User Documentation for the TRI Model

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January 17, 1995

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I. Notes for Users of the TRI calculation models

A. Introduction

The Excel template file I sent calculates the TRI for Colombia for 1989-90: what uniform tariff factor deflator is needed to maintain balance of payments and initial utility when starting from the new trade policy of 1990. This is calculated on the TRI Calc sheet. The Excel workbook will also calculate a free trade comparison treating all ntb's as nonbinding. This is done on the Fr. Tr. Calc worksheet.

The document will open by asking if you want links to unopened documents updated. You should select 'no'. (This is a minor bug --- I don't know how to get rid of the invisible links to ancestors of the file.) The setting is to automatic calculation, which means taking a couple of minutes in startup time. Once the subsidiary worksheets have been calculated it is not necessary to recalculate them (unless a parameter is changed in the parameters section, or if data is changed), so automatic calculation can be turned off. (Options Calculation to Manual.)

Cells marked with * have Notes attached, accessible by double clicking in Excel 4.0 and in Excel 5 by Tools Options View tab, check Info Window. You can also print all cell notes and cell formulae.

To solve the model, select Formula Solver. The dialog box which comes up will have a highlighted entry in the Set Cell box. This must be deleted (another minor bug which I don't know how to fix). Then hit the Solve button. At this point the model calculates pretty quickly. (Automatic calculation need not be on --- the Solver will drive the calculations on the active worksheet.)

1. Preparing a new case

First turn off automatic calculation under the Options menu. Editing is much easier then. Then enter the relevant changes to the workbook.

Data must be entered on the TRI calculation sheet for aggregate variables available from the World Development Report. Also enter substitution parameter values in the parameters section. Rent loss share parameters can be changed from the default value equal to one.

Trade data for two adjacent years is entered in the intermediate and final worksheets. Data must first be assembled with concordances between trade flows, tariffs and nontariff barriers. Tariffs are understood to be in ad valorem equivalent form. (There is a dummy variable field for ad valorem status, but it performs no role in the present version of the model.) There is a field for license premia if these are available --- the model makes appropriate use of these. I have found the 4 digit HS code level to be the finest level of disaggregation which can get the variables in concordance.

In entering trade data, care must be used to avoid over-writing calculation cells. Data is entered in named ranges, which must first be adjusted to fit the size of the data to be entered. **Expand or contract the range in the middle of the range** (to avoid incongruency of your data and the dimension of the named range). **Data ranges should**

be checked after entry to make sure no error has occurred. My experience with RAs is that this operation always generates errors at first.

Data must be in comparable units --- see the template sheet for details.

Then set calculation to Automatic and wait a couple of minutes (depending on speed of equipment, but the program is pretty efficient.) Then Solve.

2. Other uses of the model

As set up, the model calculates the TRI for a one year change in trade distortions. (TRI Calc worksheet) or for the move to free trade (Fr. Tr. Calc worksheet). It is trivial to modify the Solver model to calculate the new equilibriuim utility. Use the variable 'ut_ch_factor' to replace TRI in the 'by changing cells' box. Start from the value of TRI equal to one. Then solve.

As you get familiar with it, other experiments will suggest themselves, such as revenue constant tariff reforms and the like.

3. Some caveats

The model's data must be in equivalent units to those of the template.

The way factor service trade is handled can be problematic. I assume that the 'implied net factor service trade' is like an equity share of GDP. Thus policy changes which alter GDP will alter the payments to foreigners and lead to monopsony power effects, which can be significant for highly indebted nations. The main alternative is to assume that the ‘implied net factor service trade’ is a constant. This involves editing the cell for ‘balance of trade’ so that

-calculated_factor_payments + ‘apparent_net_services’

replaces

-calculated_factor_payments*(1-debt_ratio).

For TRI calculations in my data set this modification affects results at the third place after the decimal. For uses in which gdp changes substantially this need not be so.

Some large trade volume changes I have found for NTB-constrained goods will not be consistent with equilibrium unless the elasticity is set high enough to not imply very large changes in price. This is particularly an issue for inputs, where one might set a low elasticity.

4. Outline of Technical Notes

The notes which follow include a technical description of the TRI model, a mathematical appendix to this technical description which develops the full detail needed to link to the spreadsheet formulae, and a list of related papers. Cell note references to equations refer to equations in the technical description section. Separately I can send a table of cell linkages in the spreadsheet model which makes verification easier.

B. TRI Model

Let h denote the price of the nontraded (home) good, e the expenditure function, g the gross domestic product function and b the balance of trade function when h is still an active argument. The idea of the TRI is most simply seen by first suppressing intermediate imports.

The TRI in the tariff equivalent model is defined by:

(1.1) $\Delta(q^1,\pi^1,u^0) = \{\Delta \mid b(p^d/\Delta,\pi^1/\Delta,h(p^d/\Delta,\pi^1/\Delta,u^0),u^0) = 0\}$,

where:

(1.2) $p^d = \{p^d \mid e_p(p^d,\pi^1,h,u^0) = q^1\}$

(1.3) $h(p^d,\pi^1,u^0) = \{h \mid e_h(p^d,\pi^1,h,u^0) = g_h(h)\}$

(1.3') $h(p^d/\Delta,\pi^1/\Delta,u^0) = \{h \mid e_h(p^d/\Delta,\pi^1/\Delta,h,u^0) = g_h(h)\}$.

