior of rational, forward-looking agents. Dynamic economic models typically give rise to functional equations, in which the unknown is not simply a vector in Euclidean space, but rather an entire function defined on a continuum of points. For example, the Bellman and Euler equations that describe dynamic optima are functional equations, as often are the conditions that characterize rational expectations and arbitrate pricing market equilibria. Except in a very limited number of special cases, these functional equations lack a known closed-form solution, even though the solution can be shown theoretically to exist and to be unique.

Models that lack closed-form solution are not unique to economics. Analytically insoluble models are common in biological, physical, and engineering sciences. Since the introduction of the digital computer, scientists in these fields have turned increasingly to numerical computer methods to solve their models. In many cases where analytical approaches fail, numerical methods can often be used to successfully compute highly accurate approximate solutions. In recent years, the scope of numerical analysis applications in the biological, physical, and engineering sciences has grown dramatically. In some of these disciplines, computational model building and analysis is now recognized as a legitimate subdiscipline of specialization. Numerical analysis courses have also become standard in graduate and undergraduate training programs.

Numerical analytical methods, however, have not been as eagerly embraced by economists. Many economists have shunned numerical methods out of a belief that numerical solutions are less elegant or less general than those obtained from algebraic models. The former belief is a subjective, aesthetic judgment that is outside of scientific discourse and beyond the scope of this book. The generality of the results obtained from numerical economic models, however, is another matter. Of course, given an economic model, it is always preferable to derive an explicit algebraic solution — provided such a solution exists. However, when essential features of an economic system being studied cannot be captured neatly in an algebraically soluble model, a choice must be made. Either essential features of the system must be ignored in order to obtain an algebraically tractable model, or numerical techniques must be applied. Too often Economists chose algebraic tractability over Economic realism.

Numerical economic models are often unfairly criticized by economists on the grounds that they rest on specific assumptions regarding functional forms and parameter values. Such criticism, however, is unwarranted when strong empirical support exists for the specific functional form and parameter values used to develop a model. Moreover, even when there is some uncertainty about functional forms and parameters, a numerical model may be solved under a variety of assumptions in order to assess the robustness of its implications. Although some doubt will persist as to the implications of a model outside the range of functional forms and parameter values examined, this uncertainty must be weighed against the lack of relevance of an alternative model that is algebraically soluble, but which ignores essential features of the economic system of interest. We believe that it is better to derive economic insights from a realistic numerical model of an economic system than to derive irrelevant results, however general, from an unrealistic algebraic model.

Despite the resistance placed by the economics profession as a whole, many economists have become aware of the potential benefits of numerical approaches to economic model building and analysis. This evidenced by the recent introduction of journals and an economic society devoted to the subdiscipline of computational economics. The growth of popularity of computational economics, however, has been impeded by the absence of adequate textbooks and computer software. The methods of numerical analysis and much of the available computer software have been largely developed by non-economic disciplines, most notably by physical, mathematical, and computer sciences. The available literature can pose substantial barriers for economists, both because of the mathematical prerequisites and because the examples are unfamiliar to economists. Many available software packages are designed to solve problems that are specific to the physical sciences.

This book attempts to address, in a number of ways, the difficulties typically encountered by economists attempting to learn and apply numerical methods. First, this book emphasizes practical numerical methods, not mathematical proofs, and focuses on techniques that will be directly useful to economic analysts, not those that would be useful exclusively to physical scientists. Second, the examples used in the book are familiar to economists. In order to make our task manageable, we have chosen to emphasize stochastic dynamic optimization, rational expectations, and arbitrage pricing models in both discrete and continuous time. The methods in developed in the book, however, can be applied to a much wider range of economic problems.

We make no attempt to be encyclopedic in our coverage of numerical methods or potential economic applications. We have instead chosen to develop only a relatively small number of techniques that can be applied easily to a wide variety of economic problems, particularly dynamic economic model building. In some instances, we have deviated from the standard treatments of numerical methods in existing textbooks in order to present a simple consistent framework that may be used to a readily learned and applied by economists. In many cases we have chosen not to cover certain numerical techniques when we regarded them to be of limited use to economists, or too complicated for novices. Throughout the book, we will try to explain our choices clearly and to give references to more advanced numerical textbooks where appropriate.

The book is divided into two major sections. Basic numerical methods, including root finding, numerical integration, and function approximation methods are presented in the first half of the book. Function approximation methods are, in some sense, the key to the book because in dynamic economic analysis we must often compute an approximation to a function that satisfies certain equilibrium or optimality conditions. These equilibrium or optimality conditions typically take the form of a functional equation, such as a differential or integral equation. For the most part, we have limited our coverage of basic numerical methods to those that have direct application in solving these classes of functional equations.

The second half of the book discusses application of basic numerical methods to the solution of dynamic equilibrium and optimization models in economics and finance. Dynamic economic models can be classified by whether they treat time and states of nature as discrete or continuous. The section on dynamic economic and financial modeling consists of five chapters. The first chapter focuses on discrete time and state space models. The next chapter examines discrete time, continuous state space models. The following two chapters treat continuous time and space models. And a final chapter discusses how dynamic models can be structurally estimated econometrically.

The book is aimed at both graduate students, advanced undergraduate students, and practicing applied economists. We have attempted to write a book that can be used both as a classroom text and for self-study. The examples used in the book come from a wide range of sub-specialties of economics and finance, both in macro- and micro-economics, with particular emphasis on problems in agricultural, financial, and environmental economics. Our goal is to motivate a broad range of economists to use numerical methods in their work by demonstrating the essential unity underlying dynamic economic models across sub-disciplines.

Although we have attempted to keep the mathematical prerequisites for this book to a minimum, some mathematical training and insight is necessary to work with dynamic economic models and numerical techniques. We assume that the reader has familiarity with the principal ideas and methods of linear algebra and calculus. Appendix A provides an overview of the basic mathematics used throughout the text. Furthermore, in an attempt to make the book modular in organization, some of the mathematics used in studying specific classes of dynamic models is developed in the text as is it needed. Examples include the basic theory of Markov processes, dynamic programming, and, for continuous time models, Ito stochastic calculus.

One barrier to the use of numerical methods by economists is lack of access to functioning computer code. This presents a an apparent dilemma to us as textbook authors, given the variety of computer languages available. On the one hand, it is useful to have working examples of code in the book and to make the code available to readers for immediate use. On the other hand, using a specific language in the text could obscure the essence of the numerical routines for those unfamiliar with the chosen language. We believe, however, that the latter concern can be substantially mitigated by conforming to the syntax of a vector processing language, such as Matlab or Gauss. Vector processing languages are designed to facilitate numerical analysis and their syntax is often simple enough that the language is transparent and easily learned and implemented. Due to its facility of use and its wide availability on university campus computing systems, we have chosen to illustrate algorithms in the book using Matlab. However, we also make available similar Gauss and Fortran code for those who use these languages in their work.

It is our hope that this book will help broaden the scope of economic analysis by helping economists to solve dynamic economic and financial models that heretofore they were unable to solve within the confines of traditional mathematical economic analysis.