In this formulation, p^d is fixed by considering the compensated equilibrium defined by final goods quota q^1, tariff-ridden final import price vector π^1, and utility u^0. At this compensated equilibrium, the nontraded good h has a value given by (1.3). Then (1) calculates what uniform contraction of p^d and π^1 will achieve this compensated equilibrium. In the equilibrium given by the condition of (1), the nontraded good price takes on a new value givne by (1.3').[1] The added complication of intermediate imports, some subject to ntb and some not but subject to tariffs proceeds on similar lines. See Anderson and Neary, "A New Approach to Evaluating Trade Policy", 1993 for details.

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[1] An alternative which I considered is to define p^d by the operation:

(1.4) $p^d/\Delta = \{p^d/\Delta \mid e_p(p^d/\Delta,\pi^1/\Delta,h,u^0) = Q^1\}$

$h = \{h \mid e_h(p^d/\Delta,\pi^1/\Delta,h,u^0) = g_h(p^d/\Delta,\pi^1/\Delta,h)\}$.

The two definitions of p^d are not equivalent unless there is no nontraded good. In general the second definition would imply that the vector p^d changes nonproportionately with Δ, and even in the homothetic separable case p^d will not be invariant to Δ. Thus alternative (1.2) is the correct one.

1. The mechanics of the TRI

To calculate the tariff equivalent it is necessary as a preliminary step to solve for a compensated equilibrium in which the nontraded goods market clears and the quota-constrained markets clear with given q^1, N^1 (the intermediate imports quota and given prices π^1, m^1 (the intermediate imports tariff-ridden price), all at given utility u^0. The balance of payments is not constrained, being the means of implicit compensation. The solution value of the nontraded good's price is denoted $\bar{h}$. Based on $\bar{h}$, the solution values of the unit cost of producting the joint activity (home goods and exports) and the consumer price index are calculated, denoted $\bar{c}$ and $\bar{P}$ respectively. To conserve notation, in what follows, the superscript 1 is omitted, since π,m,q and N are understood to be at their new values. The domestic prices of quota-constrained imports in the CES/CET model are then:

$$(1.5) \quad p_j^d = \bar{P}\ \beta_j^{1/\sigma^*}\ u_0^{1/\sigma^*}\ q_j^{-1/\sigma^*}$$

$$(1.6) \quad n_k^d = \bar{c}\ \gamma_k^{1/\sigma}\bar{Z}^{1/\sigma}\ N_k^{-1/\sigma}\ .$$

Here, σ and σ^* are the input and final demand elasticities of substitution respectively, while the share parameters β and γ are obtained from base date with an initial normalization so that all domestic prices are equal to one.

The next step is to form the unit cost and consumer price index for the TRI solution module. The new prices (π, m, p^d, n^d) are deflated by the TRI, Δ, everywhere they appear as arguments. The unit cost still has the primary factor as a 'quota', while the ntb-constrained inputs are now hypothetically permitted to change to preserve the domestic price. The unit (variable) cost is now written:

$$(1.7) \quad c = \frac{1}{\Delta}\left(\frac{\Sigma\gamma_k m_k^{1-\sigma} + \Sigma\gamma_j\left(n_j^d\right)^{1-\sigma}}{1 - Z^{1/\sigma - 1}\gamma_L^{1/\sigma}L^{1 - 1/\sigma}}\right)^{1/(1-\sigma^*)}$$

$$= \frac{1}{\Delta}\left(\frac{\Sigma\gamma_k m_k^{1-\sigma} + \bar{c}^{1-\sigma}Z^{1/\sigma - 1}\Sigma\gamma_j^{1/\sigma}N_j^{1 - 1/\sigma}}{1 - Z^{1/\sigma - 1}\gamma_L^{1/\sigma}L^{1 - 1/\sigma}}\right)^{1/(1-\sigma^*)} \quad \text{,using (1.6).}$$

The numerator under the bracket is equal to one minus the 'rent share' paid to the constraining primary factor ('labor').

The consumer price index becomes

$$(1.8)\quad P = \left(\Delta^{\sigma^*-1}\Sigma\beta_k\pi_k^{1-\sigma^*} + \Delta^{\sigma^*-1}\Sigma\beta_j\left(p_j^d\right)^{1-\sigma^*} + \alpha h^{1-\sigma^*}\right)^{1/(1-\sigma^*)}$$

$$= \left(\Delta^{\sigma^*-1}\Sigma\beta_k\pi_k^{1-\sigma^*} + \Delta^{\sigma^*-1}\bar{P}^{1-\sigma^*}u_0^{1/\sigma^*-1}\Sigma\beta_j^{1/\sigma^*}q_j^{1-1/\sigma^*} + \alpha h^{1-\sigma^*}\right)^{1/(1-\sigma^*)}.$$

The second equation is obtained by using (1.5)

The supply side module determines the level of activity and the level of production of the nontraded good. The level of activity is determined by:

$$(1.9)\quad Z = \left(\frac{1-\phi^{\sigma-1}\Delta^{\sigma-1}\left(\sum_k \gamma_k m_k^{1-\sigma} + \sum_j \gamma_j (n_j^d)^{1-\sigma}\right)}{\delta_L^{1/\sigma}L^{-(1-\sigma)/\sigma}}\right)^{\sigma/(1-\sigma)}$$

$$= \left(\frac{1-\phi^{\sigma-1}\Delta^{\sigma-1}\left(\sum_k \gamma_k m_k^{1-\sigma} + \bar{Z}^{(1-\sigma)/\sigma}\bar{c}^{1-\sigma}\sum_j \gamma_j^{1/\sigma} N_j^{-(1-\sigma)/\sigma}\right)}{\delta_L^{1/\sigma}L^{-(1-\sigma)/\sigma}}\right)^{\sigma/(1-\sigma)}.$$

Here, ϕ is the price index for the joint product, and we use unit cost equal to price to solve for Z in the first line. (See the CES/CET Notes for more details.) The denominator of (1.9) is the rent share, the share of total factor payments made to fixed factors, R. The variable unit cost function (1.7) has derivatives with respect to nontraded primary factors of the same form as (1.6). Thus the payment to the primary factor is equal to:

$$(1.10)\quad G = c\,\delta_L^{1/\sigma}\, Z^{1/\sigma}\, L^{-1/\sigma+1}.$$

The balance of trade is equal to:

$$(1.11)\quad b = Pu_0 - G - TR - QR.$$

Here, TR denotes tariff revenue and QR denotes retained quota rent, if any. The last steps in the derivation lay out TR and QR.

Tariff revenue is defined so as to exclude that revenue raised by taxing quota-constrained imports. The tariff revenue term thus treats non-constrained and ntb-constrained trade differently. First, the non-ntb constrained goods have tariff revenue subdivided into final and intermediate revenue. For final revenue, using Shephard's Lemma to obtian the individual demand functions, multiplying by the tariff and summing:

$$(1.12)\quad (\pi/\Delta - \pi^*)'e_\pi \;=\; \left(\frac{\pi_k^1/\Delta}{\pi_k^*} - 1\right)\pi_k^*\;\beta_k\Delta^{\sigma^*}\left(\frac{\pi_k^1}{P}\right)^{-\sigma^*} u_0.$$

$$=\; \Delta^{\sigma^*}P^{\sigma^*}u_o\left(\frac{\pi_k^1/\Delta}{\pi_k^*} - 1\right)\pi_k^*\;\beta_k\left(\pi_k^1\right)^{-\sigma^*}.$$

For intermediate inputs the tariff revenue is,

$$(1.13)\quad \Delta^{\sigma}c^{\sigma}Z\left(\frac{m_k^1/\Delta}{m_k^*} - 1\right)m_k^*\;\gamma_k\left(m_k^1\right)^{-\sigma}.$$

Retained quota rent is equal to the sum of the hypothetical quantity demanded times the retained rent. To develop this term it is necessary to first distinguish hypothetical import demand, equal to the quantity demanded at prices $(\pi/\Delta, m/\Delta, p^d/\Delta, n^d/\Delta)$. These quantities will be denoted q^d for final goods and N^d for intermediate goods. A portion of retained rent is due to the tariff revenue from quota constrained goods. The tariff revenue arising from taxation of the hypothetical quantity of quota constrained final goods is equal to

$$(1.14)\quad t^{q\prime}q \;=\; \Sigma\tau_i\pi_i^*\;\beta_i\left(p_i^d\right)^{-\sigma^*} P^{\sigma^*}u_0\Delta^{\sigma^*}.$$

Here, the hypothetical import demand q^d is a function of p^d/Δ, the hypothetical price vector of quota constrained goods. In the computation, equation (1.5) is substituted for p^d. In (1.14) the role of Δ arises solely through deflation of p^d as an argument of the import demand function (apart from its role in P). It has no direct impact on the tariff. The reason is that a tax on a quota-constrained good is nondistortionary and is not included in the base of the tariff equivalent TRI. Similarly, for intermediates the rent retention tariff revenue is equal to

$$(1.15)\quad \Sigma\tau_i n_i^*\;\gamma_i\left(n_i^d\right)^{-\sigma} P^{\sigma}Z\Delta^{\sigma}.$$

In computation, n^d is replaced by equation (1.6).

The full quota rent retention term for final goods is equal to the sum of the tariff revenue from quota constrained goods plus the portion of rent retained which is above the tariff inclusive price of the good. Formally, full retained rent is equal to:

(1.16) $(1-\omega_q)[p^{d\prime}q^d/\Delta - (p^*+t^q)'q^d] + t^{q\prime}q^d$

(1.16') $= (1-\omega_q)[p^{d\prime}q^d/\Delta - p^{*\prime}q^d] + \omega_q t^{q\prime}q^d$,

where p^d is defined above and hypothetical import demand q^d is a function of p^d/Δ as in (1.14). Here, ω_q is the fraction of rent lost to rent seeking or to foreigners. It is convenient to break the square bracketed rent retention term into two sums. The first term yields

(1.17) $p^{d\prime}q^d/\Delta = P^{\sigma*}\,\bar{P}^{\,1-\sigma*}u_0^{1/\sigma*}\ \Sigma\beta_j^{1/\sigma*}\ q_j^{1-1/\sigma*}\ \Delta^{\sigma*-1}$.

In (1.17) the price term p^d is obtained from (1.5), with q indicating the new quota value. The second sum is equal to

(1.18) $p^{*\prime}q^d = \sum_j p_j^*\ q_j P^{\sigma*}\,\bar{P}^{\,-\sigma*}\Delta^{\sigma*}$.

For intermediate goods, the analogous terms are:

$(1-\omega_N)[n^d/\Delta - (n^*)]'N^d + \omega t^{N\prime}N^d$.

(1.19) $n^{d\prime}N^d/\Delta = c^{\sigma*}\,\bar{c}^{\,1-\sigma}Z^{1/\sigma}\ \Sigma\gamma_j^{1/\sigma}\ N_j^{1-1/\sigma}\ \Delta^{\sigma-1}$.

$n^{*\prime}N^d = \sum_j n_j^*\ N_j c^{\sigma}\,\bar{c}^{\,-\sigma}\Delta^{\sigma}$.

2. The TRI calculation

The TRI is calculated by imposing balanced trade along with the market clearance for the nontraded good. The balanced trade requirement (1.11) is a function of h and Δ after substitution of the various functions of this section. The nontraded goods market clears when h and Δ are such that:

(1.20) $\alpha_Y\left(\frac{h}{P}\right)^{-\sigma*} u_0 = \mu\left(\frac{h}{\phi}\right)^{\mu} Z.$

The model is solved on an Excel 4.0 or 5.0 spreadsheet.

C. List of Trade Restrictiveness Index papers

6/20/94

Anderson, James E. (1991), "The Coefficient of Trade Utilization: the Cheese Case" in R. Baldwin ed., Empirical Studies of Commercial Policy, NBER, Chicago: University of Chicago Press, 221-41.

Anderson, James E. (1993), "Aggregation of Trade Restrictions in a Simple CGE Model", Boston College.

Anderson, James E. (1994), "Trade Restrictiveness Benchmarks", Boston College.

Anderson, James E. (1994), "Tariff Index Theory", Review of International Economics, forthcoming.

Anderson, James E., Geoffrey Bannister and J. Peter Neary (1994), "Domestic Distortions and International Trade", International Economic Review, Feb, 1995.

Anderson, James E.. and J. Peter Neary (1990), "The Coefficient of Trade Utilization: Back to the Baldwin Envelope", in Ronald W. Jones and Anne O. Kreuger eds. The Political Economy of Trade Policy, Oxford: Basil Blackwell. (Baldwin festschrift.

Anderson, James E.. and J. Peter Neary (1992), "Trade reform with quotas, partial rent retention and tariffs", Econometrica, 60, 57-76.

Anderson, James E. and J. Peter Neary (1993), "A New Approach to Evaluating Trade Policy".

Anderson, James E. and J. Peter Neary (1994), "The Trade Restrictiveness of the Multi-Fibre Arrangement", World Bank Economic Review, 8, 171-190.

Anderson, James E.. and J. Peter Neary (1994), "Measuring the Restrictiveness of Trade Policy", World Bank Economic Review , 8, 151-170.

(Anderson and Bannister, "Measuring the Trade Restrictiveness on Mexican Agricultural Policy" WPS 874, World Bank, amplifies on the empirical work cited in "Domestic Distiortions and International Trade". It will not be separately published, but will appear in our book.)

Appendix: The CES/CET Model

For the referees, not for publication

II. Technical Appendix

A. CES Expenditure and Distorted Expenditure Functions

The representative consumer is assumed to have a CES expenditure function of the form:

$$(1.1)\quad e(p,\pi,h,u) = \left(\Sigma \beta_k p_k^{1-\sigma^*} + \Sigma \alpha_j \pi_j^{1-\sigma^*} + \alpha_y h^{1-\sigma^*}\right)^{\frac{1}{1-\sigma^*}} u,$$

where u is the level of utility, p is the domestic price of quota-constrained goods, π is the domestic price of non-quota-constrained goods, h is the price of the non-traded good. The elasticity of substitution in demand is equal to the parameter σ^*, while the α's and β's are share parameters for the non-quota constrained goods and the quota constrained goods respectively. For empirical work, it is convenient to select a benchmark year in which prices are all initially one, and the α's and β's are the initial expenditure share values in the data, and the level of expenditure is equal to u. The true cost of living index is

$$(1.2)\quad P = \left(\Sigma \beta_k p_k^{1-\sigma^*} + \Sigma \alpha_j \pi_j^{1-\sigma^*} + \alpha_y h^{1-\sigma^*}\right)^{\frac{1}{1-\sigma^*}} .$$

The quota-constrained imports are subject to fixed binding quotas equal to q_k for all k. This results in a distorted expenditure function for the unconstrained goods. The distorted expenditure function is defined by (Anderson and Neary, 1992)

$$E(\pi,h,q,u) = \max_{p} \{e(p,\pi,h,u) - p'q\}$$

The price vector p which solves this program is a virtual price vector (Neary and Roberts, 1980). In the context of quotas it is also a market clearing price vector. Using Shephard's Lemma, and solving the first order (market clearing) condition for the (virtual and market) price of each quota constrained good k, we obtain for the CES case:

$$(1.3)\quad p_k = P\left(\frac{u\beta_k}{q_k}\right)^{1/\sigma^*} ,$$

where P is the price index defined by equation (1.2). Substituting (1.2) into (1.3), the vector of virtual prices p is a implicitly defined as a function of the π's and the quotas. Fortunately, an explicit solution is available. First, substitute (1.3) into (1.2). Next, raise both right and left hand sides to the power $1\text{-}\sigma*$. Then, solve the resulting expression for $P^{1-\sigma*}$. Finally, raise both sides to the power $1/(1\text{-}\sigma*)$. The reduced form true cost of living index is:

$$(1.4) \quad P = P(\pi,h,q,u) = \left(\frac{\Sigma\alpha_j\pi_j^{1-\sigma*} + \alpha_y h^{1-\sigma*}}{1 - u^{(1-\sigma*)/\sigma*}\Sigma\beta_k^{1/\sigma*}\, q_k^{-(1-\sigma*)/\sigma*}}\right)^{1/(1-\sigma*)} .$$

The connection of (1.4) to (1.2) is clear: if consumers face fixed price vector p at the level of the virtual price vector p defined by (1.3), their cost of living is the same as when constrained by quotas q.

The distorted expenditure function is obtained by substituting (1.3) and (1.4) into the definition of E:

$$\begin{aligned}(1.5) \quad E(\pi,h,q,u) &= P(\pi,h,q,u)u - p'q \\ &= Pu\left(1 - u^{1/\sigma* - 1}\sum_k \beta_k^{1/\sigma*} q_k^{1 - 1/\sigma*}\right) .\end{aligned}$$

where P is given by (1.4). Using (1.4), equation (1.5) can be factored into:

$$(1.6) \quad E(\pi,h,q,u) = \left(\sum_j \alpha_j\pi_j^{1-\sigma*} + \alpha_y h^{1-\sigma*}\right)^{1/(1-\sigma*)}\left(1 - u^{(1-\sigma*)/\sigma*}\Sigma\beta_k^{1/\sigma*}\, q_k^{-(1-\sigma*)/\sigma*}\right)^{-\sigma*/(1-\sigma*)} u.$$

The constrained (by the presence of quotas) demand for unconstrained imports and for nontradables is obtained from use of Shephard's Lemma:

$$(1.7) \quad E_{\pi k} = \alpha_k\left(\frac{\pi_k}{P}\right)^{-\sigma*} u$$

$$E_h = \alpha_y\left(\frac{h}{P}\right)^{-\sigma*} u.$$

The virtual price vector is obtained as:

$$(1.8) \quad -E_{qk} = p_k = E\frac{\beta_k^{1/\sigma*} u_0^{(1-\sigma*)/\sigma*} q_k^{-1/\sigma*}}{1 - \sum_k \beta_k^{1/\sigma*} u_0^{(1-\sigma*)/\sigma*} q_k^{-(1-\sigma*)/\sigma*}} .$$

B. The CES/CET Cost and Product Functions

Exports and the nontraded good are jointly produced with a CES/CET technology. The level of activity of the joint process is represented by z, determined by the two outputs y, the nontraded good, and x, the export good.

1. Total and Variable Cost Functions

The cost of producing one unit of the activity z is equal to:

$$(2.1) \quad c \quad = \quad \left(\Sigma\gamma_k m_k^{1-\sigma} + \Sigma\gamma_j n_j^{1-\sigma} + \delta_L w^{1-\sigma}\right)^{1/(1-\sigma)} ,$$

where m is the price vector of imported intermediate inputs not subject to quota, n is the price vector of imported intermediate inputs subject to quota, and w is the price of the nontraded factor (the wage rate of labor). The γ's and δ_L are activity cost share parameters and σ is the elasticity of technical substitution. Nontraded intermediate goods are subsumed into the production and cost structure behind (2.1).

The input quotas are denoted N_j, and the nontraded factor is in fixed supply L. (Here I depart from the notational convention that prices and quantities are denoted with lower case letters, to adhere to the very widespread convention that fixed factors are denoted with capital letters.) Shephard's Lemma and the market clearing equations can be used to solve for the prices of the nontraded and quota constrained inputs, just as the price of quota constrained final goods was obtained in equation (1.2). Thus

$$N_j \quad = \quad \gamma_j \left(\frac{n_j}{c}\right)^{-\sigma} z$$

implies a value of n_j in terms of c and z:

$$n_j \quad = \quad c z^{1/\sigma} \; \gamma_j^{1/\sigma} \; N_j^{-1/\sigma} \quad .$$

The resulting solution for n_j may be substituted into equation (2.1), and the equation solved first for $c^{1-\sigma}$ and then c (the steps are the same as those leading from (1.2) to (1.4)) to obtain the reduced form unit cost function:

$$(2.2)\quad c \quad = \quad C(m,N,L,z) \quad = \quad \left(\frac{\Sigma \gamma_k m_k^{1-\sigma}}{1 \;-\; z^{(1-\sigma)/\sigma}\left(\Sigma \gamma_j^{1/\sigma} N_j^{-(1-\sigma)/\sigma} \;+\; \delta_L^{1/\sigma} L^{-(1-\sigma)/\sigma} \right)} \right)^{1/(1-\sigma)} .$$

Note the similarity of (2.2) to (1.3). In the reduced form cost function it is convenient to define

$$(2.3)\quad R(N,L,z) \;=\; z^{(1-\sigma)/\sigma}\left(\Sigma \gamma_j^{1/\sigma} N_j^{-(1-\sigma)/\sigma} \;+\; \delta_L^{1/\sigma} L^{-(1-\sigma)/\sigma} \right) ,$$

the share of total cost paid to fixed factors (the nontraded factor and the quota constrained inputs). To see how this interpretation arises, note that Shephard's Lemma implies that the share of cost paid to variable inputs is equal to $\Sigma \gamma_k m_k^{1-\sigma} / c^{1-\sigma}$, where c is defined by (2.1). Now raise both sides of (2.2) to the power $1-\sigma$, multiply both sides by 1-R, and divide both sides by $c^{1-\sigma}$. Then R is equal to $1- \Sigma \gamma_k m_k^{1-\sigma} / c^{1-\sigma}$, which is the share of costs paid to fixed factors.

The variable cost function[2] is the analog to the distorted expenditure function of Section II. The demand for non-quota constrained imports is, by Shephard's Lemma applied to the original unconstrained unit cost function,

$$(2.4)\quad C_k \quad = \quad \gamma_k \left(\frac{m_k}{C} \right)^{-\sigma} z.$$

The variable cost function is the sum over k of the value of spending $m_k C_k$, thus:

$$(2.5)\quad V(m,N,L,z) \quad = \quad {}_{k}\, \gamma_k m_k^{1-\sigma} \;\; C(m,N,L,z)^{\sigma} z,$$

where C(.) is given by (2.2). The variable cost function is more conveniently rewritten as:

$$(2.5')\quad V(m,N,L,z) \quad = \quad \left({}_{k}\, \gamma_k m_k^{1-\sigma} \right)^{1/(1-\sigma)} (1-R)^{-\sigma/(1-\sigma)}\, z.$$

Note the similarity of form between (2.5') and (1.6). Shephard's Lemma ensures that[3]

$$(2.6)\quad V_k \quad = \quad C_k \quad = \quad \gamma_k m_k^{-\sigma} \left({}_{k}\, \gamma_k m_k^{1-\sigma} \right)^{\sigma(1-\sigma)} (1-R)^{-\sigma/(1-\sigma)}\, z.$$

The marginal variable cost of competitive production is equal to V_z, given by:

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[2]In the present context it might better be termed the distorted variable cost function to emphasize that some of the fixed inputs are fixed by policy. This terminology was rejected in favor of simplicity and integration with standard usage in micro theory .

[3]This can be verified by differentiating (3.5') using the properties of R via (3.3).

$$(2.7)\quad V_z = \frac{V}{z}\left(1 + \frac{R}{1 - R}\right) = \frac{V}{z}\frac{1}{1 - R}$$
$$= \left(\sum_k \gamma_k m_k^{1-\sigma}\right)^{1/(1-\sigma)}(1-R)^{-1/(1-\sigma)} .$$

Finally, note that the virtual price of the constrained input is

$$(2.8)\quad -V_{Nj} = n_j = z^{(1-\sigma)/\sigma}\frac{V}{1 - R}\ \gamma_j^{1/\sigma}\ N_j^{-1/\sigma} ,$$

where n is also the domestic price of the quota constrained good. The virtual price of the primary factor is an obvious variant of (2.8). Equation (2.8) is the production analog to equation (1.8).

2. Joint Product and Gross Domestic Product Functions

For a given level of the activity z, the profit maximizing decisions of producers select y and x to maximize $hy+p_x x$ subject to a constant elasticity of transformation production frontier $f(x,y)\geq z$. Here, p_x is the export price. The value of total output in this setup is equal to

$$(2.9)\quad \phi(h,p_x)z = \left((1-\mu)p_x^{1+\eta} + \mu h^{1+\theta}\right)^{1/(1+\theta)} z,$$

where θ is the constant elasticity of transformation, $\partial\log(x/y)/\partial\log(p_x/h)$, and μ is a share parameter. Profit maximization implies that for given z

$$(2.10)\quad y = \phi_h z.$$

The determination of the activity level z follows from the profit maximizing behavior of firms. z is the implicit solution to

$$(2.11)\quad \phi(h,p_x) = V_z = \left(\sum_k \gamma_k m_k^{1-\sigma}\right)^{1/(1-\sigma)}(1-R(N,L,z))^{-1/(1-\sigma)} ,$$

where R(.) is given by equation (2.3). Equation (2.11) with (2.3) can be solved for a closed form solution for the activity level z:

$$(2.12)\quad z = \left(\frac{1 - \phi^{\sigma-1}\sum_k \gamma_k m_k^{1-\sigma}}{\Sigma\gamma_j^{1/\sigma}N_j^{-(1-\sigma)/\sigma} + \delta_L^{1/\sigma}L^{-(1-\sigma)/\sigma}}\right)^{\sigma/(1-\sigma)} .$$

Here, ϕ is given by the right hand side of (2.9).

Gross domestic product is equal to the value of payments to domestic factors. This is written:[4]

(2.13) $G \quad = \quad -LV_L(m,N,L,z)$

where the $V_L = -w$, the wage rate. The variable cost function has derivatives with respect to nontraded primary factors of the same form as (2.8). Thus the right hand side of (2.13) simplifies to

(2.13') $\frac{V}{1\ -\ R}\left(\delta_L^{1/\sigma}L^{1\text{-}1/\sigma}\right) z^{1/\sigma\ -\ 1}$

Using (2.5') for V, the gross domestic product may be rewritten as

(2.13') $G(p_X,h,m,N,L) \quad =$

$$\left(\sum_k \gamma_k m_k^{1-\sigma}\right)^{1/(1-\sigma)}(1-R)^{-1/(1-\sigma)}\, z^{1/\sigma}\left(\delta_L^{1/\sigma}L^{1-1/\sigma}\right)$$

Here, R is understood to be replaced by its value from (2.3) and the solution value of z(.) from (2.12) replaces z.

[4]It is important to note that the gross domestic product function in this setup is not an envelope function, due to the fact that the domestic value of N is lost. This imposes a terms of trade effect distortion relative to efficient production. The aggregate profit function

$$\Pi(p_X,q,m,N,L,K) = \max_Z \{\phi(p_X,q)Z - V(m,N,L,K,Z)\}.$$

This has the envelope property $\Pi_q = Y = \phi_q Z$.

C. The Balance of Trade Function and General Equilibrium

The general equilibrium of the trading economy is reached when the two endogenous variables h and u adjust to satisfy two requirements. First, the nontraded goods market must clear. Second, the external budget constraint must be met.

The nontraded goods market clears when h and u are such that:

$$(3.1) \quad E_h(h,\pi,q,u) = y, \quad \text{or}$$

$$\alpha_y\left(\frac{h}{P}\right)^{-\sigma^*} u = \phi_h(p_x,h)z = \mu\left(\frac{h}{\phi}\right)^{\theta} z,$$

applying (2.9) and (2.10).

In equation (3.1), P is determined by equation (1.4)

$$P = \left(\frac{\Sigma\alpha_j\pi_j^{1-\sigma^*} + \alpha_y h^{1-\sigma^*}}{1 - \Sigma\beta_k^{1/\sigma^*} u^{(1-\sigma^*)/\sigma^*} q_k^{-(1-\sigma^*)/\sigma^*}}\right)^{1/(1-\sigma^*)},$$

z is determined by (2.12)

$$z = \left(\frac{1 - \phi^{\sigma-1} \underset{k}{\Sigma}\gamma_k m_k^{1-\sigma}}{\Sigma\gamma_j^{1/\sigma} N_j^{-(1-\sigma)/\sigma} + \delta_L^{1/\sigma} L^{-(1-\sigma)/\sigma}}\right)^{\sigma/(1-\sigma)}$$

and ϕ is determined by (2.9)

$$\phi(h,p_x) = \left((1-\mu)p_x^{1+\eta} + \mu h^{1+\theta}\right)^{1/(1+\theta)}.$$

The balance of trade requirement as simplified here requires that payments be in balance (the inclusion of a fixed external borrowing limit is easily incorporated). The consumer's total expenditure is equal to $E - E_q'q$, since $-E_q = p$, the domestic price vector for the quota-constrained final imports. The gross domestic product provides G. In addition, the government remits all tariff revenues (quota rents are, however, all lost). The net borrowing is thus:

$$(3.2)\quad b(h,u,\pi,m,q,N,t) = E(h,\pi,q,u) - E_q(.)'q - G(h,m,N)$$
$$- \Sigma\,(\pi_k - \pi_k^{*})E_{\pi k} - \Sigma(m_k - m_k^{*})V_{mk}$$
$$- \Sigma\, t_k q_k \quad - \quad \Sigma t_j N_j$$

Here, the * denotes the fixed external price, and the gap between domestic and foreign price denotes the tariff for the goods not subject to quota constraint. It is handy for these goods to regard the domestic price as the instrument of policy. For the goods in the last row, the domestic price lies above the external price plus the specific tariff t_k. The inactive arguments are suppressed from the list in the function on the left hand side of b. The external budget constraint requires that b be equal to zero (or some exogenous value of external borrowing).

Equation (3.2) is made concrete with substitutions from previous steps. The total expenditure in domestic prices is $E\text{-}E_q'q$, and

$$E - E_q'q = Pu, \qquad \text{where P is determined by (1.4).}$$

G is determined by equation (2.13').

$$G(p_x,h,m,N) = \left({}_{k}\gamma_k m_k^{1-\sigma}\right)^{1/(1-\sigma)}(1\text{-}R(N,z;L)^{-1/(1-\sigma)}\, z^{1/\sigma}\left(\delta_L^{1/\sigma}L^{-1/\sigma}\right)$$

Here, R is determined by (2.3):

$$R = z^{(1-\sigma)/\sigma}\left(\Sigma\gamma_j^{1/\sigma}N_j^{-(1-\sigma)/\sigma} + \delta_L^{1/\sigma}L^{-(1-\sigma)/\sigma}\right).$$

Finally, the tariff revenue terms require the import demand functions. V_m is determined by (2.5).

$$V_{mk}(m,N,z;L) = \gamma_k m_k^{-\sigma}\left({}_{k}\gamma_k m_k^{1-\sigma}\right)^{\sigma(1-\sigma)}(1\text{-}R)^{-\sigma/(1-\sigma)}\, z.$$

E_π is determined by (1.7) using (1.4) for P.

$$E_{\pi k} = \alpha_k\left(\frac{\pi_k}{P}\right)^{-\sigma^*} u$$

Equations (3.1)-(3.2) constitute two equations in the two unknowns h and u. The domestic prices of non-quota constrained goods are exogenous under the small country assumption $\pi = \pi^* + t^F$ and $m = m^* + t^I$, where * denotes the exogenous external price. It is also assumed that p_x is exogenously given. Thus equations (3.1)-(3.2) are sufficient to determine the equilibrium. The interesting exogenous variables are the trade instruments: the final and intermediate good tariffs on both quota constrained goods (where they serve as a rent retention mechanism) and non-quota-constrained goods, and the final and intermediate goods quotas. The system can be perturbed with a change in any of these variables, and the new solution computed.

D. Calculating The TRI in the CES/CET Case

The TRI is calculated with one modification of the CES/CET model. The initial equilibrium level of u is fixed, and the level of the TRI (in effect previously set identically equal to one) is variable.

The TRI , Δ, is implicitly defined as

(4.1) $b(h,u^0,\pi^1/\Delta,m^1/\Delta,q^1\Delta,N^1\Delta,p-p^*,n-n^*) = 0,$

Here, u^0 is the base level of utility and the trade instruments q, N, π,m are set at their new levels. As opposed to Section 4, u is now fixed, and Δ is variable, along with h, the nontraded good price. So far as π and m are concerned, the TRI compensates for the effect of changes in the trade instrument with an equiproportionate shift in the same direction. Thus a tariff cut results in a uniform offsetting tariff factor surcharge. For quota changes, which shift q and N, the TRI offsets the change with a uniform proportional change in the quota vector in the opposite direction.

Notice that the terms n-n* (intermediate import rent retaining tariffs), and p-p* (final import rent retaining tariffs) are not included in the trade restrictiveness index. The logic is that rent retaining tariffs are not trade restrictions. Changes in such instruments do however have effects on the trade balance which are offset by changes in the TRI defined in (4.1). A rise in a rent retaining tariff reduces the foreign exchange needed to support current real income and thus acts like a reduction in tariffs or an increase in quotas, reducing the TRI.

The exogenous prices and quantities in the system below are understood to be at the new level, with the identification left out for cleaner notation. In the case of evaluation of a growing economy, the 'new' quotas are the actual quotas deflated by the growth factor. The TRI operationally comes from the solution to the 2 equation system (3.1) and (3.2) for the two variables Δ, h while keeping u constant. Let TR denote tariff revenue. The system is:

(4.1') $Pu_0 - G - TR = 0$ balance of payments

(4.2) $\alpha_y\left(\frac{h}{P}\right)^{-\sigma^*} u_0 = \mu\left(\frac{h}{\phi}\right)^{\theta} z$ nontraded goods market clearance.

P, G, TR, z and ϕ are all functions of Δ,h, using previous steps.

Computational efficiency is aided by factoring out Δ from the long sums which occur when the model is applied to thousands of tariffs and quotas. In detailing the supporting equations, Δ is thus factored out where it occurs in sums. The economic logic is clearer in the preceding step with Δ left in.

Now the system (4.1')-(4.2) will be linked to the underlying equations and variables as they appear in my computational spreadsheet model. First, the GDP deflator ϕ is given by

$$\phi = \left((1-\mu)p_x^{1+\eta} + \mu h^{1+\theta}\right)^{1/(1+\theta)} .$$

Next, G is the sum of total payments to primary factors. This works out to:

(4.3) $G(p_x,q,m,N;L) =$

$$\left(\Delta^{\sigma-1} \sum_k \gamma_k m_k^{1-\sigma}\right)^{1/(1-\sigma)} (1-R)^{-1/(1-\sigma)} z^{1/\sigma}\left(\delta_L^{1/\sigma} L^{1-1/\sigma}\right) .$$

The consumer price index P is given by:

(4.4) $$P = \left(\frac{\Delta^{\sigma^*-1}\Sigma\alpha_j\pi_j^{1-\sigma^*} + \alpha_y h^{1-\sigma^*}}{1 - \Delta^{-1/\sigma^*+1} u_0^{(1-\sigma^*)/\sigma^*}\Sigma\beta_k^{1/\sigma^*} q_k^{-(1-\sigma^*)/\sigma^*}}\right)^{1/(1-\sigma^*)}$$

The production rent share R is given by

(4.5) $$R = z^{(1-\sigma)/\sigma}\left(\Delta^{-1/\sigma+1}\Sigma\gamma_j^{1/\sigma}N_j^{-(1-\sigma)/\sigma} + \delta_L^{1/\sigma}L^{-(1-\sigma)/\sigma}\right)$$

and activity level z is given by

$$(4.6) \quad z = \left(\frac{1 - \phi^{\sigma-1}\Delta^{\sigma-1}\gamma_k m_k^{1-\sigma}}{\Delta^{-1/\sigma} + {}^1\Sigma\gamma_j^{1/\sigma} N_j^{-(1-\sigma)/\sigma} + \delta_L^{1/\sigma} L^{-(1-\sigma)/\sigma}} \right)^{\sigma/(1-\sigma)} .$$

Notice that in the equations for G,P,R and z; Δ appears multiplying four sums, one each for final and intermediate imports, both subject to nontariff barriers and not subject to nontariff barriers. These sums are nonlinear aggregators of the trade policy change. Due to separability, the complexity of the underlying distortion structure can be appropriately aggregated, independently of the general equilibrium structure of the model.

The last step is to specify tariff revenue in the hypothetical compensated situation. TR is generally written as

$$(4.7) \quad TR = \Sigma t_k^F E_{\pi k} + \Sigma t_k^I V_{mk} + \Sigma t_j^F q_j + \Sigma t_j^I N_j .$$

Denoting the new prices and quantities with a superscript 1 for clarity and compensating by the TRI, and using equation (2.7), the first term becomes:

$$(4.8) \quad \Sigma t_k^F \alpha_k \left(\frac{\pi_k^F}{P}\right)^{-\sigma*} u_0 = \sum \left(\frac{\pi_k^1/\Delta}{\pi_k^*} - 1 \right) \pi_k^* \alpha_k \Delta^{\sigma*} \left(\frac{\pi_k^1}{P}\right)^{-\sigma*} u_0.$$

$$= \Delta^{\sigma*} P^{\sigma*} u_o \sum \left(\frac{\pi_k^1/\Delta}{\pi_k^*} - 1 \right) \pi_k^* \alpha_k \left(\pi_k^1\right)^{-\sigma*} .$$

The first term in brackets is the hypothetical new tariff. It is computationally convenient to factor Δ out of this term as well. The sum reduces to two 'constant' terms multiplied by Δ to a power, and P. Moreover, the normalization which sets the initial domestic prices equal to one means that:

$$\pi_k^1 = \frac{1 + \tau_k^1}{1 + \tau_k^0} \qquad \pi_k^* = \frac{1}{1 + \tau_k^0} .$$

The second term of TR similarly becomes, using equation (3.5):

(4.9) $$(1-R)^{-\sigma/(1-\sigma)}\Delta^{\sigma}z \sum\left(\frac{m_k^1/\Delta}{m_k^*} - 1\right) m_k^* \; \gamma_k\left(m_k^1\right)^{-\sigma} .$$

The same factoring and use of the normalization of initial prices as with (4.8) complete the steps needed to operationalize (4.9)

The third and fourth terms of TR simply multiply the new quantity of intermediate and final imports by Δ. Thus the third tariff revenue term is equal to

(4.10) $$\Delta\left(\Sigma(1+\tau_k^1)p_k^* q_k^1 - \Sigma p_k^* q_k^1\right) .$$

Finally, the fourth term in TR is similarly equal to

(4.11) $$\Delta\left(\Sigma(1+\tau_k^1)n_k^* N_k^1 - \Sigma n_k^* N_k^1\right) .$$

This model is computationally quite tractable. Based on the experience reported in Anderson (1993) the model converges quickly to a unique solution despite large changes in highly distorted trading economies.